CHAPTER 11

LATERAL EARTH PRESSURE

11.1 INTRODUCTION

Structures that are built to retain vertical or nearly vertical earth banks or any other material are called *retaining walls.* Retaining walls may be constructed of masonry or sheet piles. Some of the purposes for which retaining walls are used are shown in Fig. 11.1.

Retaining walls may retain water also. The earth retained may be natural soil or fill. The principal types of retaining walls are given in Figs. 11.1 and 11.2.

Whatever may be the type of wall, all the walls listed above have to withstand lateral pressures either from earth or any other material on their faces. The pressures acting on the walls try to move the walls from their position. The walls should be so designed as to keep them stable in their position. Gravity walls resist movement because of their heavy sections. They are built of mass concrete or stone or brick masonry. No reinforcement is required in these walls. Semi-gravity walls are not as heavy as gravity walls. A small amount of reinforcement is used for reducing the mass of concrete. The stems of cantilever walls are thinner in section. The base slab is the cantilever portion. These walls are made of reinforced concrete. Counterfort walls are similar to cantilever walls except that the stem of the walls span horizontally between vertical brackets known as counterforts. The counterforts are provided on the backfill side. Buttressed walls are similar to counterfort walls except the brackets or buttress walls are provided on the opposite side of the backfill.

In all these cases, the backfill tries to move the wall from its position. The movement of the wall is partly resisted by the wall itself and partly by soil in front of the wall.

Sheet pile walls are more flexible than the other types. The earth pressure on these walls is dealt with in Chapter 20. There is another type of wall that is gaining popularity. This is mechanically stabilized reinforced earth retaining walls (MSE) which will be dealt with later on. This chapter deals with lateral earth pressures only.

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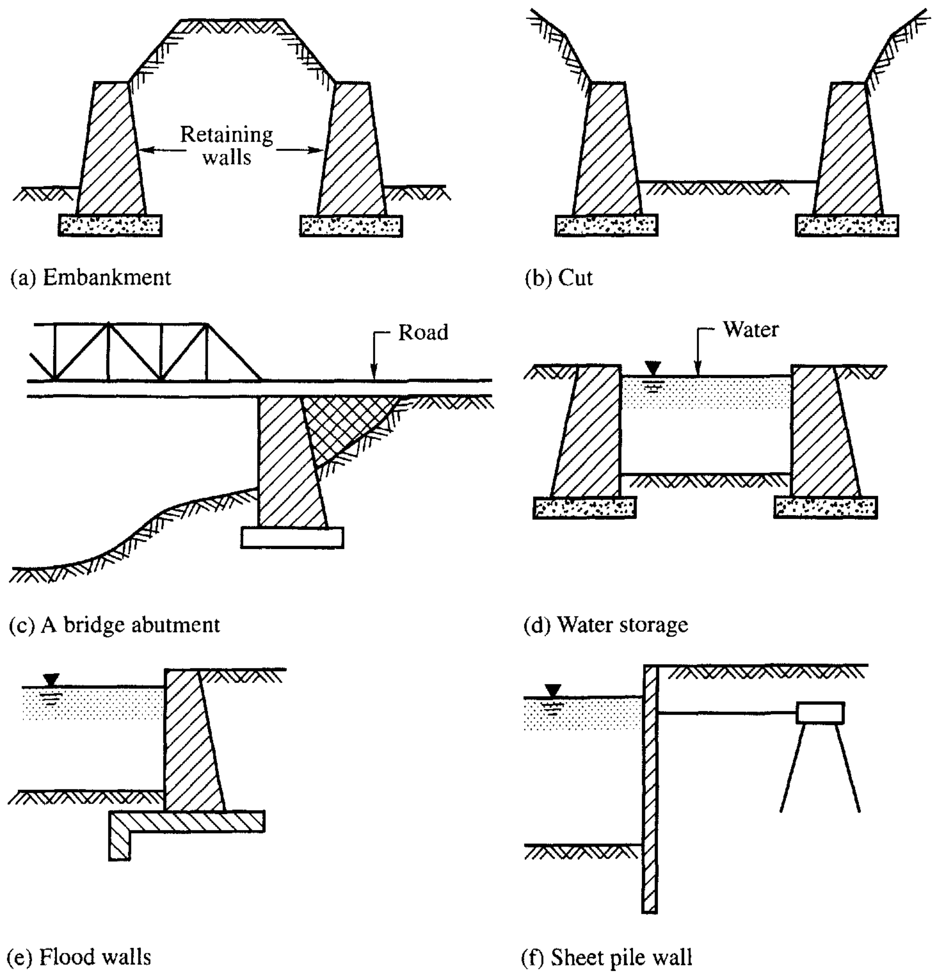

