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State-of-the-Art Report on Partially Prestressed Concrete

Reported by Joint ACI-ASCE Committee 423

Partially prestressed concrete construction uses prestressed, or a combination of prestressed and nonprestressed, reinforcement. Partially prestressed concrete falls between the limiting cases of conventionally reinforced concrete and fully prestressed concrete, which allows no flexural tension under service loads. When flexural tensile stresses and cracking are allowed under service loads, the prestressed members have historically been called partially prestressed. This report is presented as an overview of the current state of the art for partial prestressing of concrete structures. Research findings and design applications are presented. Specific topics discussed include the history of partial prestressing, behavior of partially prestressed concrete members under static loads, time-dependent effects, fatigue, and the effects of cyclic loadings.

Keywords: bridges; buildings; concrete construction; corrosion; cracking; crack widths; cyclic loading; deflections; earthquake-resistant structures; fatigue; partially prestressed concrete; post-tensioning; prestressing; prestress losses; shear; stresses; structural analysis; structural design; time-dependent effects; torsion.

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CHAPTER 1—INTRODUCTION
1.1—Historical perspective
Application of prestressing to concrete members imparts a compressive force of an appropriate magnitude at a suitable location to counteract the service-load effects and modifies the structural behavior of the members. Although the concept of prestressed concrete was introduced almost concurrently in the U.S. and in Germany before the turn of the 20th century (Lin and Burns 1981), its principle was not fully established until Freyssinet published his classical study (Freyssinet 1933). Freyssinet recognized that as the load on a prestressed member is increased, flexural cracks would appear in the tensile zones at a certain load level, which he referred to as the transformation load. Even though the cracks would close as the load was reduced and the structure would recover its original appearance, Freyssinet advocated avoiding cracks under service load so that the concrete would behave as a homogeneous material.

A different design approach, however, was proposed by von Emperger (1939) and Abeles (1940). They suggested using a small amount of tensioned high-strength steel to control deflection and crack width while permitting higher working stresses in the main reinforcement of reinforced concrete. Most of the early work in support of this design concept was done by Abeles (1945) in England. Based on his studies, Abeles determined that eliminating the tensile stress and possible cracking in the concrete is unnecessary in many designs. Abeles also realized that prestress can be applied to counteract only part of the service load so that tensile stress, or even hairline cracks, occur in the concrete under full service load. Abeles did specify that under dead load only, no flexural tension stress should be allowed at any member face where large flexural tensile stresses occurred under maximum load, so as to ensure closure of any cracks that may have occurred at maximum load. Additional bonded and well-distributed nonprestressed reinforcement could be used to help control cracking and provide the required strength. Abeles termed this design approach as “partially prestressed concrete.” Therefore, the design approach advocated by Freyssinet was then termed as “fully prestressed concrete.” In actual practice, nearly all prestressed concrete components designed today would be “partially prestressed” as viewed by Freyssinet and Abeles.

Interest in partial prestressing continued in Great Britain in the 1950s and early 1960s. Many structures were designed by Abeles based on the principle of partial prestressing, and examinations of most of these structures around 1970 revealed no evidence of distress or structural deterioration, as discussed in the technical report on Partial Prestressing published by the Concrete Society (1983). Partially prestressed concrete design was recognized in the First Report on Prestressed Concrete published by the Institution of Structural Engineers (1951). Provisions for partial prestressing were also included in the British Standard Code of Practice for Prestressed Concrete (CP 115) in 1959. In that code, a permissible tensile stress in concrete as high as 750 psi (5.2 MPa) was accepted when the maximum working load was exceptionally high in comparison with the load normally carried by the structure. Presently, the British Code (BS 8110) as well as the Model Code for Concrete Structures (1978), published by CEB-FIP, defines three classes of prestressed concrete structures:

Class 1—Structures in which no tensile stress is permitted in the concrete under full service load;

Class 2—Structures in which a limited tensile stress is permitted in the concrete under full service load, but there is no visible cracking; and
Class 3—Structures in which cracks of limited width (0.2 mm [0.008 in.]) are permitted under full service load. Calculations for Class 3 structures would be based on the hypothetical tensile stress in the concrete assuming an uncracked section. The allowable values of the hypothetical tensile stress vary with the amount, type, and distribution of the prestressed and nonprestressed reinforcement.

Elsewhere in Europe, interest in partial prestressing also developed in the 1950s and 1960s. In the mid-1950s, many prestressed concrete structures in Denmark, especially bridges, were designed using the partial prestressing concept. Their performance was reported as satisfactory after 25 years of service (Rostam and Pedersen 1980). In 1958, the first partially prestressed concrete bridge in Switzerland (Weinland Bridge) was completed near Zurich. Provisions for partial prestressing were introduced in SIA Standard 162, issued by the Swiss Society of Engineers and Architects (1968), and since 1960, more than 3000 bridges have been designed according to this concept with highly satisfactory results (Birkenmaier 1984). Unlike the British Code and CEP-FIP Model Code, the limit of partial prestressing in the Swiss Code was not defined by the hypothetical tensile stress. Instead, it was defined by the tensile stress in the prestressed and nonprestressed reinforcement, and calculated using the cracked section. Under full service load, the allowable stress in the nonprestressed reinforcement was 22,000 psi (150 MPa), and in railroad bridges, the stress increase in the prestressed reinforcement was not to exceed 1/20 of the tensile strength. This value was taken as 1/10 of the tensile strength in other structures. It was required, however, that the concrete be in compression when the structure supported only permanent load.

In the U.S., the design of prestressed concrete in the early 1950s was largely based on the Criteria for Prestressed Concrete Bridges (1954) published by the Bureau of Public Roads, which did not permit tensile stress and cracking in concrete under service loads. The ACI-ASCE Joint Committee 323 report (1958), however, recognized that “complete freedom from cracking may or may not be necessary at any particular load stage.” For bridge members, tensile stress was not allowed in concrete subjected to full service load. For building members not exposed to weather or corrosive atmosphere, a flexural tension stress limit of $6\sqrt{f'_c}$ psi* was specified with the provision that the limit may be exceeded if “it is shown by tests that the structure will behave properly under service load conditions and meet any necessary requirements for cracking load or temporary overload.” Thus, partial prestressing was permitted in that first definitive design guide for prestressed concrete, and designers were quick to embrace the idea. When the balanced load design concept was published by Lin (1963), it provided a convenient design tool and encouraged the practical application of partial prestressing.

In 1971, the first edition of the PCI Design Handbook was published. Design procedures allowing tension stresses are illustrated in that guide. The second edition (1978) mentioned the term “partial prestressing,” and by the third edition (1985), design examples of members with combined prestressed and nonprestressed reinforcement were included. Presently, ACI 318 permits a tensile stress limit of $12\sqrt{f'_c}$ psi with requirements for minimum cover and a deflection check. Section 18.4.3 of ACI 318 permits the limit to be exceeded on the basis of analysis or test results. Bridge design guidelines or recommendations, however, did not follow the development until the publication of the Final Draft LRFD Specifications for Highway Bridges Design and Commentary (1993), even though most bridge engineers had been allowing tension in their designs for many years.

The concept of partial prestressing was developed half a century ago. Over the years, partial prestressing has been accepted by engineers to the extent that it is now the normal way to design prestressed concrete structures. Bennett’s work (1984) provides a valuable historical summary of the development of partially prestressed concrete.

1.2—Definition

Despite a long history of recognition of the concept of partial prestressing, both in the U.S. and abroad, there has been a lack of a uniform and explicit definition of the term, “partial prestressing.” For example, Lin and Burns (1981) state: “When a member is designed so that under the working load there are no tensile stresses in it, then the concrete is said to be fully prestressed. If some tensile stresses will be produced in the member under working load, then it is termed partially prestressed.” On the other hand, Naaman (1982a) states: “Partial prestressing generally implies a combination of prestressed and nonprestressed reinforcement, both contributing to the resistance of the member. The aim is to allow tension and cracking under full service loads while ensuring adequate strength.” According to Nilson (1987), “Early designers of prestressed concrete focused on the complete elimination of tensile stresses in members at normal service load. This is defined as full prestressing. As experience has been gained with prestressed concrete construction, it has become evident that a solution intermediate between full prestressed concrete and ordinary reinforced concrete offers many advantages. Such an intermediate solution, in which a controlled amount of concrete tension is permitted at full service, is termed partial prestressing.”

A unified definition of the term “partial prestressing” should be based on the behavior of the prestressed member under a prescribed loading. Therefore, this report defines partial prestressing as: “An approach in design and construction in which prestressed reinforcement or a combination of prestressed and non-prestressed reinforcement is used such that tension and cracking in concrete due to flexure are allowed under service dead and live loads, while serviceability and strength requirements are satisfied.”

For the purposes of this report, fully prestressed concrete is defined as concrete with prestressed reinforcement and no flexural tension allowed in the concrete under service loads. Conventionally reinforced concrete is defined as concrete with no prestressed reinforcement and generally, there is

* In this report, when formulas or stress values are taken directly from U.S. codes and recommendations, they are left in U.S. customary units.
flexural tension in concrete under service loads. Partially prestressed concrete falls between these two limiting cases. Serviceability requirements include criteria for crack widths, deformation, long-term effects (such as creep and shrinkage), and fatigue.

By the previous definition, virtually all prestressed concrete that uses unbonded tendons is “partially prestressed,” as codes require that a certain amount of bonded reinforcement be provided to meet strength requirements. Most prestressed members used in routine applications such as building decks and frames, and bridges spanning to approximately 100 ft (30 m) will allow flexural tension under full service load. The addition of nonprestressed reinforcement is used only in special situations, such as unusually long spans or high service loads, or where camber and deflection control is particularly important.

1.3—Design philosophy of partial prestressing

The basic design philosophy for partial prestressing is not different from that of conventionally reinforced concrete or fully prestressed concrete. The primary objective is to provide adequate strength and ductility under factored load and to achieve satisfactory serviceability under full service load.

By permitting flexural tension and cracking in concrete, the designer has more latitude in deciding the amount of prestressing required to achieve the most desirable structural performance under a particular loading condition. Therefore, partial prestressing can be viewed as a means of providing adequate control of deformation and cracking of a prestressed member. If the amount of prestressed reinforcement used to provide such control is insufficient to develop the required strength, then additional nonprestressed reinforcement is used.

In the production of precast, pretensioned concrete members, serviceability can be improved by placing additional strands, as this is more economical than placing reinforcing bars. When this technique is used, the level of initial prestress in some or all of the strands is lowered. This is also a useful technique to keep transfer stresses below the maximum values prescribed by codes. At least for purposes of shear design, the ACI Building Code treats any member with effective prestress force not less than 40% of the tensile strength of the flexural reinforcement as prestressed concrete.

1.4—Advantages and disadvantages of partial prestressing

In the design of most building elements, the specified live load often exceeds the normally applied load. This is to account for exceptional loading such as those due to impact, extreme temperature and volume changes, or a peak live load substantially higher than the normal live loads. By using partial prestressing, and by allowing higher flexural tension for loading conditions rarely imposed, a more economical design is achieved with smaller sections and less reinforcement.

Where uniformity of camber among different members of a structure is important, partial prestressing will enable the designer to exercise more control of camber differentials. In multispans bridges, camber control is important in improving riding comfort as a vehicle passes from one span to the next. The relatively large mild steel bars used in partially prestressed members result in a transformed section that can be significantly stiffer than a comparable section that relies solely on prestressing strand, thus reducing both camber and deflection.

Nonprestressed reinforcement used in partially prestressed members will enhance the strength and also control crack formation and crack width. Under ultimate load, a partially prestressed member usually demonstrates greater ductility than a fully prestressed member. Therefore, it will be able to absorb more energy under extreme dynamic loading such as an earthquake or explosion.

Because mild steel does not lose strength as rapidly as prestressing strands at elevated temperature, it is sometimes added to prestressed members to improve their fire-resistance rating. See Chapter 9 of the PCI Design Handbook (1992) and Design for Fire Resistance of Precast Prestressed Concrete (1989) for more information.

Partial prestressing is not without some disadvantages. Under repeated loading, the fatigue life of a partially prestressed member can be a concern. In addition, durability is a potential problem for partially prestressed members because they can be cracked under full service load. Recent studies (Harajli and Naaman 1985a; Naaman 1989; and Naaman and Founas 1991), however, have shown that fatigue strength depends on the range of stress variation of the strand (refer to Chapter 4) and that durability is related more to cover and spacing of reinforcement than to crack width, so these concerns can be addressed with proper design and detailing of the reinforcement (Beeby 1978 and 1979).

1.5—Partial prestressing and reinforcement indexes

Several indexes have been proposed to describe the extent of prestressing in a structural member. These indexes are useful in comparing relative performances of members made with the same materials, but caution should be exercised in using them to determine absolute values of such things as deformation and crack width. Two of the most common indexes are the degree of prestress λ, and the partial prestressing ratio (PPR). These indexes are defined as

$$\lambda = \frac{M_{\text{dec}}}{M_{D} + M_{L}}$$  

where

- $M_{\text{dec}}$ = decompression moment (the moment that produces zero concrete stress at the extreme fiber of a section, nearest to the centroid of the prestressing force, when added to the action of the effective prestress alone);
- $M_{D}$ = dead-load moment; and
- $M_{L}$ = live-load moment

and
The objective of this report is to summarize the state of the art of the current knowledge as well as recent developments in partial prestressing so that engineers who are not experienced in prestressed concrete design will have a better understanding of the concept.
provide unified treatment for cracked reinforced, prestressed, and partially prestressed sections.

