Commentary on Design and Construction of Reinforced Concrete Chimneys (ACI 307-98)
Reported by ACI Committee 307

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INTRODUCTION

As industry expanded in the years immediately following World War I and as a result of the development of large pulverized coal-fired boilers for the electric power generating utilities in the 1920s, a number of rather large reinforced concrete chimneys were constructed to accommodate these new facilities. A group of interested engineers who foresaw the potential need for many more such chimneys and who were members of the American Concrete Institute decided to embark upon an effort to develop a rational design criteria for these structures. The group was organized into ACI Committee 505 (this committee was the predecessor of the present Committee 307) to develop such criteria in the early 1930s.

Committee 505 submitted to the Institute a “Proposed Standard Specification for the Design and Construction of Reinforced Concrete Chimneys,” an outline of which was published in the ACI Journal, Proceedings V. 30, Mar.-Apr. 1934. This specification was adopted as a tentative standard in February 1936. Although this tentative standard was never accepted by ACI as a regular standard, it was used as the basis for the design of many chimneys. As these chimneys aged, inspections revealed considerable cracking. When the industrial expansion began following World War II, other engineers recognized the need for developing an improved design specification for reinforced chimneys.

In May 1949, Committee 505 was reactivated to revise the tentative standard specification, embodying modifications that were found desirable during the years it had been in use. The section dealing with the temperature gradient through the chimney lining and the chimney shell was completely revised and extended to cover different types and thicknesses of linings and both unventilated and ventilated air spaces between the lining and the concrete shell. In 1954, this specification was approved as ACI 505-54.

The rapid increase in the size and height of concrete chimneys being built in the mid-1950s raised further questions about the adequacy of the 1954 version of the specification, especially as related to earthquake forces and the effects of wind.

In May 1959, the ACI Board of Direction again reactivated Committee 505 (Committee 307) to review the standard and to update portions of the specification in line with the latest design techniques and the then-current knowledge of the severity of the operating conditions that prevailed in large steam plants. The material in the standard was reorganized, charts were added, and the methods for determining loads due to wind and earthquakes were revised. The information on design and construction of various types of linings was amplified and incorporated in an appendix. That specification included criteria for working stress design. It was planned to add ultimate strength criteria in a future revision of this standard.

In preparing the earthquake design recommendations, the Committee incorporated the results of theoretical studies by adapting them to existing United States codes. The primary problems in this endeavor stemmed from the uncertainties still inherent in the definition of earthquake forces and from the difficulty of selecting the proper safety and serviceability levels that might be desirable for various classes of construction. Committee investigations revealed that with some of the modifications (such as the K factor), the base shear equations developed by the Seismology Committee of the Structural Engineers’ Association of California (SEAOC) could be applied to chimneys. Similarly, the shape of the force, shear, and moment distributions, as revised in their 1967 report, were also suitable for chimneys. A use factor (U factor) ranging from 1.3 to 2.0 was introduced in the specification and it was emphasized that the requirements of Section 4.5 of ACI 307-69 relating to seismic design could be superseded by a rational analysis based on evaluation of the seismicity of the site and modal response calculations. The modifications were approved in 1969 and the specification was designated ACI 307-69. In that specification, the commentary and derivation of equations were published separately as a supplement to ACI 307-69.

In 1970, the specification was reissued with corrections of typographical errors. This issue of ACI 307-69 was also designated ANSI A158.1-1970. At the time, as a result of numerous requests, the commentary and derivation of equations were bound together with the specification.

The 1979, revision of the specification updated its requirements to agree with the then-accepted standard practice in the design and construction of reinforced concrete chimneys. The major changes included the requirement that two layers of reinforcing steel be used in the walls of all chimneys (previously this only applied to chimney walls thicker than 18 in. [4600 mm]) and the requirement that horizontal sections through the chimney wall be designed for the radial wind pressure distribution around the chimney. Formulas were included to compute the stresses under these conditions. Many revisions of a less important nature were included to bring the specification up to date.

The editions of the specifications prior to 1979 included appendices on the subjects of chimney linings and accessories. In 1971, Committee 307 learned of buckling problems in steel chimney liners. The Committee also noted that in modern power plant and process chimneys, environmental regulations required treatment of the effluent gases that could result in extremely variable and aggressively corrosive conditions in the chimneys. In view of these facts, the Committee agreed that the task of keeping the chimney liner recommendations current was not a responsibility of an ACI committee and could be misleading to designers using the chimney specification. It was the consensus of the Committee that the reference to chimney liner construction be dropped from future editions of the specification. Recognizing this, Committee 307 made a recommendation to the Brick Manufacturers’ Association and the American Society of Civil Engineers that each appoint a task force or a committee for the development of design criteria for brick and steel liners, respectively. The Power Division of ASCE took up the recommendation and appointed a task committee that developed and published in 1975 a design guide entitled, “Design and Construction of Steel Chimney Liners.” ASTM established two task forces for chimney liners, one for brick and the other for fiberglass reinforced plastic.
The Committee had extensive discussion on the question of including strength design in the 1979 specification. The decision to exclude it was based on the lack of experimental data on hollow concrete cylinders to substantiate this form of analysis for concrete chimneys. However, the Committee continued to consider strength design and encouraged experiments in this area.

Shortly after the 1979 edition was issued, the Committee decided to incorporate strength design provisions and update the wind and earthquake design requirements.

The 1988 edition of ACI 307 incorporated significant changes in the procedures for calculating wind forces as well as requiring strength design rather than working stress. The effects of these and other revisions resulted in designs with relatively thin walls governed mainly by steel area and, in many instances, across-wind forces.

The subject of across-wind loads dominated the attention of the Committee between 1988 and 1995 and the 1995 standard introduced modified procedures to reflect more recent information and thinking.

Precast chimney design and construction techniques were introduced as this type of design became more prevalent for chimneys as tall as 300 ft (91.4 m).

The subject of noncircular shapes was also introduced in 1995. However, due to the virtually infinite array of possible configurations, only broadly defined procedures were presented.

Because of dissimilarities between the load factors required by the ACI 307 standard and ACI 318, the Committee added guidelines for determining bearing pressures and loads to size and design chimney foundations.

In summary, the following highlights the major changes that were incorporated into the 1995 standard:

- Modified procedures for calculating across-wind loads;
- Added requirements for precast concrete chimney columns;
- Added procedures for calculating loads and for designing noncircular chimney columns;
- Deleted exemptions previously granted to “smaller” chimneys regarding reinforcement and wall thickness; and
- Deleted static equivalent procedures for calculating earthquake forces.

**Synopsis of current revisions**

Revisions to the ASCE 7-95 standard relating to wind and seismic forces required that several changes be made to the 1995 edition of ACI 307. The following highlights the changes incorporated into the current standard:

- Site-specific wind loads are calculated using a “3-sec gust” speed determined from Fig. 6-1 in ASCE 7-95 instead of the previously used “fastest-mile” speed.
- Site-specific earthquake forces are calculated using the effective peak velocity-related acceleration contours determined from Contour Map 9-2 in ASCE 7-95 instead of previously designated zonal intensity.
- The vertical load factor for along-wind forces has been reduced from 1.7 to 1.3.
- The vertical load factor for seismic forces has been reduced from 1.87 to 1.43.
- The load factor for across-wind forces has been reduced from 1.40 to 1.20.
- The vertical strength reduction factor \( \phi \) has been reduced from 0.80 to 0.70.