Figure 11.1 Use of retaining walls

11.2 LATERAL EARTH PRESSURE THEORY

There are two classical earth pressure theories. They are

1. Coulomb's earth pressure theory.
2. Rankine's earth pressure theory.

The first rigorous analysis of the problem of lateral earth pressure was published by Coulomb in (1776). Rankine (1857) proposed a different approach to the problem. These theories propose to estimate the magnitudes of two pressures called *active earth pressure* and *passive earth pressure* as explained below.

Consider a rigid retaining wall with a plane vertical face, as shown in Fig. 11.3(a), is backfilled with cohesionless soil. If the wall does not move even after back filling, the pressure exerted on the wall is termed as pressure for the *at rest condition* of the wall. If suppose the wall gradually rotates about point A and moves away from the backfill, the unit pressure on the wall is gradually reduced and after a particular displacement of the wall at the top, the pressure reaches a constant value. The pressure is the minimum possible. This pressure is termed the *active pressure* since the weight of the backfill is responsible for the movement of the wall. If the wall is smooth,

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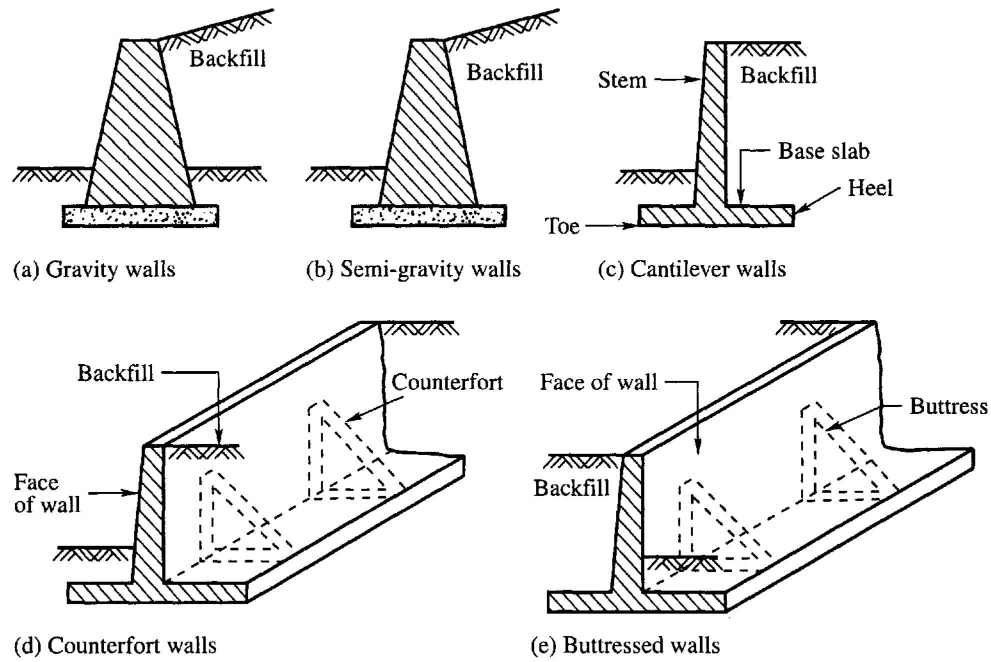

Figure 11.2 Principal types of rigid retaining walls

the resultant pressure acts normal to the face of the wall. If the wall is rough, it makes an angle δ with the normal on the wall. The angle δ is called the *angle of wall friction.* As the wall moves away from the backfill, the soil tends to move forward. When the wall movement is sufficient, a soil mass of weight W ruptures along surface ADC shown in Fig. 11.3(a). This surface is slightly curved. If the surface is assumed to be a plane surface AC, analysis would indicate that this surface would make an angle of $45° + \phi/2$ with the horizontal.