2.2.2 Strength analysis—At ultimate or nominal moment resistance, the assumptions related to the stress and strain distributions in the concrete, such as the compression block in ACI 318, or the stress and strain in the steel (such as yielding of the reinforcing steel) are identical for reinforced, prestressed, and partially prestressed concrete (Fig. 2.2). The corresponding analysis is the same and leads to the nominal moment resistance of the section. Numerous investigations have shown close correlation between the predicted (based on ACI 318) and experimental values of nominal moments. The ACI 318 analysis, however, resulted in conservative predictions of section curvatures at ultimate load, leading to erroneous estimates of deformations and deflections (Wang et al. 1978, Naaman et al. 1986). To improve the prediction of nominal moment and curvature, either a nonlinear or a simplified nonlinear analysis may be followed.

Simplified nonlinear analysis—In the simplified nonlinear analysis procedure (also called pseudo-nonlinear analysis), the actual stress-strain curve of the steel reinforcement is considered while the concrete is represented by the ACI 318 compression block. A solution can be obtained by solving two nonlinear equations with two unknowns, namely the stress and the strain in the prestressing steel at nominal moment resistance (Naaman 1977, Naaman 1983b).

Nonlinear analysis—The best accuracy in determining nominal moments and corresponding curvatures is achieved through a nonlinear analysis procedure (Cohn and Bartlett 1982, Naaman et al. 1986, Harajli and Naaman 1985b, Moustafa 1986). Nonlinear analysis requires as input an accurate analytical representation of the actual stress-strain curves of the component materials (concrete, reinforcing steel, and prestressing steel). Typical examples can be found in two references (Naaman et al. 1986, Moustafa 1986).

2.3—Cracking

Partially prestressed concrete permits cracking under service loads as a design assumption. To satisfy serviceability requirements, the maximum crack width should be equal to, or smaller than, the code-recommended limits on crack width.

The maximum allowable crack widths recommended by ACI Committee 224 (1980) for reinforced concrete members can be used, preferably with a reduction factor for prestressed and partially prestressed concrete members. To select the reduction factor, consideration should be given to the small diameter of the reinforcing elements (bars or strands), the cover, and the exposure conditions.

Only a few formulas are used in the U.S. practice to predict crack widths in concrete flexural members. Because the factors influencing crack widths are the same for reinforced and partially prestressed concrete members, existing formulas for reinforced concrete can be adapted to partially prestressed concrete. Five formulas (ACI 224 1980; Gergely and Lutz 1968; Nawy and Potyondy 1971; Nawy and Huang 1977; Nawy and Chiang 1980; Martino and Nilson 1979; and Meier and Gergely 1981) applicable to partially prestressed beams are summarized in Table 2.1 (Naaman 1985). The vari-

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Fig. 2.1—Assumed stress or strain distribution in linear elastic analysis of cracked and uncracked sections (Naaman 1985).

Fig. 2.2—Assumed strain distribution and forces in: (a) nonlinear analysis; (b) approximate nonlinear analysis; and (c) ultimate strength analysis by ACI Code (Naaman 1985).

1986). They usually are third-order equations with respect to member depth. Although they can be solved iteratively, charts, tables, and computer programs have been developed for their solution (Tadros 1982, Moustafa 1977). These equations
### Table 2.1—Crack width prediction equations applicable to partially prestressed beams (Naaman 1985)

<table>
<thead>
<tr>
<th>Source</th>
<th>Equation* with U.S. system, (in., ksi)</th>
<th>Equation* with SI system, (mm, N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gergely and Lutz (1968)</td>
<td>[ W_{\text{max}} = 7.6 \times 10^{-14} f_s^2 d_c A_b ]</td>
<td>Multiply expression by 0.1451</td>
</tr>
<tr>
<td>ACI Code (1971, 1977, and 1983)</td>
<td></td>
<td>Note: ACI Committee 224 recommends multiplication factor of 1.5 when strands, rather than deformed bars, are used nearest to beam tensile face.</td>
</tr>
<tr>
<td>ACI Committee 224 (1980)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Navy and Potyondy (1971)</td>
<td>[ W_{\text{max}} = 1.44 \times 10^{-6} (f_s - 8.3) ]</td>
<td>[ W_{\text{max}} = 5.31 \times 10^{-4} (f_s - 57.2) ]</td>
</tr>
<tr>
<td>Navy and Huang (1977)</td>
<td>[ W_{\text{max}} = \alpha \times 10^{-5} \frac{A_f}{\sum O} \Delta f_{ps} ]</td>
<td>Multiply expression by 0.1451</td>
</tr>
<tr>
<td>Navy and Chiang (1980)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Martino and Nilson (1979)</td>
<td>[ W_{\text{max}} = 14 \times 10^{-3} d_c' f_s + 0.0031 ]</td>
<td>[ W_{\text{max}} = 2 \times 10^{-5} d_c' f_s + 0.08 ]</td>
</tr>
<tr>
<td>Meier and Gergely (1981)</td>
<td>[ \begin{align*} W_{\text{max}} &amp;= C_1 \varepsilon_{ct} d_c' \ W_{\text{max}} &amp;= C_2 \varepsilon_{ct} d_c' \sqrt{A_b} \end{align*} ]</td>
<td>(1) Same equation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) Multiply by 220</td>
</tr>
</tbody>
</table>

*In the formulas shown, \( f_s \) can be replaced by \( \Delta f_{ps} \) when applied to partially prestressed concrete.

able tensile stress in the reinforcing steel \( f_s \) should be replaced by the stress change in the prestressing steel after decompression \( \Delta f_{ps} \). The ACI 318 formula initially developed by Gergely and Lutz (1968) for reinforced concrete could be used as a first approximation for partially prestressed concrete. Meier and Gergely (1981), however, suggested a modified form (shown in Table 2.1) for the case of prestressed concrete. This alternate formula uses the nominal strain at the tensile face of the concrete (instead of the stress in the steel), and the cover to the center of the steel \( d_c' \). Both the stress in the steel and the clear concrete cover are found to be the controlling variables in the regression equation derived by Martino and Nilson (1979). The two prediction equations proposed by Nawy and Huang (1977) and Nawy and Chiang (1980) contain most of the important parameters found in the cracking behavior of concrete members except the concrete cover, which is accounted for indirectly. Moreover, they are based on actual experimental results on prestressed and partially prestressed beams.

As pointed out by Siriaksorn and Naaman (1979), large differences can be observed in predicted crack widths depending on the prediction formula used. Harajli and Naaman (1989) compared predicted crack widths with observed crack widths from tests on twelve partially prestressed concrete beams. They considered the three prediction equations recommended by Gergely and Lutz (1968), Nawy and Huang (1977), and Meier and Gergely (1981). Although none of the three equations gave sufficiently good correlation with experimental data for all conditions, the following observations were made (Fig. 2.3):

- The Gergely and Lutz equation gave a lower prediction in all cases (Fig. 2.3(a));
- The Meier and Gergely equation gave the worst correlation (Fig. 2.3(c)); and
- The Nawy and Huang equation gave a higher prediction in most cases (Fig. 2.3(b)).

Although more experimental data are needed to improve the accuracy of crack-width prediction equations available in U.S. practice, there is sufficient information to judge if the serviceability, with respect to cracking or crack width under short-term loading, is satisfactory for a partially prestressed member. The effects of long-term loading and repetitive loading (fatigue) on the crack widths of partially prestressed members need to be further clarified. A research investigation provided an analytical basis to deal with the problem (Harajli and Naaman 1989); however, the proposed methodology is not amenable to a simple prediction equation that can be easily implemented for design.

### 2.4—Deflections

Fully prestressed concrete members are assumed to be uncracked and linearly elastic under service loads. Instantaneous short-term deflections are determined using general
principles of mechanics. To compute short-term deflections, customary U.S. practice is to use the gross moment of inertia \( I_g \) for pretensioned members, or the net moment of inertia \( I_n \) for members with unbonded tendons, and the modulus of elasticity of concrete at time of loading or transfer \( E_{ci} \).

Several approaches proposed by various researchers to compute short-term and long-term deflections in prestressed or partially prestressed uncracked members are summarized in Table 2.2 (Branson and Kripanarayanan 1971; Branson 1974; Branson 1977; Naaman 1982a; Naaman 1983a; Branson and Trost 1982a; Branson and Trost 1982b; Martin 1977; Tadros et al. 1975; Tadros et al. 1977; Dilger 1982; and Moustafa 1986). Although no systematic evaluation or comparison of these different approaches has been undertaken, for common cases they lead to results of the same order.

The widely accepted concept of the effective moment of inertia \( I_{eff} \), initially introduced by Branson (1977) for reinforced concrete, has been examined by several researchers and modified accordingly to compute the deflection in cracked prestressed and partially prestressed members. The modified effective moment of inertia is defined (Naaman 1982a) as

\[
I_{eff} = I_{cr} + \left( \frac{(M_{cr} - M_{dec})}{(M_a - M_{dec})} \right)^3
\]

where

\[
\begin{align*}
I_g & = \text{gross moment of inertia, in.}^4 \text{ (mm}^4) ; \\
I_{cr} & = \text{moment of inertia of cracked section, in.}^4 \text{ (mm}^4) ; \\
M_{cr} & = \text{cracking moment, in.-k (mm-N)} ; \\
M_{dec} & = \text{decompression moment, in.-k (m-N)} ; \text{ and} \\
M_a & = \text{applied moment, in.-k (m-N)} .
\end{align*}
\]

Although there is general agreement for the use of the previous expression, substantial divergence of opinion exists as to the computation of \( I_{cr} \) and \( M_{dec} \). The computation difference is whether the moment of inertia of the cracked section should be computed with respect to the neutral axis of bending or with respect to the zero-stress point, and whether the decompression moment should lead to decompression at the extreme concrete fiber or whether it should lead to a state of zero curvature in the section. The discussion of Tadros' paper (1982) by several experts in the field is quite informative on these issues. A systematic comparison between the various approaches, combined with results from experimental tests, is given in work by Watcharaumnuay (1984), who observed that the use of \( I_{cr} \) with respect to the neutral axis of bending is preferable, while the use of \( M_{dec} \) as that causing decompression at the extreme concrete fiber, is easier and leads to results similar to those obtained using the zero curvature moment.

### 2.5—Shear and torsion

#### 2.5.1 General

Nonprestressed and fully prestressed concrete (tensile stress in the concrete under full service load is zero) are the two limiting cases of steel-reinforced concrete systems. Partially prestressed concrete represents a continuous transition between the two limit cases. A unified approach in design to combined actions including partial prestressing would offer designers a sound basis to make the appropriate choice between the two limits (Thurlimann 1971).

The equivalent load concept provides a simple and efficient design of prestressed concrete structures under combined actions (Nilson 1987). For example, this approach allows the designer to calculate the shear component of the prestress anywhere in the beam, simply by drawing the shear diagram due to the equivalent load resulting from a change in the vertical alignment of the tendon (Fig. 2.4). That equivalent load, together with the prestressing forces acting at the ends of the member through the tendon anchorage, may be looked upon as just another system of external forces acting on the member. This procedure can be used for both statically determinate and indeterminate structures, and it accounts for the effects of secondary reactions due to
### Table 2.2—Deflection prediction equations for prestressed and partially prestressed beams (from Naaman 1985)