It should be noted that the reduced load factors must be used in concert with the revised strength reduction factor and the wind and seismic loads specified in ASCE 7-95.

The foregoing revisions are discussed in more detail in the following commentary.

Finally, the Committee believes that the ACI 307 standard is particularly unique in its inclusion of specific procedures to calculate wind and seismic forces on chimneys. Consequently, the Committee feels that the previous Commentary regarding these subjects should be retained wherever possible.

Similarly, the Committee believes that the Commentary regarding the assumptions and procedures for strength design and other recent revisions should also be retained for reference.

A chapter-by-chapter commentary follows.

**CHAPTER 1—GENERAL**

1.1—Scope

The scope of the 1995 standard was expanded to include precast chimney shells. Additional information may be found in PCI manuals. \(^1\) \(^2\) Warnes\(^3\) provides further guidelines on connection details for precast structures. Additional information is given in ACI 550R, “Design Recommendations for Precast Concrete Structures.”

1.4—Reference standards

The year of adoption or revision for the referenced standards has been updated.

**CHAPTER 2—MATERIALS**

No changes of note have been made in this section.

**CHAPTER 3—CONSTRUCTION REQUIREMENTS**

3.3—Strength tests

Requirements for testing precast concrete units were added in the 1995 standard.

3.4—Forms

Shear transfer within precast concrete shells must be considered in design especially if the structure has vertical as well as horizontal construction joints.

3.5—Reinforcing placement

The size, spacing, and location of vertical cores within precast concrete chimney shells will be determined by geometry and steel area requirements. It is important that the design of precast chimneys comply with the minimum spacing requirements of ACI 318 when arranging reinforcement within the cores to permit proper bar splicing and concrete placement.

**CHAPTER 4—SERVICE LOADS AND GENERAL DESIGN CRITERIA**

4.1—General

The 1995 Committee re-evaluated the previous exemptions regarding two-face reinforcement and minimum wall
thickness for chimneys 300 ft (91.4 m) or less in height and less than 20 ft (6.1 m) in diameter. Recent information has indicated that two-face circumferential reinforcement is necessary to minimize vertical cracking due to radial wind pressures and reverse thermal gradients due to the effects of solar heating. Reverse thermal gradients due to solar heating may be more pronounced when the air space between the column and lining is purged by pressurization fans and gas temperatures are low. Further, the 1995 Committee believed that two-face reinforcement should be required in all chimney columns, regardless of size, considering the aggressive environment surrounding chimneys.

4.1.3.1—A minimum wall thickness of 8 in. (200 mm) (7 in. [180 mm] if precast) is required to provide for proper concrete placement within and around two curtains of reinforcement.

4.1.3.2—The 1995 Committee expressed concern regarding edge buckling of relatively thin walls through regions where tall openings are present. The simplified procedure given in this section will give approximately the same results as the procedures of Chapter 10.10 of ACI 318.

If jamb buttresses are used, it is recommended that they be poured homogeneously with the section or adequately tied to ensure composite action.

4.1.7.2—Foundation design: The loading combinations in the 1995 version of this article have been deleted. The pseudo-bearing pressure/pile loads shall be computed by multiplying the unfactored dead and axial bending loads by their appropriate load factor from Sections 5.3.1 and 5.3.2.

4.2—Wind loads

4.2.1 General—The basic wind speed $V$ in the current standard has been revised from “fastest-mile” to a “3-sec gust” speed to reflect the changes published in ASCE 7-95. Eq. (4-1) has been modified accordingly. In Eq. (4-1), 1.47 converts wind speed from mph to ft/sec and 0.65 converts 3-sec gust speed to a mean hourly speed. The revised power law coefficient 0.154 (as an approximation of $1/6.5$) comes from Table C6-6 in the Commentary to ASCE 7-95, for Exposure C and for flexible or dynamically sensitive structures; the increase in the exponent increases the calculated pressures over the chimney height for the same speed.

The “3-sec gust” speed is always higher than the previously specified “fastest-mile” speed. A “fastest-mile” wind speed may be converted to a “3-sec gust” speed for normal speeds of interest in chimney design using the following equation

$$3-\text{sec gust } V = 1.0546 (\text{fastest mile } V + 11.94)$$

Eq. (4-1) permits the mean hourly speed at height $z$ to be determined from the basic design speed that is the “3-sec gust” speed at 33 ft (10 m) over open country. The conversion is based on the relationship recommended by Hollister. The specified wind loads presume that the chimney is located in open country. In rougher terrains the overall loads will be reduced, but for a tall chimney (height on the order of 650 ft [198 m]) the reduction is not likely to exceed 20 percent.

$V_R$ in Eq. (4-1) is the product of the square root of the importance factor $I$ and $V$, the basic wind speed as charted and defined in ASCE 7-95. It should be noted that $I$ can be used to vary probability, as well as to classify the importance of the structure. The Committee believes that all chimneys should be designed to be part of an essential facility classified as a Category IV structure. The importance factor of 1.15 for Category IV buildings and structures corresponds to a mean recurrence interval of 100 years. Additional information can be found in ASCE 7-95.

The simplified provisions of this standard do not preclude the use of more detailed methods, and the results of a full dynamic analysis employing accepted approaches and recognizing the flow profile and turbulence levels at a specific site may be used in place of the standard provisions. The approximate methods have, however, been tested against more detailed analyses, using probabilistic and deterministic approaches. These methods yielded acceptable results.

4.2.2 Along-wind loads—The recommended drag coefficients are consistent with slender chimneys [h/d(h) > 20] with a relative surface roughness on the order of $10^{-4}$ to $10^{-5}$. Some reduction in the drag coefficient $C_{dr}$ with decreasing h/d(h) can be expected but unusually rough (e.g., ribbed) chimneys would have higher values of $C_{dr}$. The variations of $C_{dr}$ with roughness and aspect ratio are discussed by Basu and Vickery and Basu.

The total load per unit length is computed as the sum of the mean component $\bar{w}(z)$ and the fluctuating component $w'(z)$. The dynamic component was evaluated using a slightly modified form of the “gust factor” equations described by Davenport, Vickery, and Simiu. The base moment is evaluated using the gust factor approach but the loads producing this moment are approximated by a triangular distribution rather than a distribution matching the mean. Eq. (4-6) is a simple empirical fit to values of $G_w'$ computed as before for a structural damping of 1.5 percent of critical. Except for referencing $V$ as the 3-sec gust speed, no revisions have been made to the procedures for calculating along-wind loads.

The natural period of the chimney may include the effect of foundation springs.

4.2.3 Across-wind loads—No revisions have been made to the procedures for calculating across-wind forces. However, Eq. (4-8a) has been rewritten for simplification and several typographical errors were corrected.