If the wall is now rotated about A *towards the backfill, the actual failure plane* ADC is also a curved surface [Fig. 11.3(b)]. However, if the failure surface is approximated as a plane AC, this makes an angle $45° - \phi/2$ with the horizontal and the pressure on the wall increases from the value of the at rest condition to the maximum value possible. The maximum pressure P_p that is developed is termed the *passive earth pressure.* The pressure is called passive because the weight of the backfill opposes the movement of the wall. It makes an angle δ with the normal if the wall is rough.

The gradual decrease or increase of pressure on the wall with the movement of the wall from the at rest condition may be depicted as shown in Fig. 11.4.

The movement Δ_p required to develop the passive state is considerably larger than Δ_a required for the active state.

11.3 LATERAL EARTH PRESSURE FOR AT REST CONDITION

If the wall is rigid and does not move with the pressure exerted on the wall, the soil behind the wall will be in a state of *elastic equilibrium.* Consider a prismatic element E in the backfill at depth z shown in Fig. 11.5.

Element E is subjected to the following pressures.

Vertical pressure = $\sigma_v = \gamma z$; lateral pressure = σ_h

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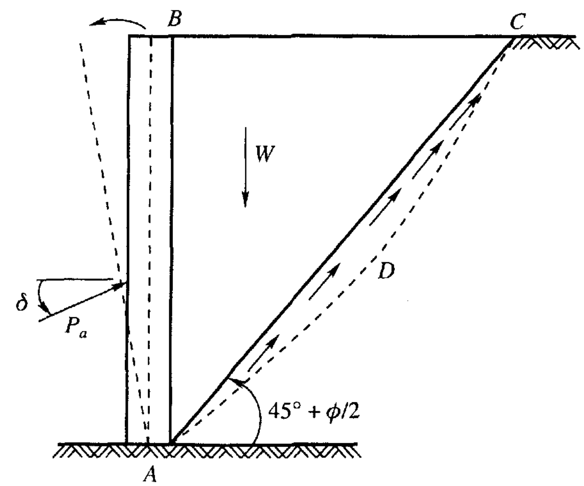
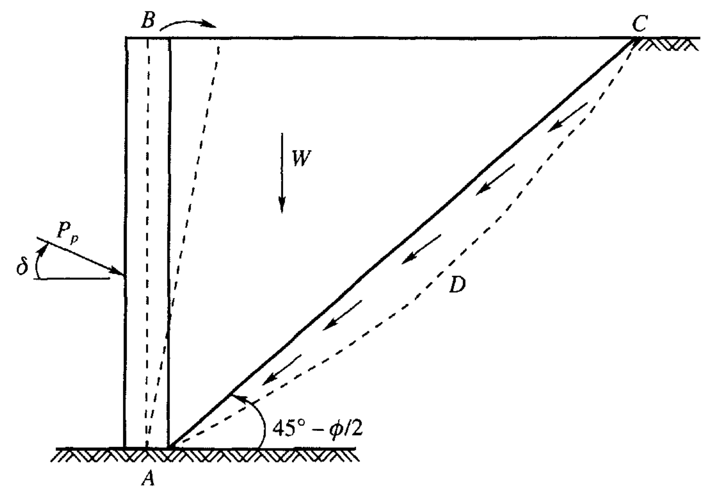


(a) Active earth pressure



(b) Passive earth pressure

Figure 11.3 Wall movement for the development of active and passive earth pressures

where γ is the effective unit weight of the soil. If we consider the backfill is homogeneous then both σ_v and σ_h increase linearly with depth z. In such a case, the ratio of σ_h to σ_v remains constant with respect to depth, that is

$$\frac{\sigma_h}{\sigma_v} = \frac{\sigma_h}{\gamma z} = \text{constant} = K_0 \tag{11.1}$$

where K_0 is called the *coefficient of earth pressure for the at rest condition or at rest earth pressure coefficient.*