<table>
<thead>
<tr>
<th>Source</th>
<th>Short-term instantaneous deflection</th>
<th>Long-term or additional long-term deflection</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| ACI 435 (1963)          | $\Delta_t$ is obtained from elastic analysis using $F_e$, $E_{ct}$, and $I_g$ | Long-term deflection obtained by integrating curvatures with due account for creep effects and prestress losses with time. | - Uncracked section; and  
- No provisions for $A_s$ and $A_s'$. |
| ACI Code Section 9.5    | $\Delta_t$ shall be obtained from elastic analysis using $I_g$ for uncracked sections. | $\Delta_{add}$ shall be computed, taking into account stresses under sustained load, including effects of creep, shrinkage, and relaxation. | - No provisions for partial prestressing (cracking, $A_s$, and $A_s'$). |
| Branson et al. (1971, 1974, and 1977) | $\Delta_t$ is obtained from elastic analysis using $E_{ct}$ and $I_g$. | $\Delta_{add} = \left[\eta - 1 + \left(\frac{1 + \eta}{2}\right) k_s C_n\right]$ where $k_s = 1/(1 + A_s/A_s')$; and $C_n = F_t/F_c$. | - Uncracked section;  
- The pressure line is assumed resulting from the sustained loadings;  
- The profile of the pressure line is assumed parabolic;  
- Prestress losses must be estimated a priori;  
- Design chart is provided for the equivalent modulus; and  
- $A_s$ and $A_s'$ are accounted for through $I_g$ and neutral axis of bending. |
| Naaman (1982 and 1983)  | $\Delta_t$ is obtained using $E_p$ and the predicted elastic modulus at time of loading $E_e(t)$. | The long-term deflection is estimated from: $\Delta(t) = \phi_1(t)\frac{I^2}{8} + \left[\phi_2(t) - \phi_1(t)\right] \frac{I^2}{48}$ where $\phi_1(t)$ = midspan curvature at time $t$; $\phi_2(t)$ = support curvature at time $t$; $\phi(t) = M/E_e(t) \times I$; and $E_{eq}(t)$ = equivalent modulus. | - Cracked members. |
| Bronson and Trost (1982) | For cracked members, the short-term deflection is computed using $I_d$ modified for partial prestressing. | Long-term deflection is not addressed but it is assumed that for a given $\Delta_t$ the earlier method is applicable. | - Uncracked sections;  
- The pressure line is assumed resulting from the sustained loadings;  
- The profile of the pressure line is assumed parabolic;  
- Prestress losses must be estimated a priori;  
- Design chart is provided for the equivalent modulus; and  
- $A_s$ and $A_s'$ were recommended; and  
- The method is adopted in PCI Design Handbook. |
| Martin (1977)           | $\Delta_t$ is obtained from elastic analysis using $E_{ct}$ and $I_g$. | $\Delta_{add} = \lambda_1(\Delta)_e + \lambda_2(\Delta)_f$ where $\lambda_1 = k_s E_{ce} \alpha$; $\lambda_2 = (2 - 1.2 A_s' / A_s) \geq 0.6$; and $\lambda_1 = \eta_1 \lambda_1$. | - $k_s$ same as Branson;  
- Uncracked section;  
- Design values of $\lambda_1$ and $\lambda_2$ were recommended; and  
- The method is adopted in PCI Design Handbook. |
| Tadros et al. (1975 and 1977) | $\Delta_t$ is obtained from elastic analysis using $E_e(t)$ and $I_g$. | The long-term deflection is obtained by integrating the curvatures modified by a creep recovery parameter and a relaxation reduction factor that are time-dependent. | - Uncracked sections; and  
- For common loading cases, only the curvatures at the support and midspan sections are needed. |
| Dilger (1982)           | $\Delta_t$ is obtained from long-term deflection expression at initial loading time. The age adjusted effective modulus and a creep transformed moment of inertia are used. | The long-term deflection is obtained by integrating the curvature along the member. The time-dependent curvature is modified by the effect of an equivalent force acting at the centroid of the prestressing steel due to creep and shrinkage strain. $\phi(t) = \phi_c C_n(t) - \frac{M_s}{I_p E_{eq}(t)}$ where $I_p$ is transformed moment of inertia; $M_s$ is moment due to equivalent transformed force; and $E_{eq}(t)$ = age adjusted modulus. | - Uncracked sections; and  
- A relaxation reduction factor is used. |
| Moustafa (1986)         | $\Delta_t$ is obtained from nonlinear analysis using actual material properties. | The nonlinear analysis takes both creep and shrinkage into account, using ACI creep and shrinkage functions and a time step method. | - A computer program is available from PCI to perform the nonlinear analysis. |
prestressing, as well. This approach allows the designer to treat a prestressed concrete member as if it was a nonprestressed concrete member. The prestressing steel is treated as mild (passive) reinforcement for ultimate conditions, with a remaining tensile capacity of \( f_{ps} - f_{pe} \), where \( f_{ps} \) is the stress in the reinforcement at nominal strength, and \( f_{pe} \) is the effective stress in the prestressed reinforcement (after allowance for all losses).

Most codes of practice (ACI 318; AASHTO Bridge Design Specifications, Eurocode 2; and CSA Design of Concrete Structures for Buildings) use sectional methods for design of conventional beams under bending, shear, and torsion. Truss models provide the basis for these sectional design procedures that often include a term for the concrete contribution (Ramirez and Breen 1991). The concrete contribution supplements the sectional truss model to reflect test results in beams and slabs with little or no shear reinforcement and to ensure economy in the practical design of such members.

In design specifications, the concrete contribution has been taken as either the shear force or torsional moment at cracking, or as the capacity of an equivalent member without transverse reinforcement. Therefore, detailed expressions have been developed in terms of parameters relevant to the strength of members without transverse reinforcement. These parameters include the influence of axial compression, member geometry, support conditions, axial tension, and prestress.

2.5.2 Shear—The following behavioral changes occur in partially prestressed members at nominal shear levels, as some of the longitudinal prestressing steel in the tension face of the member is replaced by mild reinforcement, but the same total flexural strength is maintained:

- Due to the lower effective prestress, the external load required to produce inclined cracking is reduced. This results in an earlier mobilization of the shear reinforcement; and
- After inclined cracking, there is a reduction in the concrete contribution. The reduction is less significant as the degree of prestressing decreases. This can be explained as follows:
  - The addition of mild reinforcement results in an increase in the cross-sectional area of the longitudinal

![Fig. 2.4—Equivalent loads and moments produced by prestressing tendons (Nilson 1987); \( P = \) prestressing force.](image-url)
tension reinforcement and the reinforcement stiffness; and

- The increase in stiffness of the longitudinal tension reinforcement delays the development of the cracking pattern, so that the cracks are narrower and the flexural compression zone is larger than in fully prestressed members of comparable flexural strength.

These behavioral changes are well documented in a series of shear tests carried out by Caflisch et al. (1971). In this series of tests, the only variable was the degree of prestressing. The cross sections of the prestressing steel and the reinforcing steel were selected so that all the beams had the same flexural strength. These tests also showed that for the same external load, a higher degree of prestressing delays the onset of diagonal cracking and results in a decrease in the stirrup forces. The decrease in stirrup forces can be explained by the fact that a higher degree of prestressing in the web of the member results in a lower angle of inclination of the diagonal cracks. The lower angle of inclination of the cracks leads to the mobilization of a larger number of stirrups.

In ACI 318, a cursory review of the design approach for shear indicates that partially prestressed members can be designed following the same procedure as for fully prestressed members. In ACI 318, it is assumed that flexure and shear can be handled separately for the worst combination of flexure and shear at a given section. The analysis of a beam under bending and shear using the truss approach clearly indicates that, to resist shear, the member needs both stirrups and longitudinal reinforcement. The additional longitudinal tension force due to shear can be determined from equilibrium conditions of the truss model as \( V \cot \theta \), where \( V \) is the shear force at the section, and \( \theta \) is the angle of inclination of the inclined struts with respect to the longitudinal axis of the member.

In the shear provisions of ACI 318, no explicit check of the shear-induced force in the longitudinal reinforcement is performed (Ramirez 1994). The difference between the flexural strength requirements for the prestress reinforcement and the ultimate tensile capacity of the reinforcement can be used to satisfy the longitudinal tension requirement. The 1994 AASHTO LRFD Bridge Design Specifications, in the section for shear design, includes a check for longitudinal reinforcement. These recommendations are based on a modified compression field theory (Vecchio and Collins 1986).

### 2.5.3 Torsion—ACI 318 includes design recommendation for the case of torsion or combined shear and torsion in prestressed concrete members. These provisions model the behavior of a prestressed concrete member before cracking as a thin-walled tube and after cracking using a space-truss model with compression diagonals inclined at an angle \( \theta \) around all faces of the member. For prestressed members, \( \theta \) can be taken equal to 37.5 degrees if the effective prestressing force is not less than 40% of the tensile strength of the prestressed reinforcement. For other cases, \( \theta \) can be taken equal to 45 degrees. This approach is based on the work carried out in the 1960s and 1970s by European investigators led by Thurlimann (1979). This work proposed a method supported by the theory of plasticity, in which a space truss with variable inclination of compression diagonals provides a lower-bound (static) solution.

This procedure is representative of the behavior of thin-walled tubes in torsion. For these members, the shear stresses induced by torsion can be determined using only equilibrium relationships. Because the wall of the tube is thin, a constant shear stress can be assumed across its thickness. In the longitudinal direction, equilibrium conditions dictate that the torsion-induced shear stresses be resisted by a constant shear flow around the perimeter of the section. For other sections before cracking, the strength in torsion can be computed from the elastic theory (de Saint-Venant 1956) or from the plastic theory (Nadai 1950). Rather than using these more complex approaches, an approximate procedure is used in ACI 318 based on the concept that most torsion is resisted by the high shear stresses near the outer perimeter of the section. In this approach, the actual cross section before cracking is represented by an equivalent thin-walled tube with a wall thickness \( t \) of

\[
t = 0.75 \frac{A_{cp}}{P_{cp}}
\]

where \( A_{cp} \) = area enclosed by outside perimeter of concrete cross section, and \( P_{cp} \) = outside perimeter of the concrete cross section. While the area enclosed by the shear flow path, \( A_p \), could be calculated from the external dimensions and wall thickness of the equivalent tube, it is reasonable to approximate it as equal to \( 2A_{cp}/3 \). Cracking is assumed to occur when the principal tensile stress reaches \( 4\sqrt{f_c'} \). For prestressed members, the cracking torque is increased by the prestress. A Mohr’s Circle analysis based on average stresses indicates that the torque required to cause a principal tensile stress equal to \( 4\sqrt{f_c'} \) is the corresponding cracking torque of a nonprestressed beam times

\[
\frac{1 + \frac{f_{pc}}{4\sqrt{f_c'}}}{\frac{1 + f_{pc}}{4\sqrt{f_c'}}}
\]

where \( f_{pc} \) in psi is the average precompression due to prestress at the centroid of the cross section resisting the externally applied loads or at the junction of web and flange if the centroid lies within the flange.

In ACI 318, the design approach for combined actions does not explicitly consider the change in conditions from one side of the beam to the other. Instead, it considers the side of the beam where shear and torsional effects are additive. After diagonal cracking, the concrete contribution of the shear strength \( V_c' \) remains constant at the value it has when there is no torsion, and the torsion carried by the concrete is taken as zero. The approach in ACI 318 has been compared with test results by MacGregor and Ghoneim (1995).

In the AASHTO LRFD Specifications, the modified compression field theory proposed for members under shear has
been extended to include the effects of torsion. Similar to the ACI 318 procedure, the AASHTO approach concentrates on the design of the side of the beam where the shear and torsional stresses are additive.

As in the case of shear, torsion leads to an increase in the tensile force on the longitudinal reinforcement. The longitudinal reinforcement requirement for torsion should be superimposed with the longitudinal reinforcement requirement for bending that acts simultaneously with the torsion. In ACI 318, the longitudinal tension due to torsion can be reduced by the compressive force in the flexural compression zone of the member. Furthermore, in prestressed beams, the total longitudinal reinforcement, including tendons at each section, can be used to resist the factored bending moment plus the additional tension induced by torsion at that section.

ACI 318 and AASHTO Specifications recognize that, in many statically indeterminate structures, the magnitude of the torsional moment in a given member will depend on its torsional stiffness. Tests have shown (Hsu 1968) that when a member cracks in torsion, its torsional stiffness immediately after cracking drops to approximately 1/5 of the value before cracking, and at failure can be as low as 1/16 of the value before cracking. This drastic drop in torsional stiffness allows a significant redistribution of torsion in certain indeterminate beam systems. In recognizing the reduction of torsional moment that will take place after cracking in the case of indeterminate members subjected to compatibility-induced torsion, ACI 318R states that a maximum factored torsional moment equal to the cracking torque can be assumed to occur at the critical sections near the edges of supports. This limit has been established to control the width of the torsional cracks at service loads.

**CHAPTER 3—TIME-DEPENDENT BEHAVIOR**

### 3.1—Prestress losses

The stress in the tendons of prestressed concrete structures decreases continuously with time. The total reduction in stress during the life span of the structure is termed “total prestress loss.” The total prestress loss consists of several components, which are generally grouped into instantaneous and time-dependent losses.

Instantaneous losses are due to elastic shortening, anchorage seating, and friction. They can be computed for partially prestressed concrete in a manner similar to that for fully prestressed concrete.

Time-dependent losses are due to shrinkage and creep of concrete and relaxation of prestressing steel. Several methods are available to determine time-dependent losses in prestressed concrete members: lump-sum estimate of total losses, lump-sum estimates of separate losses (such as loss due to shrinkage or creep), and calculation of losses by the time-step method. Because the numerical expressions—described, for instance, in the AASHTO’s Standard Specifications for Highway Bridges or the PCI Design Handbook (1992)—for the first two methods were developed assuming full prestressing, they are not applicable to partially prestressed members.

The combined presence of nonprestressed reinforcing bars and a lower level of prestress in partially prestressed concrete should lead to smaller time-dependent prestress losses than for fully prestressed concrete. Another significant factor is that a partially prestressed member can be designed as a cracked member under sustained loading. A computerized time-step analysis of the concrete and steel stresses along partially prestressed sections shows that the effect of creep of concrete on the stress redistribution between the concrete and steel tends to counteract the effect of prestress losses over time. This is particularly significant for members that are cracked under permanent loads. The result is illustrated in Fig. 3.1 (Watcharaumnuay and Naaman 1985), which was derived from the analysis of 132 beams with various values of the partially prestressed ratio (PPR) and the reinforcement index $\omega$. While time-dependent prestress losses can be as high as 14% for uncracked fully prestressed sections, they remain low for cracked sections up to relatively high values of the PPR. Fig. 3.1 also shows that for cracked sections, time-dependent prestress losses decrease with a decrease in PPR. Creep redistribution of force to reinforcing steel may reduce the precompression in the concrete.

An investigation under NCHRP Project 12-33 has addressed prestress losses in partially prestressed normal and high-strength concrete beams. This work was described by Naaman and Hamza (1993) and was adopted in the Final Draft LRFD Specifications for Highway Bridge Design and Commentary (Transportation Research Board 1993). It led to lump-sum estimates of time-dependent losses for partially prestressed beams that are assumed uncracked under the design sustained loading. These are summarized in Table 3.1 and can be used as a first approximation in design.

### 3.2—Cracking

Crack widths in reinforced and cracked partially prestressed concrete members subjected to sustained loads are known to increase with time. Bennett and Lee (1985) reported that crack widths increase at a fast rate during the early stages of loading then tend toward a slow steady rate of increase. This is not surprising because deflections and
camber increase with time. Crack widths, however, represent only localized effects, and their relative increase with time is not proportional to the increase of deflections.