The 1995 Committee had numerous user comments and discussions regarding the procedures included in the 1988 standard for across-wind forces. Virtually all of the com-
### Table 4.2.3—Comparison of results: along- plus across-wind moments, 1988 versus 1995 procedures

#### Description of chimneys

<table>
<thead>
<tr>
<th>Chimney</th>
<th>Height, ft</th>
<th>TOD, ft</th>
<th>BOD, ft</th>
<th>Tapers</th>
<th>VI, mph</th>
<th>h/d at 5/6h</th>
<th>Frequency, hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>485</td>
<td>47.67</td>
<td>53.50</td>
<td>3</td>
<td>85.0</td>
<td>10.17</td>
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<td>13</td>
<td>500</td>
<td>52.17</td>
<td>52.17</td>
<td>1</td>
<td>76.8</td>
<td>9.58</td>
<td>0.428</td>
</tr>
<tr>
<td>7</td>
<td>534</td>
<td>51.09</td>
<td>61.55</td>
<td>1</td>
<td>74.9</td>
<td>10.11</td>
<td>0.591</td>
</tr>
<tr>
<td>8</td>
<td>545</td>
<td>33.00</td>
<td>55.00</td>
<td>1</td>
<td>85.6</td>
<td>14.86</td>
<td>0.432</td>
</tr>
<tr>
<td>9</td>
<td>613</td>
<td>73.00</td>
<td>73.00</td>
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<td>74.9</td>
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<tr>
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<td>85.6</td>
<td>17.05</td>
<td>0.529</td>
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</table>

#### Calculated wind speeds

<table>
<thead>
<tr>
<th>Chimney</th>
<th>Per ACI 307-88</th>
<th>Per ACI 307-95</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_cr, mph</td>
<td>V(z_cr), mph</td>
<td>V(z_cr), mph</td>
</tr>
<tr>
<td>6</td>
<td>78.9</td>
<td>93.9</td>
</tr>
<tr>
<td>13</td>
<td>76.2</td>
<td>84.0</td>
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<tr>
<td>7</td>
<td>106.4</td>
<td>84.8</td>
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<td>8</td>
<td>54.0</td>
<td>96.0</td>
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<td>9</td>
<td>101.1</td>
<td>86.4</td>
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<tr>
<td>12</td>
<td>72.0</td>
<td>92.3</td>
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<tr>
<td>2</td>
<td>71.8</td>
<td>87.2</td>
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<tr>
<td>4</td>
<td>39.7</td>
<td>91.1</td>
</tr>
</tbody>
</table>

#### Factored base wind moments in ft-tons

<table>
<thead>
<tr>
<th>Chimney</th>
<th>Per ACI 307-88, RMS combined along- and across-wind: B_r = 0.015; LF = 1.40</th>
<th>Per ACI 307-95, RMS combined along- and across-wind: B_r = 0.010; LF = 1.40</th>
<th>Per ACI 307-88 and ACI 307-95 along-wind only: LF = 1.70</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>270,600</td>
<td>209,200</td>
<td>160,900</td>
</tr>
<tr>
<td>13</td>
<td>283,500</td>
<td>224,100</td>
<td>148,000</td>
</tr>
<tr>
<td>7</td>
<td>447,800</td>
<td>238,100</td>
<td>165,100</td>
</tr>
<tr>
<td>8</td>
<td>117,500</td>
<td>79,400</td>
<td>161,200</td>
</tr>
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<td>9</td>
<td>971,700</td>
<td>459,100</td>
<td>320,700</td>
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<td>12</td>
<td>1,475,800</td>
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<td>865,300</td>
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<tr>
<td>2</td>
<td>39,800</td>
<td>34,100</td>
<td>28,600</td>
</tr>
<tr>
<td>4</td>
<td>16,500</td>
<td>11,600</td>
<td>43,800</td>
</tr>
</tbody>
</table>

Mentators felt that the 1988 procedures were unduly conservative, especially in the absence of any record of structural failure. As a result of these discussions, and with the availability of new data and full-scale observations, the procedures for calculating across-wind loads were extensively revised.

A general solution for the across-wind response of circular chimneys with any geometry was developed by Vickery. These procedures, based on Vickery’s general solution, were simplified to some extent, which requires that their application be restricted to certain geometries. Similar models have provided the basis for vortex-induced forces incorporated by the National Building Code of Canada, and the ASME/ANSI STS-1-1992 Steel Stack Standard.

Circular chimneys outside the bounds of these procedures, or where a flare or strong taper (nozzle) exists for more than one diameter near the top, may be conservatively analyzed using the procedures of Section 4.2.3.3 of ACI 307-88 or by the general approach put forth by Vickery. It should be noted, however, that the procedures for determining shedding forces are not materially affected by the configuration of the lower third of the shell, which may range from plumb to any degree of taper.

However, it should also be noted that noncircular shapes may be more sensitive to across-wind forces and may require analyses beyond the scope of this standard.

Eq. (4-16) establishes a basis for increasing structural damping from a minimum of 1.0 percent to a maximum of 4.0 percent when the wind speed $V$ exceeds $V(z_{cr})$. Structural damping of 1 percent of critical is consistent with measured values and moderate stress levels with little cracking. Damping of 4.0 percent, which would be permitted when $V = 1.30 V(z_{cr})$, is more consistent with damping values permitted in seismic design.
Eight sample chimneys were studied using the 1988 procedures and the 1995 procedures. Fatigue damage was also considered using the procedures put forth by Vickery. It was concluded that a case-by-case analysis of fatigue in circular chimneys that would require a supplemental working stress analysis was not necessary, as fatigue stresses in the sample chimneys were within acceptable limits.

Results using the 1988 and the 1995 procedures are compared in Table 4.2.3. These chimneys were selected from a group of projects where the aspect ratio \( h/d \) is at or near 10, where peak excitation is normally found. Note that for Chimneys 7 and 9 the critical wind speed exceeds the design wind speed, permitting modification of both damping \( \zeta \) and \( M_a \) which significantly reduces the base moments.

4.2.3.4 Grouped chimneys—Interactions between closely spaced cylindrical objects have been studied in considerable detail but virtually all the test results are for subcritical values of Reynolds Numbers and their applicability to chimneys is highly questionable. However, even with the scale effects introduced by the inequality of the Reynolds Number, the wind tunnel is presently the only tool that will provide guidance as to the likely magnitude of interference effects. A review of interference effects is given by Zdravkovsk. Vickery attributes the amplification of shedding forces to increased turbulence and additional buffeting effects, which formed the basis for revisions made to this section.

At center-to-center spacings \( s \), in excess of 2 to 3 diameters, the prime interference effect is related to across-wind excitation due to shedding. The recommendations in Section 4.2.3.4 are based on the results of Vickery and Daly, and were obtained at subcritical values of the Reynolds Number. The first term in modifier \( c \) is an enhancement factor to account for buffeting due to vortices shed by the upstream structure; the second term accounts for small-scale turbulence. The same reference also contains results for two cylindrical liners for earthquakes by the Task Committee of the American Society of Civil Engineers. To obtain the design response spectrum, the normalized spectrum must be scaled down to the effective peak velocity \( EPV \) related ground acceleration.