The lateral earth pressure σ_h acting on the wall at any depth z may be expressed as

$$\sigma_h = K_0 \gamma z \tag{11.1a}$$

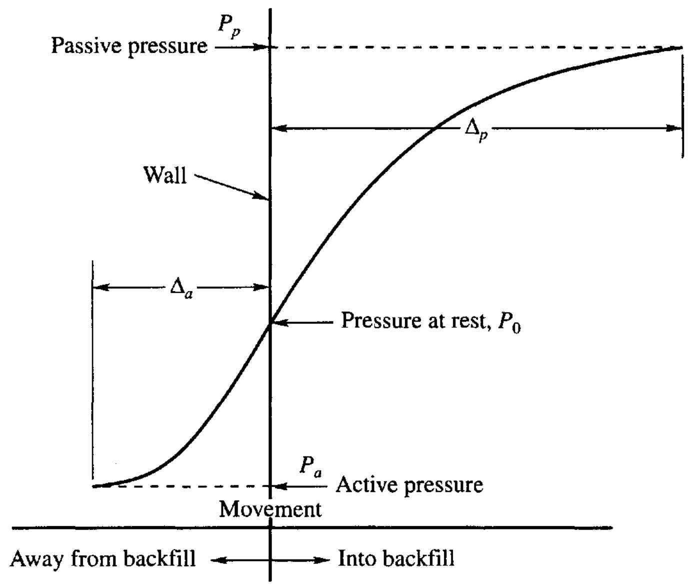


Figure 11.4 Development of active and passive earth pressures

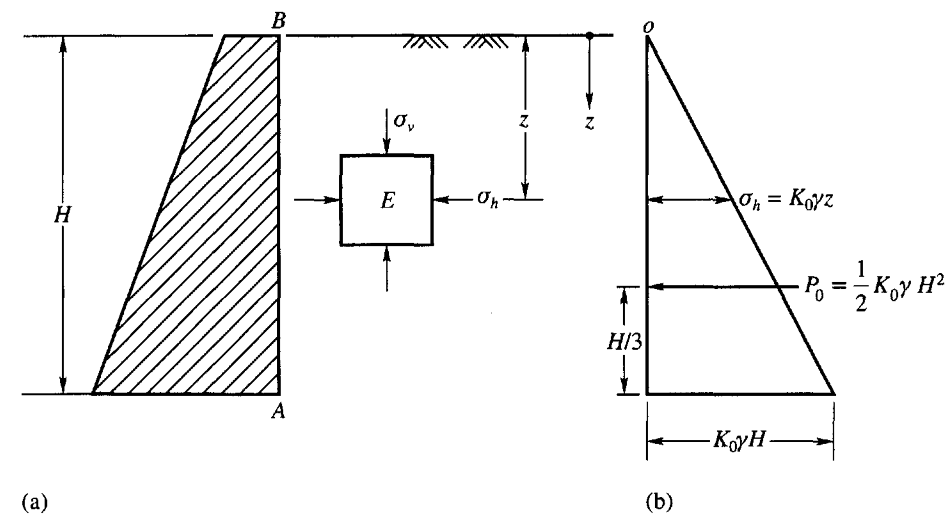


Figure 11.5 Lateral earth pressure for at rest condition

The expression for σ_h at depth H, the height of the wall, is

$$\sigma_h = K_0 \gamma H \tag{11.1b}$$

The distribution of σ_h on the wall is given in Fig. 11.5(b).
The total pressure P_0 for the soil for the at rest condition is

$$P_0 = \frac{1}{2} K_0 \gamma H^2 \tag{11.1c}$$

Table 11.1 Coefficients of earth pressure for at rest condition

Type of soil	I_p	K_0
Loose sand, saturated	–	0.46
Dense sand, saturated	–	0.36
Dense sand, dry (e = 0.6)	–	0.49
Loose sand, dry (e = 0.8)	–	0.64
Compacted clay	9	0.42
Compacted clay	31	0.60
Organic silty clay, undisturbed (w_l = 74%)	45	0.57

The value of K_0 depends upon the relative density of the sand and the process by which the deposit was formed. If this process does not involve artificial tamping the value of K_0 ranges from about 0.40 for loose sand to 0.6 for dense sand. Tamping the layers may increase it to 0.8.