No studies have been reported where an analytical model of crack width increase with time was developed. An investigation by Harajli and Naaman (1989), however, discussed in Chapter 4 of this report, has led to the development of a model to predict the increase in crack width under cyclic fatigue loading. The model accounts for the effect of change in steel stress due to cyclic creep of concrete in compression, the increase in slip due to bond redistribution, and concrete shrinkage.

3.3—Deflections

Several experimental investigations have dealt with the time-dependent deflection of partially prestressed concrete members (Bennett and Lee 1985; Bruggeling 1977; Jittawait and Tadros 1979; Lambotte and Van Nieuwenburg 1986; Abeles 1965; and Watcharaumnuay and Naaman 1985). Deflections and cambers in partially prestressed concrete members are expected to vary with time similarly to reinforced or fully prestressed concrete. When positive deflection (opposite to camber) is present, in all cases the deflection in partially prestressed beams falls between those of reinforced concrete and fully prestressed concrete beams (Fig. 3.2). A limited experimental study by Jittawait and Tadros (1979) also seems to confirm this observation.

A study of the long-term behavior of partially prestressed beams has been conducted at the Magnel laboratory in Belgium (Lambotte and Van Nieuwenburg 1986). Twelve partially prestressed beams with PPR of 0.8, 0.65, and 0.5 were either kept unloaded or were loaded with an equivalent full service load. For the unloaded beams, increased camber with time was generally observed for PPR = 0.8; for PPR = 0.65, the camber reached a peak value and then decreased with time, resulting in a practically level beam; and for PPR = 0.5, the initial camber decreased with time, resulting in a final downward deflection. For the loaded beams, deflections were observed in all cases and increased with time. These observations are illustrated in Fig. 3.3. After 2 years of loading, the ratio of additional deflection to the initial deflection was about 1.25 for pretensioned members and 1.5 for post-tensioned members.

Several analytical investigations have dealt with the time-dependent deflections of prestressed and partially prestressed beams assumed to be uncracked (Table 2.2). The evaluation of deflections for cracked, partially prestressed members has been conducted by several researchers (Branson and Shaikh 1985; Ghali and Tadros 1985; Tadros et al. 1985; Ghali and Favre 1986; Al-Zaid et al. 1988; Elbadry and Ghali 1989; Ghali 1989; and Founas 1989). Watcharaumnuay and Naaman (1985) proposed a method to determine time- and cyclic-dependent deflections in simply supported, partially prestressed beams in both the cracked and uncracked state. The time-dependent deflection is treated as a special case of cyclic deflection. Compared with the time-step method where the deflection is obtained from the summation of deflection increments over several time steps, this method involves the solution of a differential equation that accounts for the effects of time and cyclic loading.
intervals, this method leads to the deflection at any time \( t \) and cycle \( N \) directly. The method satisfies equilibrium and strain compatibility. Using a slightly different approach, the method proposed by Watcharaumnuay and Naaman (1985) was generalized by Al-Zaid et al. (1988) and Founas (1989), and was extended to include composite beams as well as noncomposite beams.

In dealing with time-dependent deflections, several time-dependent variables should be determined. These include the prestressing force, the moment of inertia of the section, and the equivalent modulus of elasticity of the concrete. The expression for the effective moment of inertia described in Eq. (2-1) can be used. In this expression, however, \( M_{cr} \) and \( I_{cr} \) are time-dependent variables because they depend on the value of the prestressing force and the location of the neutral axis (zero stress point along the section), both of which vary with time. The equivalent modulus of elasticity of the concrete depends on the variation of creep strain with time.

The following method can be used to estimate deflection at any time \( t \) in a cracked, simply supported beam

\[
\Delta(t) = \frac{K_D M}{E_{ce}(t)I_{eff}(t)} + \frac{K_F F(t)}{E_{ce}(t)I_{eff}(t)}
\]

where

\( K_D, K_F = \) constants depending on type of loading and steel profile;
\( M = \) sustained external moment at midspan;
\( F(t) = \) prestressing force at time \( t \);
\( I_{eff}(t) = \) effective moment of inertia of cracked section at time \( t \); and
\( E_{ce}(t) = \) equivalent elastic modulus of concrete at time \( t \).

The equivalent elastic modulus of concrete can be approximated by the following equation

\[
E_{ce}(t) = \frac{E_c(t)}{1 + C_c(t - t_A)}
\]

where

\( t = \) time or age of concrete;
\( t_A = \) age of concrete at time of loading;
\( E_c(t) = \) instantaneous elastic modulus of concrete at time \( t \);
and
\( C_c(t - t_A) = \) creep coefficient of concrete at time \( t \) when loaded at time \( t_A \).

Several expressions are available to predict the creep coefficient of concrete. The recommendations of ACI Committee 209 (1982) can be followed in most common applications.

### 3.4—Corrosion

The reinforcement in a fully prestressed member is better protected against corrosion than the reinforcement in a partially prestressed member. Cracks in partially prestressed beams are potential paths for the passage of corrosive agents. Although corrosion also occurs along uncracked sections, cracking can facilitate corrosion. Abeles (1945) suggested that corrosion of the prestressing steel in partially prestressed members can be mitigated by requiring that the member faces that are cracked under full service load be in compression under permanent (dead) loads. He demonstrated the effectiveness of this strategy in the behavior of beams partially prestressed using small-diameter wires that were used in the roof of an engine shed for steam locomotives. These beams successfully resisted a very corrosive atmosphere caused by the mixture of smoke and steam ejected onto them from the funnels of the locomotives.

Limiting the size of crack widths to reduce the probability of corrosion has been common practice in design. Later studies (ACI 222R-89), however, move away from this approach by pointing out that corrosion is due to many causes, most of which can proceed with or without cracking to be activated. Corrosion in prestressing steels is much more serious than corrosion in nonprestressed reinforcing steels. Prestressing steel is generally stressed to over 50% of its strength, making it susceptible to stress corrosion, and the diameter of individual prestressing steel wires is relatively small. Even a small, uniform corrosive layer or a corroded spot can progressively reduce the cross-sectional area of the steel and lead to wire failure.
Corrosion is mostly an electrochemical problem and should be treated accordingly. Precautions should be taken to prevent or to reduce prestressing steel corrosion.

ACI 423.3R addresses the historical causes of corrosion in unbonded tendons. The Post-Tensioning Manual (Post-Tensioning Institute 1990) provides guidance for corrosion protection for bonded and unbonded tendons. Occasionally, for pretensioned concrete, epoxy-coated prestressing strands have been specified for corrosive environments. High curing temperatures, however, could adversely affect the bond of epoxy-coated strand. Guidelines for the Use of Epoxy-Coated Strand (PCI Ad Hoc Committee 1993) contains recommendations for its use.

Lenschow (1986) reported that crack widths less than 0.004 to 0.006 in. (0.1 to 0.15 mm), which develop under maximum load, will heal under long-term compression. Crack widths that increase to less than 0.01 in. (0.3 mm) under rare overload (every 1 to 3 years) can reduce to 0.004 to 0.006 in. (0.1 to 0.15 mm) under sustained compression. Keeping crack widths under such limits should avoid problems with corrosion.

CHAPTER 4—EFFECTS OF REPEATED LOADING (FATIGUE)

4.1—Background

Two major requirements should be considered when designing members subjected to repeated loads: member strength and serviceability. The static strength and fatigue strength of the member should exceed loads imposed and adequate serviceability requirements (deflection and crack control) should be provided.

The fatigue strength of a member is affected primarily by the stress range (difference between maximum and minimum stress), the number of load applications or cycles, and the applied stress levels. The fatigue life of a member is defined as the number of load cycles before failure. The higher the stress range imposed on the member, the shorter the fatigue life.

Reliability analyses indicate that the probability of fatigue failure of reinforcement in partially prestressed beams is higher on average than failure by any other common serviceability or ultimate limit state criterion (Naaman 1985, Naaman and Siriaksorn 1982). Fatigue can be a critical loading condition for partially prestressed concrete beams because high stress ranges can be imposed on the member in the service-load range (Naaman and Siriaksorn 1979).

Partially prestressed concrete beams generally crack upon first application of live load (Naaman 1982b). Subsequent applications of live load cause the cracks to reopen at the decompression load (when the stress at the extreme tensile face is zero), which is less than the load that caused first cracking. To maintain equilibrium in the section after cracking, the neutral axis shifts toward the extreme compression fiber. This shift generates higher strains (stresses) in the tensile reinforcement.

Under repeated loads, the larger stress changes (created by opening and closing of the cracks) cause fatigue damage in the constituent materials, bond deterioration, and increased crack widths and deflections under service loads (Naaman 1982b; Shakawi and Batchelor 1986; and ACI Committee 215 1974). The increase in crack widths in partially prestressed beams, however, has been smaller than that generated in similarly loaded, precracked, fully prestressed beams (Harajli and Naaman 1984).

Because the proportion of dead to total load often increases as the span length increases, the significance of fatigue as a critical limit state tends to diminish as span lengths increase (Freyermuth 1985).

Abeles demonstrated the practicability of using partially prestressed concrete members when fatigue resistance is a serious consideration (Abeles 1954). He persuaded British Railways to consider the use of partial prestressing in the reconstruction of highway bridges over the London to Manchester line, when it was electrified around 1950. British Railways financed extensive cyclic loading tests of full-scale members, which were designed to allow 550 psi (approximately 8½f′c or 3.8 MPa) tension under full service load and 50 psi (0.34 MPa) compression under dead load only. These members were cracked under static load and were then subjected to 3 million cycles of load producing the design range of stress, 50 psi (0.34 MPa) compression to 550 psi (3.8 MPa) tension at the flexural tension face. Behavior was satisfactory, with essentially complete closure of cracks and recovery of deflection after 3 million cycles of load. The strength under static loading was not decreased by the cyclic loading. Many relatively short-span bridges were constructed using such partially prestressed members and they performed satisfactorily.

Recently, Roller et al. (1995) conducted an experimental program including four full-size, pretensioned, bulb-tee girders made with high-strength concrete and pretensioned. The girders were 70 ft (21.3 m) long and 54 in. (1.4 m) deep with a concrete compressive strength of 10,000 psi (69 MPa). One of the four test girders with a simple span of 69 ft (21.0 m) was subjected to cyclic (fatigue) flexural loading using two point loads spaced 12 ft (3.66 m) apart at midspan. A concrete deck 10 ft (3.05 m) wide and 9.5 in. (250 mm) thick had been cast on the girder to represent the effective flange of the composite girder in a bridge.

During the cyclic flexural loading, the upper limit of the load produced a midspan tensile stress at the extreme fiber of the lower flange equal to 6½f′c. The lower limit of the load was selected such that a steel stress range of 10,000 psi (69 MPa) would be produced. After each million cycles of loading, the girder was tested statically to determine its stiffness. Slight reductions in stiffness and camber were observed, but there was no significant change in prestress loss. The girder performed satisfactorily for 5 million cycles of fatigue loading. After completion of the long-term fatigue load test, the girder was tested under static load to determine its ultimate flexural strength. It developed an ultimate moment equal to 94% of the ultimate moment capacity of a companion girder that had been under long-term sustained load. The measured moment capacity also exceeded the calculated moment capacity by 7.5% based on the AASHTO Standard Specifications for Highway Bridges.
Tests have been conducted on ordinary reinforcement, both in-air and embedded in concrete, to determine its fatigue properties. These tests have yielded varying results (Rehm 1960, Soretz 1965). For straight deformed bars, ACI Committee 215 (1974), Model Code for Concrete Structures (CEB-FIP 1978), FIP Commission on Model Code (1984), and Ontario Highway Bridge Design Code (Ministry of Transportation and Communications 1983) recommend stress range limits of 20, 22, and 18 ksi (138, 152, and 124 MPa), respectively. The lowest stress range found to cause fatigue failure in a hot-rolled bar is 21 ksi (145 MPa) (ACI Committee 215 1974).

ACI 343R recommends limiting the reinforcement stress range in terms of the minimum stress and reinforcement deformation geometry:

\[ f_f = 21 - 0.33 f_{\text{min}} + 8(r/h) \]  

where 
- \( f_f \) = safe stress range, ksi; 
- \( f_{\text{min}} \) = minimum applied stress, ksi; and 
- \( r/h \) = ratio of base radius-to-height of rolled-on transverse deformation (a value of 0.3 can be used in the absence of specific data).

The fatigue strength of prestressing reinforcement depends upon the steel type (bar, wire, strand), anchorage (unbonded post-tensioned reinforcement), extent of bond (ACI Committee 215 1974), and steel treatment. Paulson et al. (1983) conducted fatigue tests (in-air) of 50 seven-wire strand samples obtained from six different manufacturers. All of the strands conformed with ASTM A 416 requirements. The minimum stresses applied in the tests ranged from 75 to 165 ksi (517 to 1138 MPa), and the stress ranges varied from 22 to 81 ksi (152 to 559 MPa). A significant variation was observed in results from even two samples of the same product produced by the same manufacturer. The effect of the end grips dominated the fatigue curves in the region of long-life, low-stress-range.

The following relationship was found to lie above 95 to 97.5% of the failure points:

\[ \log N = 11 - 3.5 \log f_{sr} \]  

where 
- \( N \) = number of cycles; and 
- \( f_{sr} \) = maximum stress range for a fatigue life of \( N \) cycles, ksi.

The researchers did not find the effect of minimum stress on fatigue life great enough to warrant inclusion in the equation.