The ASCE 7-95 map for the \( EPV \)-related acceleration coefficient is used in this standard. This map differs from those used in the Uniform Building Code, which was based on the maximum recorded intensity of shaking without regard to the frequency with which earthquake shaking might occur. The ASCE 7-95 map, on the other hand, has a more uniform probability of earthquake occurrence, and is based on those given by the NEHRP (see Reference 21). For example, in Zone 4 seismic area, the \( EPV \)-related acceleration is 0.4g and the probability of not exceeding this peak \( EPV \) ground acceleration within 50 years is estimated to be 90 percent. This is equivalent to a mean recurrence interval of 475 years, or an average annual risk of 0.002 events per year. The peak \( EPV \)-related ground acceleration at a site can be determined either by using this contour map and the recommended scale factors given in Table 4.3.2 or from the specific seismic record available at the site. It should be noted that a ductility factor of 1.33 is built into the scale factors of Table 4.3.2. For instance, instead of 0.40, a scale factor of 0.30 is used for a site with an \( A_0 \) of 0.4.

It should also be pointed out that the recommended design response spectrum is based on firm soil sites. Soil conditions at the firm site consist of bedrock with shear wave velocity greater than 2500 ft/sec (762.0 m/sec) or stiff soils with deposits less than 200 ft (61.0 m). For chimneys to be built on shallow and soft or medium-stiff clays and sands, a greater design response spectrum is anticipated. Guidelines provided in NEHRP to obtain a modified design response spectrum and the soil-structure interaction may be used.

In place of a dynamic response spectrum analysis, a time history dynamic analysis is permitted, provided a reliable time history of earthquake ground motion is used.
In the design of a chimney for horizontal earthquake forces, only one horizontal direction need be considered. Unlike building structures, chimneys are generally axisymmetric, and the orthogonal effects from two horizontal earthquakes acting simultaneously in the two principal directions are negligible.

The effect of the vertical component of the earthquake on the chimney has been determined to be of no design significance. An extensive time history analysis made by the Committee shows that the vertical stresses from dead load and horizontal seismic excitation are increased by at most a few percent by the effects of vertical seismic excitations. This is principally because the psa responses to vertical and horizontal acceleration do not occur simultaneously.

Design based on SRSS of vertical and horizontal earthquake forces will be unduly conservative. Therefore, the inclusion of vertical seismic effects is not recommended by the Committee.

For cases in which the chimney lining (brick, steel, or other materials) is supported by the concrete chimney shell, either at the top of the chimney shell or at other intermediate points, a dynamic analysis including both concrete shell and liner should be used. Appropriate damping values should be used for the liner depending on its construction (e.g., 1.5 percent for steel liners, 4.0 percent for brick liners, and 2.0 percent for fiber reinforced plastic liners).

4.5—Deflection criteria

The incorporation of the strength design method into the standard will generally result in chimneys with thinner walls in the lower portion and with higher deflections. The Committee felt that deflections under service loads should be checked and that the deflections of chimneys designed by the working stress method should not vary greatly from the deflections of existing chimneys designed by the working stress method. Limiting deflections also serves to reduce the effects of secondary bending moments.

However, the procedures in the ACI 307 1988 edition were found to be too restrictive for shorter chimneys and were modified in the 1995 standard. The deflection limit is compared against the deflection calculated using uncracked concrete sections and a fixed base.

Operation, access for inspection, lining type, etc., as well as wind or earthquake-induced deflection, should be considered when establishing shell geometry.

CHAPTER 5—DESIGN OF CHIMNEY SHELL: STRENGTH METHOD

5.1—General

Several significant revisions were made to this section, most notably the load factors specified in 5.3 and the strength reduction factor \( \phi \) specified in 5.4. Portions of previous commentary are, however, retained for reference.

5.1.2 The maximum compressive strain in the concrete is assumed to be 0.003, or the maximum tensile strain in the steel is assumed to be the fracture limit of 0.07, whichever is reached first. If the steel fracture limit is reached first, the maximum concrete strain computed from the linear strain diagram is below 0.003. This deviates from the design assumptions of ACI 318. For a given total vertical steel ratio, this may occur when the ratio of the vertical load to the moment is below a certain value. A total vertical steel ratio in the chimney cross section less than that per the minimum requirement of ACI 318 for flexural members is permitted.

Even when the maximum concrete compressive strain \( \varepsilon_m \) is less than 0.003, the stress block is still considered rectangular. However, in these instances, the stress level is modified by a correction factor called the parameter \( Q \). See commentary on Section 5.5.1.

5.3—Required strength

5.3.1—The Committee noted that the “fastest-mile” provisions in the 1988 edition of ACI 307 resulted in an increase in wind moments of between 0 and 50 percent when compared with ACI 307-79. The use of a “3-sec gust” wind speed results in further increases in axial bending moments, which are 10 to 20 percent higher than moments calculated using “fastest-mile” speeds. Since the Committee has no data or information concerning axial bending failures of chimney shells designed using previously established procedures, it was decided that the load factor for along-wind loads can be safely reduced from 1.7 to 1.3 when “3-sec gust” wind speeds are used. It should be noted that a wind load factor of 1.3 is consistent with that recommended by ASCE 7-95.

Similarly, the Committee has determined that the wind load factor for along, plus across-wind loads can be reduced from 1.4 to 1.2.

It should be noted that the vertical load factor reductions incorporated in the current standard must be accompanied by a decrease in the strength reduction factor \( \phi \) from 0.80 to 0.70, as described in Article 5.4.1. The net effect of the revision to the vertical load factors, coupled with the change in the strength factor, is relatively minor. Table 5.3.1 summarizes the effects of the revisions on 12 sample chimney shells over a range of wind speeds. The geometry of the chimneys studied is as follows

<table>
<thead>
<tr>
<th>Chimney no.</th>
<th>Height, ft</th>
<th>TOD, ft</th>
<th>BOD, ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>13.50</td>
<td>19.75</td>
</tr>
<tr>
<td>2</td>
<td>275</td>
<td>28.00</td>
<td>28.00</td>
</tr>
<tr>
<td>3</td>
<td>325</td>
<td>15.00</td>
<td>20.00</td>
</tr>
<tr>
<td>4</td>
<td>375</td>
<td>20.00</td>
<td>32.00</td>
</tr>
<tr>
<td>5</td>
<td>425</td>
<td>35.00</td>
<td>39.00</td>
</tr>
<tr>
<td>6</td>
<td>485</td>
<td>47.67</td>
<td>53.50</td>
</tr>
<tr>
<td>7</td>
<td>534</td>
<td>51.09</td>
<td>61.55</td>
</tr>
<tr>
<td>8</td>
<td>545</td>
<td>33.00</td>
<td>55.00</td>
</tr>
<tr>
<td>9</td>
<td>613</td>
<td>73.00</td>
<td>73.00</td>
</tr>
<tr>
<td>10</td>
<td>700</td>
<td>60.00</td>
<td>78.00</td>
</tr>
<tr>
<td>11</td>
<td>773</td>
<td>43.00</td>
<td>70.00</td>
</tr>
<tr>
<td>12</td>
<td>978</td>
<td>73.00</td>
<td>114.78</td>
</tr>
</tbody>
</table>

5.3.2—The Committee has determined that, based on the required use of velocity-related acceleration contours coupled with a re-evaluation of the ductility inherent in chimney shells, a decrease in the ratio of the load factor to the strength reduction factor for earthquake forces from 2.34 to 2.04 is warranted.
The load factor for determining the circumferential strength required to resist wind load has not been revised, although the reinforcement necessary to satisfy the higher moments may increase up to 15 percent on large-diameter chimneys. However, the Committee believes that this additional reinforcement is justified to minimize vertical cracking of chimney shells.