The value of K_0 may also be obtained on the basis of elastic theory. If a cylindrical sample of soil is acted upon by vertical stress σ_v and horizontal stress σ_h, the lateral strain ε_1 may be expressed as

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_h - \mu(\sigma_h + \sigma_v)\right] \tag{11.2}$$

where E = Young's modulus, μ = Poisson's ratio.

The lateral strain $\varepsilon_1 = 0$ when the earth is in the at rest condition. For this condition, we may write

$$\frac{1}{E}\left[\sigma_h - \mu(\sigma_h + \sigma_v)\right] = 0 \quad \text{or} \quad \frac{\sigma_h}{\sigma_v} = \frac{\mu}{1-\mu} \tag{11.3}$$

$$\text{or} \quad \sigma_h = \left(\frac{\mu}{1-\mu}\right)\sigma_v = K_0\sigma_v = K_0\gamma z$$

$$\text{where} \quad \frac{\mu}{1-\mu} = K_0, \quad \sigma_v = \gamma z \tag{11.4}$$

According to Jaky (1944), a good approximation for K_0 is given by Eq. (11.5).

$$K_0 = 1 - \sin\phi \tag{11.5}$$

which fits most of the experimental data.

Numerical values of K_0 for some soils are given in Table 11.1.

Example 11.1

If a retaining wall 5 m high is restrained from yielding, what will be the at-rest earth pressure per meter length of the wall? Given: the backfill is cohesionless soil having ϕ = 30° and γ = 18 kN/m^3. Also determine the resultant force for the at-rest condition.

Solution

From Eq. (11.5)

$$K_0 = 1 - \sin\phi = 1 - \sin 30° = 0.5$$

From Eq. (11.1b), $\sigma_h = K_o\gamma H = 0.5 \times 18 \times 5 = 45$ kN/m^2

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From Eq. (11.1c)

$$P_0 = \frac{1}{2}K_0 \gamma H^2 = \frac{1}{2} \times 0.5 \times 18 \times 5^2 = 112.5 \text{ kN/m length of wall}$$

11.4 RANKINE'S STATES OF PLASTIC EQUILIBRIUM FOR COHESIONLESS SOILS

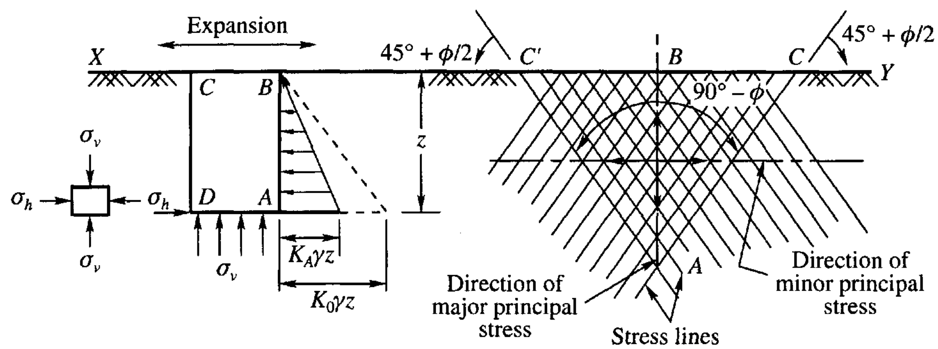
Let *XY* in Fig. 11.6(a) represent the horizontal surface of a semi-infinite mass of cohesionless soil with a unit weight γ. The soil is in an initial state of elastic equilibrium. Consider a prismatic block *ABCD*. The depth of the block is z and the cross-sectional area of the block is unity. Since the element is symmetrical with respect to a vertical plane, the normal stress on the base *AD* is