The FIP Commission on Prestressing Steel (1976) recommends a stress range of 15% of \( f_{pu} \) with a minimum applied stress not greater than 75% of \( f_{pu} \) for a fatigue life of 2 million cycles. For the same fatigue life of two million cycles, however, Naaman (1982b) recommends a reduced stress range of 10% \( f_{pu} \) with a minimum applied stress not greater than 60% of \( f_{pu} \), to better correlate with test results (Fig. 4.1). The following equation can be used to predict other maximum safe stress ranges.
\[ \frac{f_{sr}}{f_{pu}} = -0.123 \log N_f + 0.87 \]  

(4-4)

where

- \( f_{sr} \) = maximum safe stress range for a fatigue life of \( N \) cycles;
- \( f_{pu} \) = specified tensile strength of the prestressing strand; and
- \( N_f \) = number of cycles to failure.

The endurance limit (stress range for which the reinforcement will not fail for an infinite number of cycles) has not been found for prestressing steel (Naaman 1982b); however, a fatigue life of 2 million cycles is considered to be sufficient for most applications.

The previous discussion applies to pretensioned strands. For post-tensioned tendons, two more levels of fatigue strength have to be considered: the strand/duct assembly and the tendon anchorages. For the strand/duct assembly, fretting fatigue may govern if high contact stresses between strand and corrugated steel duct are combined with small relative movements at cracks. Under such circumstances, the fatigue strength of the strand/duct assembly can drop to as low as 14,300 psi (100 MPa). Fatigue strengths of anchorages are in the order of 14,300 psi (100 MPa), according to FIP Commission on Prestressing Steel and Systems (1992).

Designers typically place tendon anchorages away from areas with high stress variations and avoid fatigue problems at the anchorages. A similar approach normally will not work to avoid fretting fatigue because maximum stresses often occur at sections with maximum tendon curvature and maximum contact stresses between strand and duct. Fretting fatigue between strand and duct, however, can be avoided by using thick-walled plastic ducts rather than corrugated steel ducts (Oertle 1988). With a thick-walled plastic duct, the strand reaches fatigue strengths comparable to those of strand in air. Fig. 4.2 shows the fatigue performance of tendons with steel and plastic ducts in simply supported beams under four-point loading. In the specimen with a steel duct, 50% of the tendons failed at a fatigue amplitude of 25,000 psi (175 MPa); in contrast, only 18% of the tendons in the specimen with a plastic duct failed at a fatigue amplitude of 39,400 psi (275 MPa).

4.3—Fatigue in partially prestressed beams

To illustrate the relative importance of fatigue for partially prestressed beams compared with that for ordinary reinforced or fully prestressed beams, Naaman (1982b) analyzed three concrete beams, identical except for the partially prestressed reinforcement ratio (PPR = 0, 0.72 and 1.0). Note that PPR = 0 represents an ordinary reinforced beam; PPR = 1.0 represents a fully prestressed beam; and PPR = 0.72 represents a partially prestressed beam. All of the beams were designed to provide the same ultimate moment capacity. Material properties and relevant data are given by Naaman and Siriaksorn (1979).

For each beam, computed stress ranges in ordinary and prestressed steel were plotted with respect to the applied load (in excess of the dead load) varying from zero to the specified live load (Fig. 4.3). For the same type of beam section, the effect of the PPR was plotted with respect to the reinforcement stress range due to the application of live loads (Fig. 4.4). The discontinuity in the plots corresponds with first cracking of the concrete in the beams. It is evident from the figures that higher stress ranges are associated with partially prestressed sections. Thus, fatigue problems are more significant in partially prestressed sections than in their ordinary reinforced or fully prestressed counterparts.

4.4—Prediction of fatigue strength

The studies described have a common conclusion summarized by Naaman (1982b) and Warner and Hulsbos (1966b). The critical limit state (fatigue failure) of partially prestressed concrete beams is generally due to failure of the reinforcement. The fatigue life of the member can be predicted from the smaller of the fatigue lives of the reinforcing steel or the prestressing steel. Many of these investigations have indicated that in-air test results of reinforcement provide a good indication of the member fatigue life.

Naaman therefore recommends using Eq. (4-4) or Fig. 4.1 to estimate the fatigue life of stress-relieved seven-wire strand for the appropriate stress range. A strand subjected to a minimum stress less than 60% of its tensile strength with a stress range of 10% of the tensile strength should provide a fatigue life of approximately two million cycles.

For ordinary reinforcement, Naaman recommends using Eq. (4-2) to determine safe stress ranges that provide fatigue lives in excess of 2 million cycles.

ACI Committee 215 (1974) and Venuti (1965) recommend conducting a statistical investigation of at least six to 12 reinforcement samples at appropriate stress levels to establish the fatigue characteristics of the material. At least three stress levels are required to establish the finite-life portion of the S-N diagram: one stress level near the static strength, one near the fatigue limit, and one in between.

The choice of the PPR and relative placement of the reinforcement have a significant effect on the fatigue response of the members. Naaman (1982b) states that proper selection of these variables can maintain the stress ranges in the reinforcement to within acceptable limits.
Balaguru (1981) and Balaguru and Shah (1982) have presented a method and a numerical example for predicting the fatigue serviceability of partially prestressed members. The method compares the stress ranges in the beam constituents to the fatigue limits of each individual component (concrete, prestressing steel and nonprestressing steel) using the equations derived by Naaman and Siriaksorn (1979) to calculate stresses for both uncracked and cracked sections.

Naaman and Founas (1991) also presented models to calculate the structural responses that account for shrinkage, static and cyclic creep, and relaxation of prestressing steel. For any time $t$ and cycle $N$, the models can be used to compute stresses, strains, curvatures, and deflections.

4.5—Serviceability aspects

In a cracked concrete member, whether nonprestressed, partially prestressed, or fully prestressed, the crack widths and deflections generally increase under repeated loadings (Naaman 1982b).

The increase in crack widths and deflections in concrete members is mostly attributed to the cyclic creep of concrete and bond deterioration accompanied by slip between the reinforcement and concrete on either side of existing cracks. ACI Committee 224 (1980) notes that 1 million cycles of load can double the crack widths.

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Fig. 4.3—Typical comparison of stress changes in steel for reinforced, prestressed, and partially prestressed beams (Naaman 1982a).

Fig. 4.4—Typical stress changes in steel at different levels of prestressing (Naaman 1982a).
Harajli and Naaman (1989) developed an analytical model for cyclic slip and stresses to compute crack widths in partially prestressed concrete beams. In the analysis, equilibrium was assumed between the reinforcement bond stresses and the concrete tensile stresses in a concrete tensile prism joining two successive cracks. In addition, a local bond stress-slip relationship was assumed for the reinforcement.

The results of the analyses were in agreement with the experimental results shown in Fig. 4.5 and 4.6 for monotonic and cyclic loading, respectively. Agreement was also obtained with tests conducted by Lovegrove and El Din (1982) for which the steel stresses ranged from 7 to 43 ksi (48 to 297 MPa).

The analytical models were also useful in predicting trends in the crack patterns in terms of growth and mechanisms of growth. From the analyses, Harajli and Naaman (1989) attributed increased crack widths to increases in steel stresses caused by cyclic creep of concrete in compression and bond redistribution between cracks.

They indicated that partially prestressed beams have smaller crack spacings and less crack growth than fully prestressed beams subjected to fatigue loads. The presence of ordinary reinforcement in prestressed beams helps to control cracking in static and fatigue tests. Ordinary reinforcement reduces the increased steel stresses attributed to cyclic creep of concrete. In addition, it minimizes bond redistribution.

Crack widths were observed to be a function of reinforcement stress and crack spacing. In tests that generated similar crack spacings (3 to 4.5 in., [76 to 114 mm]), a nearly linear relationship was observed between crack widths and reinforcement stress (Fig. 4.7). To estimate the crack spacing \( a_{cs} \) and crack width \( W \) the following equations were given

\[
a_{cs} = 1.20A_t/P \quad (4-5)
\]

\[
W = 2S_oB \quad (4-6)
\]

\[
W_N = 2S_{o,N}B_N + [(f_{so,N} - f_{so})a_{cs}B_N/E_s] \quad (4-7)
\]

where

- \( a_{cs} \) = crack spacing, in. (mm);
- \( A_t \) = effective area of concrete tension zone surrounding all reinforcement having the same centroid as the reinforcement, in.\(^2\) (mm\(^2\));
- \( P \) = sum of the perimeters of all tension reinforcement, in. (mm);
- \( W \) = crack width at the first loading cycle;
- \( W_N \) = crack width at the \( N \)th loading cycle;
- \( S_o \) = slip of the reinforcement at the first cycle;
- \( S_{o,N} \) = slip of the reinforcement at the \( N \)th cycle;
- \( B \) = distance ratio at the first cycle;
- \( B_N \) = distance ratio at the \( N \)th cycle;
- \( f_{so} \) = steel stress at the primary crack at the first cycle;
- \( f_{so,N} \) = steel stress at the primary crack at cycle \( N \);
- \( N \) = number of load cycles; and
- \( E_s \) = modulus of elasticity of steel.

Tests by Shahawi and Batchelor (1986) indicated a similar linear relationship between crack width and reinforcement stress. They recommend the use of the FIP-CEB equation (Eq. 4-8) for predicting the maximum crack widths in partially prestressed beams subjected to fatigue

\[
W_{max} = f_s \times 10^{-3} \quad (4-8)
\]

where

- \( W_{max} \) = maximum crack width, mm; and
- \( f_s \) = change in steel stress from decompression at the extreme tensile fiber, MPa.

The previous FIP-CEB equation accounts for partial debonding, but does not consider cyclic creep of the concrete. Balaguru (1981) and Balaguru and Shah (1982) developed an equation to estimate maximum crack widths that incorporates the effects of cyclic creep and progressive deterioration of flexural stiffness. Comparing the results of this equation with those of the FIP-CEB crack width formula using data by Bhuvasarakul (1974), Balaguru determined the following ratios of maximum crack width after \( N \) cycles to initial maximum crack width: \( W_{max,N}/W_{max} = 2.45 \) (experimental) = 1.003 (FIP-CEB) = 2.15 (Balaguru). He attributed the disparity in the result obtained by the FIP-CEB equation to the lack of consideration of cyclic creep of concrete.

### 4.6—Summary of serviceability

In all types of concrete members, crack widths and deflections generally increase under repeated loadings (Naaman 1982b). These increases occur during early load stages and then generally stabilize until approximately 90% of their fatigue life. Researchers (Harajli and Naaman 1989,
Shahawi and Batchelor (1986) show a nearly linear relationship between maximum crack width and reinforcement stress. Deflections can increase to twice the static deflection of the first load cycle.

The importance of correct reinforcement detailing for crack control should be emphasized. Pretensioned wires or nonprestressed bars can be placed near the tensile face to help limit crack widths according to the report *Partial Prestressing* (Concrete Society 1983). The presence of ordinary reinforcement in prestressed beams helps to control cracking in static and fatigue tests. Ordinary reinforcement reduces the increased steel stresses attributed to cyclic creep of concrete. In addition, it minimizes bond redistribution.

Balaguru (1981) and Balaguru and Shah (1982) have derived a refined approach that incorporates the effects of cyclic creep of the concrete and tension stiffening effects to estimate maximum crack widths and deflections at a given load cycle. Naaman and Founas (1991) also presented models to compute stresses, strains, and deflections in partially prestressed beams for static and cyclic loading incorporating effects of shrinkage, creep, and relaxation.

**CHAPTER 5—EFFECTS OF LOAD REVERSALS**

**5.1—Introduction**

Cyclic load reversals can result from dynamic effects of earthquake, shock, wind, or blast loadings. Earthquake loadings are imparted to the base of the structure through ground motion, whereas wind loadings transmit forces to the aboveground portions of the structure. Shock or blast loadings can involve both of these effects: air overpressure forces similar...
Partially Prestressed Concrete

5.2—Design philosophy for seismic loadings

In design for seismic effects, the primary concern is safety of the occupants (Bertero 1986), and a secondary concern is economics. Even for an earthquake with reasonable probability of occurrence; however, it is not possible to guarantee with absolute certainty complete life safety and no structural damage. The Federal Emergency Management Agency, in their document *NEHRP 1997 Recommended Provisions* (1998), give minimum requirements to provide “reasonable and prudent life safety.” To achieve these objectives, structures should be proportioned to resist design forces with adequate strength and stiffness to limit damage to acceptable levels. Design forces are determined by using a response-modification coefficient $R$ to reduce the forces obtained, assuming that the structure responds linear elastically to the ground motion. The reduction factor accounts for the reduced strength demand due to a number of effects, including energy dissipation through hysteretic damping and lengthening of the structural period as the structure becomes inelastic.

In simplistic terms, there are two basic options in seismic design: (1) make the structure strong enough so that it will respond elastically; or (2) permit the structure to deform inelastically while ensuring adequate ductility and energy dissipation capacity. The second option permits the structure to be designed for considerably lower forces than required for the first option.

Designing partially prestressed concrete structures in accordance with the second approach, raises the same concerns as for the case of designing with conventional reinforced concrete; that is, the designer must ensure:

- Ductility at plastic hinges; and
- Adequate energy dissipation through damping and hysteresis.

This chapter concentrates on these characteristics as they relate to partially prestressed concrete.

5.3—Ductility

Ductility allows the redistribution of forces in indeterminate systems and ensures gradual rather than brittle failure, providing a warning to the occupants before collapse. If adequate ductility is not provided at the critical regions (plastic hinge zones), the member will be unable to develop the required inelastic rotation.