5.4—Design strength

5.4.1—In the calculation of limit-state bending moments, allowance needs to be made for the moment caused by the weight of the chimney in its deformed shaped. The deflection will be less than that calculated by standard methods due to the stiffening effect of the concrete in the cracked tension zone. The stiffening effect needs to be investigated further.

The strength reduction factor for vertical strength has been reduced from 0.80 to 0.70. A $\phi$ factor of 0.70 was chosen because it was found that the dead-load axial stress on the gross area exceeds 0.10 $f'_c$ in the lower portions of some sample chimneys. The effects of this revision are discussed more fully in Section 5.3.

The formulas are also derived for cross sections with one or two openings in, or partly in, the compression zone. No reduction in the forces and moments due to reinforcing steel is made for the reduction in the distance of the additional vertical reinforcement on each side of the opening, provided per Section 4.4.6.

5.5—Nominal moment strength

The formulas for the nominal moment strength of chimney cross sections are obtained based on the design assumptions of ACI 318, except as modified under Section 5.1.2 of this standard. The derivations of the formulas are given in Appendix A.

The formulas are derived for circular hollow cross sections with a uniform distribution of vertical reinforcing steel around the circumference.

5.5.1 The parameter $Q$—The use of a rectangular compression stress block for rectangular and T-shaped reinforced concrete beams came to be accepted after extensive comparative study between the analytical results using this stress-strain relationship and the test data. The acceptability of the rectangular stress block was based on the closeness between the results of the analyses and the tests, comparing the following: a) concrete compression; and b) moment of the compression about the neutral axis (for a rectangular section this is equivalent to the distance of the center of gravity of the compression stress block from the neutral axis).

The preceding comparative study was based on the limited test data available on reinforced concrete members of hollow circular sections subjected to axial and transverse loads.22

Another special problem in arriving at the compressive stress block for the analysis of reinforced concrete chimneys was the fact that the maximum concrete compressive strain is less than 0.003 when the fracture limit of steel is reached. That is, the compressive stress block is not fully developed (see commentary on Section 5.1.2). Thus, the previous attempts at specifying the rectangular stress block for chimney cross sections needed to be modified.

A numerical study was undertaken by the 1988 Committee to find an equivalent rectangular stress block for the calculation of the strength of chimney cross sections.

For a given value of $\alpha$, the results of the rectangular concrete compression stress block, expressed by dimensionless modifications of (a) and (b) previously stated, were compared with the corresponding results using a more exact concrete stress-strain relationship23 given by Hoggestad24 using a limiting strain of 0.003. The comparisons were made for hollow circular sections without openings and with single openings with values of $\beta$ of 10, 20, and 30 deg.

It was concluded that for values of $\alpha$ above 20 deg, or when the limiting strain of concrete is reached first, an equivalence between the two approaches is reached if the stress level of the rectangular compression block is reduced by a factor of 0.89. For values of $\alpha$ below about 20 deg, a further correction is required, leading to the values of the parameter $Q$ defined in Section 5.5.1.

Thus the correction factor, or the parameter $Q$, achieves a close equivalence between the resulting values of (a) and (b) previously stated for the “thereby corrected” rectangular stress block and the stress block based on the Hoggestad stress-strain relationship, yet retains the simplicity of the rectangular stress block.

5.5.6 Due to thermal exposure of the concrete chimneys, the temperature drop across the wall reduces the nominal strength of chimney sections. This effect is accounted for by reducing the specified yield strength of steel and specified compressive strength of concrete.

The derivation of equations is included in Appendix A.

5.6—Design for circumferential bending

5.6.2 The commentary on Section 5.5.6 applies equally to this section.

Table 5.3.1—Comparison of along-wind design moments

<table>
<thead>
<tr>
<th>Chimney no.</th>
<th>$90(3sg)/70(fm)$</th>
<th>$120(3sg)/100(fm)$</th>
<th>$150(3sg)/130(fm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.054</td>
<td>0.973</td>
<td>0.940</td>
</tr>
<tr>
<td>2</td>
<td>1.058</td>
<td>0.976</td>
<td>0.944</td>
</tr>
<tr>
<td>3</td>
<td>1.062</td>
<td>0.980</td>
<td>0.947</td>
</tr>
<tr>
<td>4</td>
<td>1.065</td>
<td>0.983</td>
<td>0.950</td>
</tr>
<tr>
<td>5</td>
<td>1.069</td>
<td>0.988</td>
<td>0.955</td>
</tr>
<tr>
<td>6</td>
<td>1.072</td>
<td>0.991</td>
<td>0.958</td>
</tr>
<tr>
<td>7</td>
<td>1.073</td>
<td>0.993</td>
<td>0.960</td>
</tr>
<tr>
<td>8</td>
<td>1.074</td>
<td>0.993</td>
<td>0.960</td>
</tr>
<tr>
<td>9</td>
<td>1.079</td>
<td>0.998</td>
<td>0.965</td>
</tr>
<tr>
<td>10</td>
<td>1.082</td>
<td>1.000</td>
<td>0.967</td>
</tr>
<tr>
<td>11</td>
<td>1.084</td>
<td>1.002</td>
<td>0.969</td>
</tr>
<tr>
<td>12</td>
<td>1.090</td>
<td>1.008</td>
<td>0.976</td>
</tr>
</tbody>
</table>

{Values of $[1.3 \times M(3sg)/0.7]/1.7 \times M(fm)/0.8$ for sample chimneys]
CHAPTER 6—THERMAL STRESSES

6.1—General
The derivations of the formulas for the vertical and horizontal stresses in concrete and steel, due to a temperature drop only across the chimney wall, are given in Appendix B. No revisions have been made to this section.

6.2—Vertical temperature stresses

6.2.2 The research data available to establish the coefficients of heat transfer through chimney lining and shell, especially as they concern the heat transfer from gases to the surfaces and through ventilated air spaces between lining and shell, are somewhat meager. Unless complete heat balance studies are made for the particular chimney, it is permissible to use constants as determined or stated in this standard.

APPENDIX A—DERIVATION OF EQUATIONS FOR NOMINAL STRENGTH

Equations for the nominal strength of concrete chimney sections, with and without openings, are derived in this Appendix.

The factored vertical load \( P_u \) and the corresponding nominal moment strength \( M_n \) are expressed in dimensionless form, as given in Section 5.5.1 by Eq. (5-2) and (5-10), respectively.

Also a procedure to account for the temperature effects in the vertical and horizontal directions is outlined.