$$\sigma_v = \gamma z \tag{11.6}$$

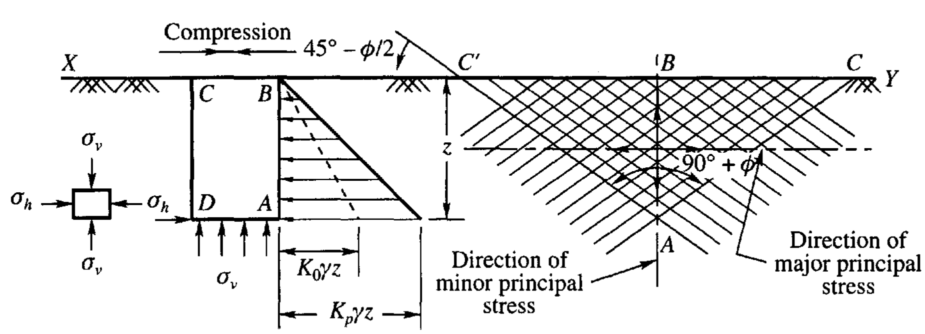
σ_v is a principal stress. The normal stress σ_h on the vertical planes *AB* or *DC* at depth z may be expressed as a function of vertical stress.

$$\sigma_h = f(\sigma_v) = K_0 \gamma z \tag{11.7}$$

where K_0 is the coefficient of earth pressure for the at rest condition which is assumed as a constant for a particular soil. The horizontal stress σ_h varies from zero at the ground surface to $K_0\gamma z$ at depth z.



(a) Active state



(b) Passive state

Figure 11.6(a, b) Rankine's condition for active and passive failures in a semi-infinite mass of cohesionless soil

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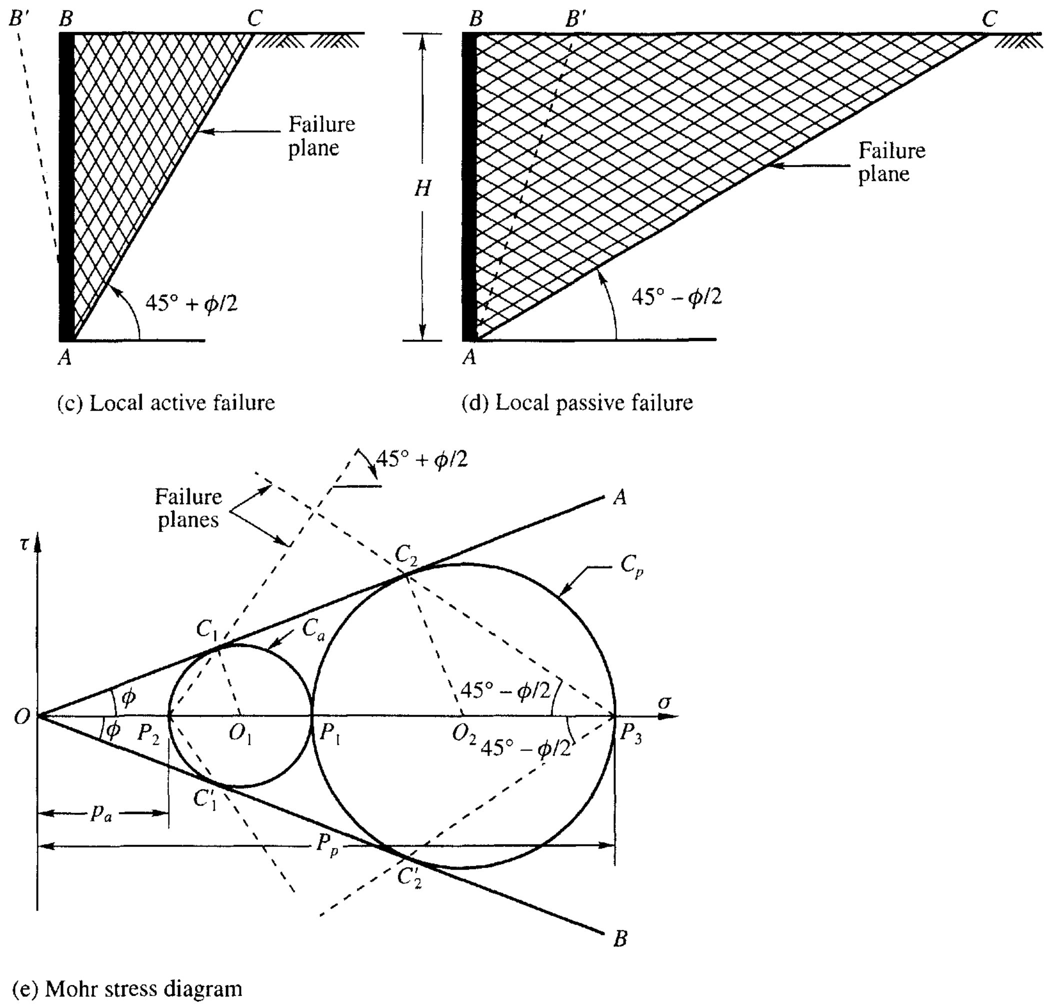