Ductility is usually expressed for structural members and systems in terms of a deformation ductility ratio, where the deformation is described in terms of displacement, rotation, or curvature (Naaman et al. 1986, Thompson and Park 1980a, Giannini et al. 1986). Deflection and rotation ductilities give an indication of drift (ratio of lateral displacement to height) and are used in structural analysis. Curvature ductility is used to define member or section behavior at plastic hinges.

The ductility ratio is defined as the ratio of the ultimate deformation to the yield deformation. In the case of partially

Fig. 4.7—Crack width variation versus reinforcing steel stress at different PPR (Harajli and Naaman 1985); 1 ksi = 6.9 MPa, 1 in. = 25.4 mm.
prestressed concrete structures, the definition of yield deformation can be quite arbitrary (Naaman et al. 1986; Thompson and Park 1980a; Giannini et al. 1986). The section contains both prestressed and nonprestressed reinforcement that can have different yield stresses; prestressing steel does not have a definite yield point, and reinforcement located in different layers will yield at different member deformations. The yield deformation, therefore, can be defined in many ways.

Among numerous investigations on partially prestressed concrete, there has not been a consistent definition of yield deformation (Fig. 5.1). Cohn and Bartlett’s study (1982) on partially prestressed concrete assumed that the yield curvature corresponded to yielding of ordinary reinforcement. This approach gives a relatively high value of ductility because the prestressing reinforcement yields at a much larger deformation. Thompson and Park (1980a) defined first yield as the intersection of the tangent to the elastic portion of the load-deformation curve and a horizontal line at ultimate load. In Park and Falconer’s study (1983) on prestressed piles, the yield deformation was taken at the intersection of the secant from zero to 75% of the ultimate moment capacity and the postelastic slope. Naaman et al. (1986) defined the yield deformation as the intersection of the secant from zero to the proportional limit of the prestressing steel and the postelastic slope. Although these latter definitions give slightly different results, they are all based on the ductility of the member section rather than yielding of a particular component in the cross section.

Local section ductility demands can be much more severe than overall deflection ductilities of frames. Tests by Muguruma et al. (1980) on partially prestressed concrete frames indicated that the ratios of section curvature ductilities to deflection ductilities of the frame were 1.24 and 1.53 for partially prestressed concrete systems with bonded and unbonded tendons, respectively.

5.3.1 Factors affecting ductility—Numerous investigations have been conducted to determine the effect of different parameters on ductility. Generally, factors that affect the ductility of ordinary-reinforced concrete sections affect the ductility of partially prestressed concrete sections as well. Effects of anchorage, bond, transfer lengths, grouting, characteristics of high-strength prestressing steels, and level of prestressing are additional parameters uniquely important to prestressed and partially prestressed concrete members (Bertero 1986).

Investigations were conducted to determine the effects of the reinforcement ratio (Cohn and Ghosh 1972; MacGregor 1974; Scott et al. 1982; and Park and Thompson 1977), concrete confinement (Park and Falconer 1983; Scott et al. 1982; Burns 1979; Kent and Park 1966; Wight and Sozen 1975; Sheikh and Uzumeri 1982; and Sheikh 1982), material strengths and stress-strain properties (Wight and Sozen 1975, Wang 1977), section geometry, ratio of compression reinforcement (Park and Thompson 1977, Burns and Seiss 1962), PPR, and axial load (Lin and Burns 1981). Another investigation by Thompson and Park (1980a) was conducted to determine the effects of the content and distribution of longitudinal reinforcement, transverse reinforcement, and concrete cover. Naaman et al. (1986) analyzed beams with four types of sections (rectangular, T, I, and box sections), five different concrete strengths, three types of prestressing steel, four levels of PPR, and seven reinforcing indexes $\omega$. For the rectangular and T-sections, five levels of effective prestress $f_{pe}/f_{pu}$ were analyzed, and in selected cases, four levels of compression reinforcement ratios and the effect of confinement were investigated. The following observations were made from the parametric evaluation:

- **Reinforcement index**—Section ductility decreases with increasing $\omega$ because of the significant reduction in ultimate curvatures associated with increased reinforcement ratios (Fig. 5.2 and 5.3).
- **Effective prestress**—When the effective prestress was decreased, it led to an increase in ultimate curvature for reinforcing indices $\omega$ greater than approximately 0.12. It also led to an increase in yield curvature irrespective of the reinforcement index. Fig. 5.2 illustrates the influence of effective prestress on ductility factor or the ratio of these two quantities. The increase in yield curvature tended to dominate the results and caused a reduction in ductility with decreased effective prestress irrespective of the reinforcing indices. This effect was more significant for fully prestressed beams.
- **Partially prestressed ratio**—A decrease in PPR led to an increase in both ultimate and yield curvatures, but in terms of ductility, there was no consistent trend. The effect of PPR on ductility became negligible with increased concrete compressive strengths.
- **Transverse reinforcement**—Increases in concrete confinement resulted in corresponding increases in sectional ductility irrespective of the reinforcement index. This effect was also observed in tests by several
other investigators (Watanabe et al. 1980; Muguruma et al. 1982b; Iyengar et al. 1970; Okamoto 1980; and Hawkins 1977).

- **Concrete strength**—For high values of reinforcement index ($\omega > 0.25$), the concrete compressive strength did not have a significant effect on ductility. For low values of $\omega (< 0.25)$, however, the use of higher-strength concrete reduced section ductility as much as 30%, particularly in cases of low PPR (ordinary reinforced concrete). This trend was also more pronounced for rectangular sections as compared with box, T- and I-sections. Tests by Muguruma et al. (1983) indicate that high-strength concrete is effective in improving ductility, but that the consequences of high-strength tendon fracture and brittle compressive failure of high-strength concrete should be considered.

- **Section geometry**—The section geometry did not have an appreciable effect on ductility, provided that the ductility index was expressed as a function of

reinforcement index $\omega$ (computed using the web for T-sections).

Naaman et al. (1986) found the reinforcement index $\omega$ to be an excellent independent variable for describing section ductility, as it is proportional to $c/d_{eff}$ ($c =$ distance from extreme compression fiber to neutral axis; and $d_{eff} =$ depth to centroid of tensile steel). Based on the reinforcement ratio and concrete compressive strength, it can be used to describe all three types of systems: reinforced concrete, prestressed concrete, and partially prestressed concrete.

Based on the results of analysis and experimental tests on twelve partially prestressed concrete beams (Harajli and Naaman 1984) described in the previous chapter, Naaman
et al. developed prediction equations for sectional ductility and plastic rotation as a function of the reinforcement index $\omega$. Three equations were derived for both the sectional ductility and plastic rotation to give upper limit, lower limit, and average values for ranges of reinforcement index of 0.05 to 0.3 (Fig. 5.3). The upper and lower limits accommodate the effects of parameters described previously, other than reinforcement index $\omega$ which were shown to have an effect on ductility. For example, Naaman et al. suggest the lower-bound equation be used to describe cases with high-strength concrete, low effective prestress, and high PPR. The upper-bound equations are suggested for cases with normal strength concrete, high effective prestress, and low PPR. The equations are given as follows

$$\frac{1}{\omega - 0.045} \left[ \frac{1.05 - \omega}{850\omega - 35} \right] \frac{L_p}{d_{ctf}^2}$$  \hspace{1cm} (5-1)

$$\frac{1}{1.94\omega - 0.086} \left[ \frac{1.05 - 1.65\omega}{1300\omega - 40} \right] \frac{L_p}{d_{ctf}^2}$$  \hspace{1cm} (5-2)

$$\frac{1}{1.5\omega - 0.075} \left[ \frac{1.07 - 1.58\omega}{1050\omega - 45} \right] \frac{L_p}{d_{ctf}^2}$$  \hspace{1cm} (5-3)

where

$\omega = \text{reinforcement index};$

$d_{ctf} = \text{depth to centroid of tensile force in steel};$ and

$L_p = \text{equivalent plastic hinge length}.$

In Fig. 5.4, the equations are compared with the results of several investigations (Kent and Park 1966; Corley 1966; Mattock 1964; Mattock 1967; Harajli and Naaman 1985c; Bishara and Brar 1974; and Baker and Amarakone 1964). The equations show fairly good correlation with the results (assuming a plastic hinge length $L_p$ of $d_{ctf}/2$), and are recommended by Naaman et al. for use as a first approximation in the design and detailing of ordinary reinforced, fully prestressed or partially prestressed members.

In a discussion of this work, Loov et al. (1987) recommend specifying $c/d$ rather than $\omega$ to ensure ductility, as is done in the Standards Association of Australia’s Draft Unified Concrete Structure for Buildings (1984) and the CSA’s Design of Concrete Structures for Buildings. The use of $c/d$ ensures consistent ductility demands from sections independent of geometry and concrete compressive strength. To accommodate this, Naaman et al. also provided the expressions in terms of $c/d_{ctf}$, valid for $c/d_{ctf}$ between 0.08 and 0.42:

$$\frac{1}{0.73c/d_{ctf} - 0.053} \left[ \frac{1.86 - 0.73c/d_{ctf}}{621c/d_{ctf} - 42} \right] \frac{L_p}{d_{ctf}^2}$$  \hspace{1cm} (5-4)

$$\frac{1}{1.42c/d_{ctf} - 0.102} \left[ \frac{1.86 - 1.28c/d_{ctf}}{949c/d_{ctf} - 50} \right] \frac{L_p}{d_{ctf}^2}$$  \hspace{1cm} (5-5)

$$\frac{1}{1.95c/d_{ctf} - 0.087} \left[ \frac{1.88 - 1.15c/d_{ctf}}{766c/d_{ctf} - 53} \right] \frac{L_p}{d_{ctf}^2}$$  \hspace{1cm} (5-6)

where

$c = \text{depth to neutral axis from extreme compression fiber};$

$d_{ctf} = \text{depth to centroid of tensile force in steel};$ and

$L_p = \text{equivalent plastic hinge length}.$

Thompson and Park (1980a) agreed with a form of $c/d$ as a measure of the ductility in an investigation in which a moment-curvature relation was developed applicable to ordinary reinforced, prestressed, and partially prestressed concrete beams, and verified with experimental tests on symmetrically reinforced (prestressed and ordinary reinforced) rectangular beam-column assemblies subjected to cyclic load reversals. The model was subsequently used to conduct a parametric study. Their recommendations are as follows:

- **Effect of prestressing steel content**—ACI 318 limits the prestressing steel content by requiring $\omega_0 \leq 0.36 \beta_1$, where $\beta_1$ is a parameter used to modify the neutral axis depth $c$ to determine the depth of the equivalent rectangular stress block $a$. Although the code limits the prestressing steel content, ACI 318 allows this value to be exceeded if the additional reinforcement is not considered in the flexural strength calculation. Although this can result in sufficient ductility for gravity loading, it can be insufficient for seismic loading. To ensure adequate ductility, one can limit $\omega_0$ to less than 0.2 (rather than 0.3 for the case of $\beta_1 = 0.85$). Alternatively, rather than using the previous equation, one can also limit $a/h \leq 0.2$ ($a = \text{depth of rectangular stress block}$; and $h = \text{depth of section}$). This provides a given amount of
ductility by specifying the location of the neutral axis independent of tendon positions. This relationship has been adopted by FIP (Cement and Concrete Association of England 1977) and the Standards Association of New Zealand’s Code of Practice for General Structural Design and Design Loading for Buildings (1980).

**Effect of prestressing steel distribution on ductility**—Sections with only one tendon have reduced moment capacity as the concrete in compression deteriorates. Thompson and Park (1980a) recommend the use of at least two grouted tendons (one near the top and one near the bottom of the section) for seismic loading, and suggested locating a third tendon towards the center of the section. The use of unbonded tendons was generally not recommended for primary earthquake-resistant members because of lack of information on the performance of the anchorages (Muguruma 1986). In cases where partially prestressed beams are designed with nonprestressed reinforcement in the extreme fibers providing at least 80% of the seismic resistance, however, they indicated that the prestress can be provided by one or more grouted or unbonded tendons in the middle third of the beam depth. The centrally located prestressing steel is recommended because it can delay cracking and enhance strength without much reduction in ductility. Studies at the National Institute of Standards and Technology (NIST) and as part of the National Science Foundation Precast Seismic Structural Systems (PRESSS) program (Cheok and Lew 1990) have explored the use of unbonded prestressing reinforcement through the connections. In the studies at NIST, central post-tensioning was used in conjunction with mild steel reinforcement at the top and bottom of the beam-column interface. The central post-tensioning maintained integrity of the joint, while energy dissipation was accommodated with the mild steel reinforcement. A study by Priestley and Tao (1993) proposed the use of lightly stressed, unbonded prestressing steel through the joint (nonlinear-elastic concept). The system offers ductile behavior through crack opening at the interface, while maintaining a “self-restoring” force. Because the strands do not yield, they work to bring the connection back to its originally undeformed position. This system has been studied further with other proposed connection systems as part of the PRESSS program (Cheok and Lew 1990).

**Effect of transverse reinforcement on ductility**—The degree of confinement provided by the transverse reinforcement to the compression regions significantly increases ductility. The confinement enhances the performance of the concrete within the core and also inhibits buckling of the compression reinforcement (Park and Thompson 1977; Park and Paulay 1975; and Thompson 1975). Thompson and Park recommend that the stirrups be spaced less than \( d/4 \) or 6 in. (150 mm).

**Effect of cover thickness**—For ductility, Thompson and Park suggest that the cover be made as small as possible, because loss of cover under cyclic loads causes a reduction in capacity. The effect of cover spalling on member strength is not as great for deeper members where cover is a smaller percentage of member depth.