Forces are designated as follows:

- \( M_{DS} \) = design moment strength of the section
- \( M_n \) = nominal moment strength of the section
- \( M_u \) = factored moment acting on the section
- \( P \) = total force in the concrete compressive stress block
- \( P^* \) = factored vertical load acting on section
- \( S_1^*, S_1^*, S_2^*, S_3^*, S_4^* \) = moments of \( P, S_1, S_2, S_3, S_4 \) about neutral axis, respectively
- \( S_1 \) = tensile force where steel stress is below yield point, from \( \alpha \) to \( \psi \)
- \( S_2 \) = tensile force where steel stress is at yield point, from \( \psi \) to \( \pi \)
- \( S_3 \) = compressive force in steel where stress is below yield point, from \( \mu \) to \( \alpha \)
- \( S_4 \) = compressive force in steel where stress is at yield point, from \( 0 \) to \( \mu \)
- \( \phi \) = capacity-reduction factor

From Fig. 5.5.1(a) and 5.5.1(b)

- \( \cos \mu = \cos \alpha + [(1 - \cos \alpha)/\varepsilon_m](f_f/E_f) \)
- \( \cos \psi = \cos \alpha - [(1 - \cos \alpha)/\varepsilon_m](f_f/E_f) \)
- \( K_e = E_d f_f \)
- \( \eta = \text{number of openings in the compression zone} \)
- \( \beta = \text{one-half opening angle} \)
- \( \varepsilon_m = 0.07(1 - \cos \alpha)/(1 + \cos \alpha) \leq 0.003 \)
- \( \gamma = \text{one-half angle between center lines for two openings} \)
- \( \theta = \text{variable function of } \alpha \)

\[ \omega = \rho_1 f_f f'_c \text{, therefore } \omega f'_c = \rho_1 f_f \]

\[ S_1 = 2 \int_0^\psi r(\cos \alpha - \cos \theta) \frac{\varepsilon_m E_f}{r(1 - \cos \alpha)} \rho_1 r d \theta \]

\[ = \frac{2 \varepsilon_m E_f \rho_1 r}{(1 - \cos \alpha)} \left( \frac{\cos \alpha - \sin \theta}{\psi} \right) \]

\[ = \frac{2 \varepsilon_m E_f \rho_1 r}{(1 - \cos \alpha)} \left[ \frac{\cos \alpha - \sin \psi + \sin \alpha}{\psi} \right] \]

but

\[ E_s \rho_1 = E_s \rho_1 \cdot (\omega f'_c/\rho_1 f_f) \]

\[ = E_s f_f f'_c \cdot \omega f'_c \]

\[ K_e \omega f'_c \]

therefore

\[ S_1 = 2 \varepsilon_m K_e \omega_1 r f'_c \frac{[(\psi - \alpha)\cos \alpha - \sin \psi + \sin \alpha]}{(1 - \cos \alpha)} \]

or

\[ S_1 = 2 \varepsilon_m K_e \omega_1 r f'_c \cdot Q'_1 \]

but

\[ \rho_1 f_f = \omega f'_c \]

\[ S_2 = 2(\pi - \psi) \rho_1 r f_f \]

\[ P = 2(\tau - n_1 \beta) r \cdot 0.85 f'_c \]

\[ = 1.7 r f'_c (\tau - n_1 \beta) \]

\[ = 1.7 r f'_c \lambda \]

where

\[ \lambda = \tau - n_1 \beta \]

\[ S_3 = 2 \int_0^\alpha r(\cos \theta - \cos \alpha) \frac{\varepsilon_m E_f}{r(1 - \cos \alpha)} \rho_1 r d \theta \]

\[ = \frac{2 \varepsilon_m E_f \rho_1 r}{(1 - \cos \alpha)} \left( \sin \theta - \theta \cos \alpha \right) \]

\[ = \frac{2 \varepsilon_m K_e \omega_1 r f'_c}{(1 - \cos \alpha)} \cdot \left[ \frac{\sin \alpha - \sin \mu - (\alpha - \mu) \cos \alpha}{1 - \cos \alpha} \right] \]

\[ = 2 \varepsilon_m K_e \omega_1 r f'_c \cdot Q_3 \]
\[ S_4 = 2 \mu \rho \cdot r t f_y \]
\[ = 2 \omega rt f_c' \cdot \mu \]

Sum of vertical forces must equal zero, therefore

\[ P_u = P + S_3 + S_4 - S_1 - S_2 \]
\[ = 1.70 rt f_c' \lambda + 2 \epsilon_m K_c \omega rt f_c' Q_3 + 2 \omega rt f_c' \mu \]
\[ - 2 \epsilon_m K_c \omega rt f_c' Q' + 2 \omega rt f_c'(\pi - \psi) \]
\[ P_u / rt f_c' = K_1 \]
\[ = 1.70 \lambda + 2 \epsilon_m K_c \omega_1 Q_1 + 2 \omega_1 \lambda_1 \]

where
\[ \lambda = \tau - n_1 \beta \]
\[ Q_1 = \frac{\sin \psi - \sin \mu - (\psi - \mu) \cos \alpha}{1 - \cos \alpha} \]
\[ \lambda_1 = \mu + \psi - \pi \]
\[ K_c = E / f_y \]
\[ \omega_1 = \rho f_y / f_c' \]

\[ S_1' = 2 \int_{\psi} \rho (\cos \alpha - \cos \theta) \cdot r (1 - \cos \alpha) \cdot \epsilon_m E \cdot \rho rt f \theta d \theta \]
\[ = 2 \epsilon_m \int_{\psi} \rho (\cos \alpha - \cos \theta)^2 \cdot r (1 - \cos \alpha) d \theta \]
\[ = 2 \epsilon_m \int_{\psi} \rho (\cos \alpha - \cos \theta)^2 \cdot \left( \theta \cos^2 \alpha - 2 \cos \alpha \sin \theta + \frac{\sin 2 \theta}{4} \right) d \theta \]
\[ = 2 \epsilon_m \int_{\psi} \rho (\cos \alpha - \cos \theta)^2 d \theta \]
\[ \left[ (\psi - \alpha) \cos^2 \alpha - 2 \cos \alpha (\sin \psi - \sin \alpha) + (1/2) (\psi - \alpha) \right. \]
\[ \left. + (1/4) (\sin^2 2 \alpha - \sin 2 \alpha) \right] \]

Let \( J = [ ] \)

\[ = (\psi - \alpha) \cos^2 \alpha + 2 \sin \alpha \cos \alpha - 2 \cos \alpha \sin \psi \]
\[ + (1/2) \sin \psi \cos \psi - (1/2) \sin \alpha \cos \alpha + (1/2) (\psi - \alpha) \]

or

\[ 2J = 2(\psi - \alpha) \cos^2 \alpha + 3 \sin \alpha \cos \alpha - 4 \cos \alpha \sin \psi \]
\[ + \sin \psi \cos \psi + (\psi - \alpha) \]

therefore

\[ S_1' = \epsilon_m r^2 t f_c' K_c \omega J \]

where

\[ J_1 = 2J / (1 - \cos \alpha) \]

or

\[ J_1 = [2(\psi - \alpha) \cos^2 \alpha + 3 \sin \alpha \cos \alpha - 4 \cos \alpha \sin \psi \]
\[ + \sin \psi \cos \psi + (\psi - \alpha)] / (1 - \cos \alpha) \]

\[ S_2' = 2 \int_{\omega} \rho rt f_y \cdot r (\cos \alpha - \cos \theta) d \theta \]
\[ = 2 \epsilon_m \int_{\omega} \rho (\cos \alpha - \cos \theta) d \theta \]
\[ = 2 \epsilon_m \int_{\omega} \rho (\cos \alpha - \sin \theta)^2 d \theta \]
\[ = 2 \epsilon_m \int_{\omega} \rho (\cos \alpha - \sin \theta)^2 d \theta \]

but

\[ \rho f_y = \omega_1 f_c' \]

therefore

\[ S_2' = 2 r^2 t f_c' \omega J \]

where

\[ J_2 = (\pi - \psi) \cos \alpha + \sin \psi \]

\[ S_3' = 2 \int_{\mu} \rho r^2 \cdot \epsilon_m E \cdot \rho rt f \theta d \theta \]
\[ = 2 \epsilon_m \int_{\mu} (\cos \theta - \cos \alpha)^2 r (1 - \cos \alpha) d \theta \]
\[ = 2 \epsilon_m \int_{\mu} (\cos \theta - \cos \alpha)^2 d \theta \]
\[ \left[ (1/2) (\alpha - \mu) (1/4) (\sin^2 2 \alpha - \sin 2 \alpha) + (\alpha - \mu) \cos^2 \alpha \right] \]
\[ + (\pi - \psi) \cos \alpha + \sin \psi \]
\[ + \sin \psi \cos \psi + (\psi - \alpha) \]