Holding all other parameters constant, ductility decreases with an increase in reinforcement index. ACI 318 gives a maximum value for the reinforcement index of \( \omega \leq 0.36 \beta_1 \). Maintaining this limit would give section ductilities on the order of 1.5 to three. With Park’s suggested limit of 0.2, ductilities in excess of four or five can be obtained. Tests by Muguruma et al. (1982a) indicate that if the sections do not contain compression reinforcement, \( \omega < 0.2 \) can give ductilities of only about three. If the reinforcement index is maintained at less than 0.1, ductilities in excess of 10 can be achieved (Naaman et al. 1986).

Tests by Nakano and Okamoto (1978) on beam-column subassemblies indicated that even when tension reinforcement is adequate (for example, \( \omega < 0.15 \) to provide ductility of five), hoop reinforcement (\( \omega > 0.5\% \)) is required to provide the required deflection ductility. Okamoto (1980) carried out similar tests on eleven simply supported beams and nine cantilever beams and reached similar conclusions.

### 5.3.2 Special cases

**Compression members**—Relatively little information is available on partially prestressed concrete columns (Bertero 1986). Tertea and Onet (1983) analyzed 20 columns with different PPR ranging from zero to 100%. As observed in the beam tests, curvature ductility of columns improved with the addition of nonprestressed reinforcement. Curvature ductility factors of 3.9, 6.1, and 9.8 corresponded with PPR of 100, 50, and 0%. Irrespective of the degree of prestressing, column ductilities improved with increased confinement.

Tests by Park and Falconer (1983) on prestressed piles found that the curvature ductility increased with reduced axial load levels and increased confinement by spiral reinforcement. They discourage the use of hard-drawn reinforcing wire for spirals as it commenced fracture at displacement ductility factors of four to six. They recommend requiring the wire to have high fracture strains.

**Shear**—Beams should be proportioned to ensure ductile flexural behavior and to avoid brittle shear behavior (Bertero 1986). To accomplish this objective, the FIP Commission (Cement and Concrete Association of England 1977) recommends assuming material overstrengths on the order of 15% in calculating the design shear force from the plastic hinge moments. The New Zealand Code of Practice for General Structural Design and Design Loading for Buildings (Standards Association of New Zealand 1980) recommend neglecting the concrete contribution to shear resistance when the axial compression is less than 0.1 \( f_c' \).

Muguruma et al. (1983) determined that concrete resistance to shear can be improved by prestressing. Experiments were conducted on 7 x 10 in. (180 x 250 mm) simply supported beams with span lengths of 7.22 ft. (2.20 m) to investigate shear in prestressed beams. The flexural shear cracking load was found to increase in direct proportion to the increase in flexural cracking load (due to an increase in prestressing). Muguruma (1986) suggests in the design of partially prestressed concrete beams that the concrete contribution to shear resistance can be increased by the shear force corresponding to the decompression moment at the critical section. This effect is taken into consideration in ACI 318.

**Beam-column joints**—Beam-column joints should be designed and detailed so that the plastic-hinge zones in the beams can develop their full plastic capacities. Muguruma (1986) indicates that there are no differences in joint design
for ordinary reinforced or partially prestressed systems, except that the prestressing tendons do not bond to the concrete as well as ordinary deformed reinforcement. Research on the performance of bond between tendons and concrete in joints (Muguruma and Tominaga 1972) has indicated that tendons undergo bond deterioration at early load stages. Tendon slip causes a reduction in energy dissipation capacity by pinching the hysteresis loops.

Park’s investigation of beam-column subassemblages with grouted post-tensioned tendons (Applied Technology Council 1981) recommended minimizing the possibility of failure in the joint. To avoid this type of failure, the hinge zones should be displaced away from the beam-column interface. This can be accomplished by launching the beams, adding additional reinforcement at the interface, or using cruciform shapes (Bertero 1986). Displacing the hinge from the interface has the added advantage of locating the failure in a region in which nonprestressed reinforcement is located, increasing ductility and energy dissipation. Displacing the hinge further into the beam results in increased shear at the interface when the hinge reaches its capacity and produces increased ductility demands relative to hinges at the interface when subjected to similar drift levels. Using post-tensioned tendons through the joint core at middepth can reduce the amount of required joint shear reinforcement as it serves to maintain integrity of the joint (Park and Thompson 1977).

5.4—Energy dissipation

Energy dissipation can be attributed to viscous and hysteretic damping described separately below. Energy dissipation of prestressed concrete is as little as 0.15 times that of conventional reinforced concrete of comparable size and strength (Bertero 1986).

5.4.1 Viscous damping—In a 1978 state-of-the-art report by Hawkins (1977) on seismic resistance of prestressed and precast structures, a thorough review of the research on damping was presented. Tests by Penzien (1964) indicated that the level of prestress and concrete strength affects damping by affecting cracking. Decreases in cracking cause a reduction in damping.

Tests by Spencer (1969) on centrally prestressed beams indicated that damping ratios were not frequency dependent. Damping ratios tended to increase with end rotations and prestress. Spencer also found damping to be higher for beams loaded in uniform shear rather than uniform moment and attributed this result to the higher effects of interface shear and bond slip in the case of uniform shear loading.

Tests by Brondum-Nielsen (1973) on centrally prestressed beams indicated damping decreased with increased prestress levels, which is the opposite of what Spencer observed. Values of damping obtained by Nakano (1965) from tests of 1/3 scale four-story frames were 1% before cracking, 3% at cracking, and 7% before inelastic behavior.

5.4.2 Hysteretic behavior—The energy dissipation (area enclosed within the force-deformation hysteresis loops) of prestressed concrete members is lower than that of reinforced concrete members designed to develop similar strengths (Thompson and Park 1980b). Prestressed concrete members can achieve significant elastic recoveries even at large deformations (Schoeder 1977) because of the initial tensile stress in the strand due to prestressing. In elastic recoveries, the strand unloads its tensile force and is capable of achieving large recoveries. The member will not achieve significant energy dissipation unless the prestressing steel yields, the concrete crushes, or the nonprestressed deformed bar yields (Nakano 1965; Blakeley and Park 1973b; Guyon 1965; Blakeley and Park 1971; and Bertero 1973). The addition of nonprestressed reinforcement in partially prestressed concrete members has been shown to provide a marked improvement in energy dissipation characteristics (Thompson and Park 1980b, Inomata 1986). This is evident in Fig. 5.5, which shows typical hysteresis loops for cyclically loaded fully prestressed, ordinary reinforced, and partially prestressed concrete members. In addition, by reducing the strength degradation, the presence of longitudinal nonprestressed reinforcing steel improves ductility by acting as compression reinforcement (Park and Thompson 1977, Inomata 1986). As mentioned previously, Cheok and Lew (1990) have investigated the use of lightly prestressed unbonded reinforcement. The performance of the PRESSS connections has the advantage of a self-restoring force (nonlinear-elastic system). The stiffness reduction compared with that of a monolithic counterpart can compensate for the reduced energy dissipation. Hybrid connections (Cheok and Lew 1991) have been investigated that combine the nonlinear elastic system with elements that yield or dissipate energy.

Thompson and Park (1980a, 1980b) developed idealized moment-curvature relations for partially prestressed concrete sections by combining relations obtained by Blakeley (1971) and Blakeley and Park (1973a) for prestressed concrete and by Ramberg and Osgood for reinforced concrete (Iwan 1973). The idealized hysteresis
5.5—Dynamic analyses

Giannini et al. (1986) presented an analytical investigation on the seismic behavior of fully prestressed and partially prestressed concrete. The influences of the fundamental period, $PPR$, and seismic intensity were considered. The response was obtained in the form of time histories and peak values in terms of displacements, curvatures, local strains (in the constituents), forces, and dissipated energy.

Partially prestressed concrete seems to couple the advantages of fully prestressed concrete with those of ordinary reinforced concrete in resisting loads. Fully prestressed concrete has a tendency to greater strain and deformation demands than ordinary reinforced concrete. After a strong impulse, the displacements of the reinforced concrete and partially prestressed concrete systems tended to damp out, whereas those of the fully prestressed concrete system did not. The analysis showed that fully prestressed concrete resists the design seismic intensity, but for greater intensities, it rapidly approaches collapse. Fully prestressed concrete structures, however, can behave better for low intensity seismic excitation.

Fig. 5.6—Comparison of idealized and experimental moment-curvature relations for prestressed concrete section (Thompson and Park 1980b).

Fig. 5.7—Comparison of idealized and experimental moment-curvature relations for reinforced concrete section (Thompson and Park 1980b).

Fig. 5.8—Comparison of idealized and experimental moment-curvature relations for partially prestressed concrete section (Thompson and Park 1980b).
Similar conclusions were obtained from a study by Thompson and Park (1980b). They used the idealized moment-curvature relations (discussed previously) to conduct a nonlinear dynamic analysis of ordinary reinforced, fully prestressed, and partially prestressed concrete single-degree-of-freedom systems. Displacements of fully prestressed concrete systems are larger than those of ordinary reinforced and partially prestressed concrete systems because of the reduced energy dissipation associated with prestressing.

Fully prestressed systems also undergo a significant reduction in stiffness after cracking that counteracts the increased displacements under dynamic loading. The reduction in stiffness causes a reduction in the effective period of vibration that can reduce the displacement response. Under the combination of both of these effects, the analyses indicate that fully prestressed frames will have, on average, 30% greater maximum lateral displacements than reinforced concrete systems designed to have the same strength, viscous damping ratio, and initial stiffness (Thompson and Park 1980b).

Similar analytical studies have been conducted as part of the NSF PRESSS program (Palmieri et al. 1996, El Sheikh et al. 1999) that indicated systems incorporating unbonded post-tensioned connections experienced moderate increases in displacements in comparison with those expected from monolithic concrete frame systems. An advantage of the unbonded post-tensioned connections was the reduction in residual drift associated with the nonlinear elastic behavior of the system in comparison with the permanent system deformations associated with the inelastic response of the ordinary reinforced frames.

Fully prestressed concrete members can be detailed to accommodate increased displacements in a ductile manner; however, these higher displacements can result in greater nonstructural damage and P-delta effects. An advantage of using full prestressing and partial prestressing is the reduction in residual displacements due to the self-restoring nature of these systems. The addition of nonprestressed reinforcement can reduce these deflections.

5.6—Connections

Results of tests (Berg and Degenkolb 1973; Elliott 1972; Kunze et al. 1965; Sutherland 1965; Ozaka 1968; FIP 1970; FIP 1974; Arya 1970; and Architectural Institute of Japan 1970) and observations on the past performance of prestressed concrete in earthquakes indicate that most prestressed concrete members perform well. Most failures have been found to occur in connections between elements.

Several investigations have been conducted to study the performance of connections between fully prestressed and partially prestressed concrete precast elements. As mentioned previously, ductile connection concepts have been tested as part of the PRESSS program. French et al. (1989a, 1989b), Amu and French (1985), Hafner and French (1986), Jayashankar and French (1987), and Tarzikhan and French (1987) investigated post-tensioned, welded, bolted, composite, threaded coupler, and tapered-threaded splice connections. The most viable alternatives were continuous post-tensioned connections displacing the plastic-hinge region from the beam-column interface, and tapered-threaded splice connections that emulate the performance of monolithically cast beam-column connections. It was also suggested that advantage be taken of composite action between the beam and the slab.

5.7—Summary

If members are detailed properly, it is possible to ensure ductile behavior in fully prestressed and partially prestressed concrete beams (Applied Technology Council 1981).

Ordinary reinforced concrete has an advantage in energy-dissipation characteristics. Prestressed concrete has an advantage in its ability to reach large displacements with only a small loss in load-carrying capacity and almost complete recovery of displacements. Joint performance is improved by the use of continuous prestressed strand through the joint.

Partially prestressed concrete couples the advantages of prestressed concrete with those of ordinary reinforced concrete (Giannini et al 1986). The sections can be detailed so that energy dissipation and deformation demands are on the same order as those of ordinary reinforced concrete. Partially prestressed concrete also offers the possibility of calibrating the subsequent yielding of both prestressed and nonprestressed reinforcement to optimize stiffness and energy-dissipation properties (Giannini et al 1986).

A recommended design concept is to balance all or part of the gravity loads, satisfy serviceability criteria with draped or blanket ed prestressing strands, and place sufficient nonprestressed reinforcement in the section to meet the additional requirements for seismic design strength, ductility, and energy dissipation (Hawkins 1977; Inomata 1986; and Thompson and Park 1980b). Stirrup ties should be located in hinging regions to prevent bar buckling, confine concrete, and increase shear strength (Park and Thompson 1977, Park and Paulay 1975).

CHAPTER 6—APPLICATIONS

6.1—Early applications

Partial prestressing has been used in the design of structures since the early 1950s. In 1952, partially prestressed beams with post-tensioned cables were first used in the roof of a freight depot at Bury St. Edmunds, England (Bennet 1984). Lower concrete tensile stresses were specified initially, but with successful experience and an increasing amount of test data, the permissible fictitious tensile stresses were increased to 750 psi (5.2 N/mm²) for pretensioning and 650 psi (4.5 N/mm²) for post-tensioning (Bennet 1984). Other early applications of partial prestressing included the development of a concrete mast for overhead power lines and structures designed to withstand mining collapse; with both applications, the most severe design load was usually a temporarily unbalanced load (Bennet 1984).
6.2—Pretensioned concrete components

In the early 1980s, a survey was conducted among the pretensioned, precast-concrete plants in the U.S. and Canada (Jenny 1985) concerning the use of partial prestressing in the industry. Sixty-five plants responded to the survey. Four of the plants manufactured only bridge girders, and partial prestressing was rarely considered in their production. The survey revealed, however, that there was a general familiarity with partial prestressing and the concept was used for meeting special design situations rather than as a routine production standard. Only seven of the 61 remaining plants responding to the survey indicated that they did not allow any product to exceed a nominal tensile stress of 6$\sqrt{f_c'}$ psi (6$\sqrt{f_c'}$/2 MPa) for certain members, such as stemmed units, inverted tees, spandrel beams, hollow-core slabs, or columns (Jenny 1985).

The nominal stress levels reported by the plants were: 7.5 to 12$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/1.6$ to $\sqrt{f_c'}/2$ MPa) for stemmed units; 7.5 to 12$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/1.6$ to $\sqrt{f_c'}/2$ MPa) for beams; 7.5 to 9$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/1.6$ to $\sqrt{f_c'}/2$ MPa) for hollow-core slabs; and 7.5$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/1.6$ MPa) for columns. Reasons for designing these members with a nominal tensile stress greater than 6$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/2$ MPa) were to meet special design loads, reduce camber, save steel and lower the cost, reduce the required concrete strength at release, and achieve competitive span-depth ratios (Jenny 1985).

6.3—Post-tensioned building construction

The majority of post-tensioned building construction in the U.S. is based on the use of unbonded tendons in conjunction with code-specified minimum amounts of bonded non-prestressed reinforcement. Maximum design tensile stress levels range from 6 to 12$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/2$ to $\sqrt{f_c'}/2$ MPa) (Feyermuth 1985). For designs where durability considerations do not govern, service-load nominal tensile stress levels of 6 to 12$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/2$ to $\sqrt{f_c'}/2$ MPa) are now routine for one-way, post-tensioned systems. According to Feyermuth, “The use of partial prestressing, especially as applied to flat slabs, has reduced the incidence of significant cracking problems in post-tensioning buildings. This results from lower prestress levels, inducing proportionately smaller elastic and creep shortening movements, and the presence of bonded non-prestressed reinforcement to control crack widths” (1985).

In the design of office and hotel structures, the use of partial prestressing is often cost-effective. Post-tensioned beams, as well as one-way slabs, designed to an extreme nominal fiber stress in tension between 6 and 12$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/2$ and $\sqrt{f_c'}/2$ MPa) have performed well in practice. The use of partially prestressed design is especially effective when certain designated areas of a building are required to have a higher design live load than the majority of the floor area. The 1100 Peachtree Office Tower (shown in Fig. 6.1) is a 27-story, cast-in-place concrete office building located in Atlanta, Ga. The slab is 5 in. (127 mm) thick, conventionally reinforced, and supported by post-tensioned beams 14 in. (356 mm) wide and 22 in. (559 mm) deep. By allowing the nominal tensile stress in the beams to reach 10$\sqrt{f_c'}$ psi ($\sqrt{f_c'}/1.2$ MPa) under service loads, economy was achieved by reducing the overall number of tendons and by reducing the overall height of the structure, owing to the shallower beam depths.

For two-way flat-plate designs, ACI 318 limits the stress at column locations as calculated by the equivalent frame method or other approximate methods to 6$\sqrt{f_c'}$ ($\sqrt{f_c'}/2$ MPa). This is in recognition of the gross approximation of the actual strains in the vicinity of the column. Higher allowable stresses, however, are permitted in conjunction with more precise analytical procedures.

6.4—Bridges

Prestressed concrete bridges designed to permit cracking under service loads are still rare in the U.S., even though it is common to allow flexural tension in extreme fibers under service load. The primary concerns about bridge members being cracked under service loads have to do with fatigue and durability. Partially prestressed concrete bridges have been built in Europe and Japan.

Taerwe (1990) has described a number of partially prestressed concrete bridges constructed in Belgium. These highway bridges were designed according to the Class 2 conditions described in Chapter 1 of this report. As of 1990, the bridges described by Taerwe (built between 1965 and 1975) performed satisfactorily.

The Kannonji Viaduct in Japan was completed in 1991 (Soda and Kazuo 1994). The bridge has a total length of 2464 ft (751 m), with an average span length of 92 ft (28 m). The bridge superstructure units are continuous for three or four spans each. It was designed to allow flexural cracking under service loads at midspan but not over interior supports. Four other partially prestressed concrete highway bridges also have been constructed in Japan (Soda and Kazuo 1994).

AASHTO’s Standard Specifications for Highway Bridges does not currently allow service load cracking in prestressed concrete bridges. AASHTO, however, has adopted a load-
and resistance-factor design specification for highway bridges (AASHTO LRFD Bridge Design Specifications) that does specifically address partially prestressed concrete. This can lead to a wider use of partially prestressed concrete for highway bridges in the U.S.

6.5—Other applications

Dolan reports that the prismatic beams on the Walt Disney World Monorail were designed for tensile stresses of nearly $12\sqrt{f'c}$ psi ($\sqrt{f'c}$ MPa) for certain load combinations. The beams were precast and post-tensioned to provide continuity. Reinforcing bars were used to control cracking at the continuity joint.

Partial prestressing also has applications in nuclear-containment structures, cylindrical shells, and rectangular water vessels. In the 1960s, the first prestressed concrete containment structures for nuclear reactors were partially prestressed in the vertical direction only with nonprestressed reinforcement in the circumferential (hoop) direction of the cylinder and in the dome. Two examples are the containment structures at R.E. Ginna and H.B. Robinson Unit 2 nuclear stations (Ashar et al. 1994). Similarly, partial prestressing is used vertically in the walls of cylindrical shell and rectangular water-storage structures. If these structures are designed with restrained bases, high negative or positive vertical bending moments, or both, will be present. An economical design can be produced when a small amount of vertical pre-stressing in the range of 200 psi (1.4 MPa) is provided and nonprestressed reinforcement is added to resist any net flexural tensile stresses. These high bending moments occur only at one point in the wall. Therefore, the economy is achieved by not overreinforcing the entire height of wall. This method of design has been used since the 1970s.

CHAPTER 7—REFERENCES

7.1—Referenced standards and reports

American Concrete Institute
222R Corrosion of Metals in Concrete
318 Building Code Requirements for Structural Concrete and Commentary
343R Analysis and Design of Reinforced Concrete Bridge Structures
423.3R Recommendations for Concrete Members Prestressed with Unbonded Tendons

ASTM
A 416 Standard Specification for Uncoated Seven-Wire Stress-Relieved Strand for Prestressed Concrete

American Association of State Highway and Transportation Officials (AASHTO)
AASHTO/ LRFD Bridge Design Specifications
AASHTO Standard Specification for Highway Bridges

The reports and standards listed above were the latest editions at the time this document was prepared. Because these documents are revised frequently, the reader is advised to contact the proper sponsoring group if it is desired to refer to the latest version.

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Suite 249
Washington D.C. 20001

American Concrete Institute
P.O. Box 9094
Farmington Hills, Mich. 48339-9094

ASTM
100 Barr Harbor Drive
West Conshohocken, PA 19428

7.2—Cited references

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The Concrete Society, 1983, Partial Prestressing, Techni-


**APPENDIX—NOTATIONS**

- \( a \) = depth of equivalent rectangular compressive stress block
- \( a_{cs} \) = crack spacing
- \( A_{cp} \) = area enclosed by outside perimeter of concrete cross section
- \( A_{ps} \) = area of prestressed reinforcement in tension zone
- \( A_{n/p} \) = area of nonprestressed tension reinforcement
- \( A_{pe} \) = area of nonprestressed compression reinforcement
- \( A_{t} \) = effective area of concrete tension zone surrounding all reinforcement having the same centroid as reinforcement
- \( b \) = width of compression face of member
- \( B \) = distance ratio at first loading cycle
- \( B_N \) = distance ratio at the Nth loading cycle
- \( c \) = depth to neutral axis from extreme compression fiber
- \( c_N \) = depth of neutral axis after Nth cycle of loading
- \( c_2 \) = constant
- \( C_{ct}(t-t_A) \) = creep coefficient of concrete at time \( t \) when loaded at time \( t_A \)
- \( d \) = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement
- \( d_c \) = cover to center of reinforcing steel
- \( d_c' \) = concrete clear cover
- \( d_{cf} \) = depth to centroid of tensile force in steel
- \( d_e \) = depth to centroid of nonprestressed tension reinforcement
- \( d_{max,N} \) = maximum deflection after Nth cycle
- \( d_p \) = distance from extreme compression fiber to centroid of prestressed reinforcement
- \( e_{cr,n} \) = average strain in concrete between the cracks after \( N \) cycles of loading
- \( e_{s,n} \) = steel strain at crack calculated using cracked section elastic analysis after \( N \) cycles of loading
- \( E_{ci}(t) \) = instantaneous elastic modulus of concrete at time \( t \)
- \( E_{ce}(t) \) = equivalent elastic modulus of concrete at time \( t \)
- \( E_{ci} \) = elastic modulus of concrete at time of loading or transfer
- \( E_A(t) \) = instantaneous elastic modulus of concrete at time \( t \)
- \( E_N \) = cyclic modulus of elasticity after \( N \) cycles
- \( E_s \) = modulus of elasticity of steel
- \( f_c' \) = concrete compressive strength
- \( f_{cr} \) = maximum recommended stress range for concrete
- \( f_f \) = safe stress range
- \( f_{min} \) = minimum applied stress (in concrete or reinforcing steel, as applicable)
- \( f_{pc} \) = average precompression in concrete due to prestress at centroid of cross-section
- \( f_{pe} \) = effective stress in prestressing steel
- \( f_{ps} \) = stress in prestressed reinforcement at nominal bending strength
- \( f_{pu} \) = ultimate strength of the prestressing steel
- \( f_s' \) = tensile stress in reinforcing steel, or change in steel stress from decompression
- \( f_{so} \) = steel stress at primary crack at first cycle
- \( f_{so,N} \) = steel stress at primary crack at cycle \( N \)
- \( f_{sr} \) = maximum recommended stress range in reinforcing steel (prestressed or nonprestressed, as applicable)
- \( f_y \) = yield strength of nonprestressed reinforcement
- \( F \) = prestressing force
- \( F(t) \) = prestressing force at time \( t \)
- \( h \) = depth of member

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\( b = \) width of compression face of member
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\( c_2 = \) constant
\( C_{ct}(t-t_A) = \) creep coefficient of concrete at time \( t \) when loaded at time \( t_A \)
\( d = \) distance from extreme compression fiber to centroid of nonprestressed tension reinforcement
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\( f_{sr} = \) maximum recommended stress range in reinforcing steel (prestressed or nonprestressed, as applicable)
\( f_y = \) yield strength of nonprestressed reinforcement
\( F = \) prestressing force
\( F(t) = \) prestressing force at time \( t \)
\( h = \) depth of member
$h_1$ = distance from neutral axis to tension face

$h_2$ = distance from neutral axis to centroid of reinforcement

$I_{cr}$ = moment of inertia of cracked section

$I_{ef}f(t)$ = effective moment of inertia of cracked section at time $t$

$I_{en}$ = effective moment of inertia after $N$ cycles

$I_g$ = gross moment of inertia

$K_D, K_F$ = constants depending on type of loading and steel profile

$L_p$ = equivalent plastic hinge length

$M = M_s$ = sustained external moment at midspan

$M_D = M_{ed}$ = applied moment

$M_{dec} = M_{cr}$ = decompression moment (moment which produces zero concrete stress at extreme fiber of a section, nearest to centroid of the prestressing force, when added to action of effective prestress alone)

$M_{cr}$ = cracking moment

$M_L = M_{ed}$ = live-load moment

$M_{u}$ = total nominal moment capacity

$M_{np}$ = moment capacity provided by prestressed reinforcement

$N = N_{cy}$ = number of load cycles

$N_{f}$ = number of cycles to failure

$P = P_s$ = sum of perimeters of all tension reinforcement

$P_{cp}$ = outside perimeter of concrete cross section

$P\overline{PR}$ = partial prestressing ratio

$Q = Q_{ser}$ = applied service load

$Q_{ser} = \text{applied service load}$

$r/h = \text{ratio of base radius to height of rolled-on transverse deformation (value of 0.3 may be used in absence of specific data)}$

$S_o = \text{slip of reinforcement at first cycle}$

$S_{o,N} = \text{slip of reinforcement at Nth cycle}$

$t = \text{time or age of concrete}$

$t_A = \text{age of concrete at time of loading}$

$V_c = \text{concrete contribution of shear strength}$

$W = \text{crack width at the first loading cycle}$

$W_N = \text{crack width at the Nth loading cycle}$

$W_{max} = \text{maximum crack width}$

$W_{max,N} = \text{maximum crack width after N cycles}$

$\beta_1 = \text{parameter relating neutral axis depth to depth of equivalent rectangular compression block}$

$\Delta f_{ps} = \text{change in stress in prestressing steel}$

$\Delta(t) = \text{deflection at time t}$

$\lambda = \text{degree of prestress}$

$\epsilon = \text{strain (in general)}$

$\rho = \text{reinforcement ratio of tensile nonprestressed reinforcement}$

$\rho' = \text{reinforcement ratio of compressive nonprestressed reinforcement}$

$P_{p} = \text{reinforcement ratio of tensile prestressed reinforcement}$

$\Sigma_o = \text{sum of perimeters of all tension reinforcement}$

$\omega = \text{reinforcing index}$

$\omega_p = \text{reinforcing index for prestressing steel}$