Let

\[ J_3 = 2J / (1 - \cos \alpha) \]

or

\[ J_3 = [\alpha - \mu + \sin \alpha \cos \mu - 4 \cos \alpha (\sin \alpha - \sin \mu) \]
\[ + 2(\alpha - \mu) \cos^2 \alpha] / (1 - \cos \alpha) \]

therefore

\[ S_3' = \epsilon_m r^2 t f_c' K_c \omega J \]

\[ S_4' = 2 \int_{\omega} \rho rt f_y \cdot r (\cos \theta - \cos \alpha) d \theta \]
therefore

\[ S'_4 = 2r^2 t'_f \omega J_4 \]

where

\[ J_4 = \sin \mu - \mu \cos \alpha \]

For \( P' \) with one opening in compression zone [Fig. 5.5.1(a)]

\[
P' = 2r^2 t^2 t'_f \cdot \left[ \frac{\tau (r \sin \tau - r \cos \alpha)}{\tau} - \int_0^\beta \rho (r \cos \theta - \cos \alpha) d\theta \right] \]

\[ = 1.70r^2 t'_f (\sin \tau - \tau \cos \alpha - \sin \beta + \beta \cos \alpha) \]

therefore

\[
P' = 1.70r^2 t'_f [\sin \tau - (\tau - \beta) \cos \alpha - \sin \beta] \]

For \( P' \) with two openings in compression zone [Fig. 5.5.1(b)]

\[
P' = 2r^2 t^2 t'_f \cdot \left[ \frac{\tau (r \sin \tau - r \cos \alpha)}{\tau} - \int_{\gamma - \beta}^{\gamma + \beta} \rho (r \cos \theta - \cos \alpha) d\theta \right] \]

\[ = 1.70r^2 t'_f [\sin \tau - \tau \cos \alpha - \sin (\gamma + \beta) + \sin (\gamma - \beta) + 2\beta \cos \alpha] \]

therefore

\[
P' = 1.70r^2 t'_f [\sin \tau - (\tau - 2\beta) \cos \alpha - \sin (\gamma + \beta) + \sin (\gamma - \beta)] \]

Generalizing

\[
P' = 1.70r^2 t'_f \cdot R \]

where

\[ R = \sin \tau - (\tau - n_1 \beta) \cos \alpha - (n_1/2)[\sin (\gamma + \beta) - \sin (\gamma - \beta)] \]

For no openings

\[ n_1 = \gamma = \beta = 0 \]

For one opening in compression zone

\[ n_1 = 1 \]

\[ \gamma = 0 \]

For two openings in compression zone

\[ n_1 = 2 \]

Sum of moments about neutral axis must equal zero, therefore

\[
M_n = P_n r \cos \alpha + P'_1 + S'_1 + S'_2 + S'_3 + S'_4
\]

\[
= P_n r \cos \alpha + 1.70r^2 t'_f R + \varepsilon_m r^2 t'_f K_\omega \omega J_1 + 2r^2 t'_f \omega J_2
\]

\[
+ \varepsilon_m r^2 t'_f K_\omega \omega J_3 + 2r^2 t'_f \omega J_4
\]

\[
= P_n r \cos \alpha + 1.70r^2 t'_f R + \varepsilon_m r^2 t'_f K_\omega \omega (J_1 + J_3)
\]

\[
+ 2r^2 t'_f \omega (J_2 + J_4)
\]

therefore

\[
M_n / r^2 t'_f = (P_n r \cos \alpha / r t'_f) + K_2
\]

where

\[
K_2 = 1.70R + \varepsilon_m K_\omega \omega (J_1 + J_3) + 2\omega (J_2 + J_4)
\]

or

\[
K_2 = 1.70R + \varepsilon_m K_\omega \omega Q_2 + 2\omega K
\]

\[
Q_2 = \frac{[(\psi - \mu)(1 + 2 \cos \gamma \alpha) + (\gamma/2)(4 \sin 2\alpha + \sin 2\psi - \sin 2\mu)}{4 \cos \alpha (\sin \alpha + \sin \psi - \sin \mu)](1 - \cos \alpha)
\]

and

\[ K = \sin \psi + \sin \mu + (\pi - \psi - \mu) \cos \alpha \]

Multiply both sides of the equation by 1/K_1 = r t'_f / P_a

\[
rt_f / P_a \cdot M_n / r^2 t'_f = rt_f / P_a \cdot P_n r \cos \alpha / r t'_f + 1/K_1 \cdot K_2
\]

therefore

\[
K_3 = M_n / P_a = \cos \alpha + K_2 / K_1
\]

or

\[
M_n = K_3 P_a d' \]

and require

\[ M_{DS} = \phi M_n \geq M_u \]

For two symmetric openings partly in compression zone [Fig. 5.5.1(c)]

\[ \gamma + \beta > \tau \]

and

\[ \gamma - \beta < \tau \]

let

\[ \delta = \gamma - \beta \]

The situation is the same as for no openings in the compression zone with

\[ \tau = \delta \]
\[ \lambda = \delta \]

\[ R = \sin \delta - \delta \cos \alpha \]

and all other values are the same as before.

Openings in the tension zone—Openings in the tension zone are ignored since the tensile strength of the concrete is neglected, and the bars cut by the openings are replaced at the sides of the openings.

Openings in the compression zone—Openings in the compression zone are ignored in calculations of the forces in the compression reinforcement only, since the cut bars are replaced at the sides of the openings.

Vertical temperature stresses in reinforcement; effect on \( f_y \)

\[ f_{STV} = \] tensile temperature stress in outside steel

\[ f_{STV}'' = \] compressive temperature stress in inside steel

\[ f_{STV} \text{ and } f_{STV}'' \text{ at service loads} \]

\[
\frac{\rho}{\rho(1 + \gamma_1)} = \frac{1}{1 + \gamma_1}
\]

= ratio, outside steel area to total steel area

\[
\frac{\gamma_1 \rho}{\rho(1 + \gamma_1)} = \frac{\gamma_1}{1 + \gamma_1}
\]

= ratio, inside steel area to total steel area

\[ F_y(v) = \text{load factor for temperature combined with } W \text{ or } E \]

\[ = 1.4 \]

At ultimate, effect on \( f_y \) on windward side

Usable yield force = yield force - \( F_y(v) \cdot \text{tensile force in outside steel + } F_y(v) \cdot \text{compressive force in inside steel} \)

Dividing by total steel area \( A_s \)

\[
f_y'(v) = f_y - \frac{F_y(v)}{1 + \gamma_1} \cdot A_s \cdot f_{STV} \]

\[
= \frac{F_y(v) \cdot \gamma_1}{1 + \gamma_1} \cdot A_s \cdot f_{STV}''
\]

\[ \frac{A_s}{f_y'} \]

therefore

\[ f_y'(v) = f_y - \frac{F_y(v)}{1 + \gamma_1} (f_{STV} - \gamma_1 f_{STV}'') \]

It is conservative and convenient to use the same value for \( f_y' \) on the leeward side as well.

Vertical temperature stresses in concrete effect on \( f_c' \)

\[ f_{CTV}' = \] concrete compressive stress due to temperature alone at service loads

At ultimate, effect on \( f_c' \)

\[ f_c''(v) = f_c' - F_y(v) \cdot f_{CTV}' \]

Nominal strength for circumferential bending (compression on inside)

\[ f_c'(v) = f_c' - 1.05 f_{STC} \]

\[ f_c''(v) = f_c' - 1.05 f_{CTC}'' \]

\[ \rho' = \text{ratio outside steel area to total area} \]

\[ \gamma_1' = \text{ratio inside steel area to outside steel area} \]

\[ \rho' \gamma_1' = \text{area outside steel, in.} \]

\[ \gamma_1' \gamma_1' = \text{area inside steel, in.} \]

Stress in compression steel

\[ f_{CS} = \frac{(a/b_1) - (1 - \gamma_2')}{a/b_1} \cdot 0.003 E_s \]

\[ f_{CS} = \frac{a - b_1(1 - \gamma_2)}{a} \cdot 0.003 E_s \leq f_y'(c) \quad (A-1) \]

Stress in tensile steel

\[ f_{TS} = \frac{\gamma_2' - (a/b_1)}{a/b_1} \cdot 0.003 E_s \]
Load in compression steel

\[ P_{CS} = f_{CS} \gamma_{1} \rho' t \]  \hspace{1cm} (A-3)

Load in tensile steel

\[ P_{TS} = f_{TS} \rho' t \]  \hspace{1cm} (A-4)

Load in concrete compression block

\[ P_{CB} = 0.85 f''(c) t a \]  \hspace{1cm} (A-5)

\[ \Sigma V = 0, P_{CB} + P_{CS} - P_{TS} = 0 \]  \hspace{1cm} (A-6)

Find the value of \( a \) that satisfies this equation.

\[ \Sigma M \text{ about } P_{TS}, M_{n} = \{ P_{CB}[\gamma_{2} - (a/2)] + P_{CS}(2 \gamma_{2} - 1)\} t \]

\[ M_{DS} = \phi M_{n} \geq M_{u} \]  \hspace{1cm} (A-7)

**APPENDIX B—DERIVATION OF EQUATIONS FOR TEMPERATURE STRESSES**

The equations for maximum vertical stresses in concrete and steel due to a temperature drop only across the concrete wall with two layers on reinforcement are derived as follows.

Unrestrained rotation caused by a temperature differential of \( T_{x} \)

\[ \theta_{u} = \alpha_{u} T_{x} / t \]

Since rotation is prevented, corresponding stresses are induced

- In concrete (inside)
  \[ \varepsilon_{c} = \theta_{ct} c t = \alpha_{u} T_{c} c \]

- In outside reinforcement
  \[ f''_{CTV} = \alpha_{u} (\gamma_{2} - c) t \]

\[ \rho = \text{ratio of total area of vertical outside face reinforcement to total area of concrete chimney shell at section under consideration} \]

\[ \gamma_{1} = \text{ratio of inside face vertical reinforcement area to outside face vertical reinforcement area} \]

\[ f''_{STV} = \alpha_{u} (\gamma_{2} - c) T_{n} E_{c} \]

For \( c \)

\[ \Sigma V = 0, f''_{CTV} (ct/2) + f''_{STV} \gamma_{1} \rho t \]

\[ \phi = \frac{f''_{STV} (c - 1 + \gamma_{2}) n}{c} f''_{CTV} \]

\[ \alpha_{u} c T_{c} E_{c} (ct/2) + \alpha_{u} (c - 1 + \gamma_{2}) T_{n} E_{c} \gamma_{1} \rho t \]

\[ c^{2} + 2 \rho \gamma_{1} \rho c + 2 \rho \gamma_{1} \rho (\gamma_{2} - 1) + 2 \rho n \rho \gamma_{2} = 0 \]

\[ c^{2} + 2 \rho n (\gamma_{1} + 1) c + 2 \rho n [\gamma_{1}(\gamma_{2} - 1) - \gamma_{2}] = 0 \]

\[ c^{2} + 2 \rho n (\gamma_{1} + 1) c - 2 \rho n [\gamma_{1} + \gamma_{1}(1 - \gamma_{2})] = 0 \]

\[ c = -\rho n (\gamma_{1} + 1) + \]
The derivation for the equations for the maximum horizontal stresses in concrete and steel due to a temperature drop only, across the concrete wall with two layers of reinforcement, is similar to that for the vertical temperature stresses. Replace $\rho$ with $\rho'$, $f'_{CTV}$ with $f''_{CTC}$, $f_{STV}$ with $f'_{STC}$, $c$ with $c'$, $\gamma_1$ with $\gamma_1'$, $\gamma_2$ with $\gamma_2'$, then

$$f'_{CTC} = \alpha_{ctc}T_sE_c$$

$$f_{STC} = \alpha_{stc}(\gamma'_2 - c')T_sE_s$$

$$c' = -\rho'\gamma'_1(1 + \frac{2\rho'(\gamma_1' + 1)^2}{\sqrt{[\rho'(\gamma_1' + 1)]^2 + 2\rho'(\gamma_2' + \gamma_1'(1 - \gamma_2')}}}$$

### APPENDIX C—REFERENCES

#### C.1—Recommended references

**American Concrete Institute**

307R-69 Specification for the Design and Construction of Reinforced Concrete Chimneys
307R-88 Standard Practice for the Design and Construction of Cast-in-Place Reinforced Concrete Chimneys
318 Building Code Requirements for Structural Concrete
505-54 Standard Specification for the Design and Construction of Reinforced Concrete Chimneys
550R-93 Design Recommendations for Precast Concrete Structures

**American Society of Civil Engineers**

ASCE 7-88 Minimum Design Loads for Buildings and Other Structures (formerly ANSI A58.1)
ASCE 7-95 Minimum Design Loads for Buildings and Other Structures

**American Concrete Institute**

P.O. Box 9094
Farmington Hills, Mich. 48333-9094

**American Society of Civil Engineers**

1801 Alexander Bell Drive
Reston, Va. 20191

#### C.2—Cited references

1. PCI Manual for Structural Design of Architectural Precast Concrete, Prestressed Concrete Institute, 1977.
3. Warnes, C. E., “Precast Concrete Connection Details for All Seismic Zones,” Concrete International, V. 14, No. 11, Nov. 1992, pp. 36-44.