Figure 11.6(c, d, e) Rankine's condition for active and passive failures in a semi-infinite mass of cohesionless soil

If we imagine that the entire mass is subjected to horizontal deformation, such deformation is a plane deformation. Every vertical section through the mass represents a plane of symmetry for the entire mass. Therefore, the shear stresses on vertical and horizontal sides of the prism are equal to zero.

Due to the stretching, the pressure on vertical sides *AB* and *CD* of the prism decreases until the conditions of *plastic equilibrium* are satisfied, while the pressure on the base *AD* remains unchanged. Any further stretching merely causes a plastic flow without changing the state of stress. The transition from the state of *plastic equilibrium* to the state of *plastic flow* represents the failure of the mass. Since the weight of the mass assists in producing an expansion in a horizontal direction, the subsequent failure is called *active failure.*

If, on the other hand, the mass of soil is compressed, as shown in Fig. 11.6(b), in a horizontal direction, the pressure on vertical sides *AB* and *CD* of the prism increases while the pressure on its base remains unchanged at γz. Since the lateral compression of the soil is resisted by the weight of the soil, the subsequent failure by plastic flow is called a *passive failure.*

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The problem now consists of determining the stresses associated with the states of plastic equilibrium in the semi-infinite mass and the orientation of the surface of sliding. The problem was solved by Rankine (1857).

The plastic states which are produced by stretching or by compressing a semi-infinite mass of soil parallel to its surface are called *active* and *passive Rankine states* respectively. The orientation of the planes may be found by Mohr's diagram.

Horizontal stretching or compressing of a semi-infinite mass to develop a state of plastic equilibrium is only a concept. However, local states of plastic equilibrium in a soil mass can be created by rotating a retaining wall about its base either away from the backfill for an active state or into the backfill for a passive state in the way shown in Figs. 11.3(c) and (d) respectively. In both cases, the soil within wedge *ABC* will be in a state of plastic equilibrium and line *AC* represents the rupture plane.

Mohr Circle for Active and Passive States of Equilibrium in Granular Soils

Point P_1 on the σ-axis in Fig. 11.6(e) represents the state of stress on base *AD* of prismatic element *ABCD* in Fig. 11.6(a). Since the shear stress on *AD* is zero, the vertical stress on the base

$$\sigma_v = \gamma z \tag{11.8}$$

is a principal stress. *OA* and *OB* are the two Mohr envelopes which satisfy the Coulomb equation of shear strength

$$s = \sigma \tan \phi \tag{11.9}$$

Two circles C_a and C_p can be drawn passing through P_1 and at the same time tangential to the Mohr envelopes *OA* and *OB*. When the semi-infinite mass is stretched horizontally, the horizontal stress on vertical faces *AB* and *CD* (Fig. 11.6 a) at depth z is reduced to the minimum possible and this stress is less than vertical stress σ_v. Mohr circle C_a gives the state of stress on the prismatic element at depth z when the mass is in active failure. The intercepts OP_1 and OP_2 are the major and minor principal stresses respectively.

When the semi-infinite mass is compressed (Fig. 11.6 b), the horizontal stress on the vertical face of the prismatic element reaches the maximum value OP_3 and circle C_p is the Mohr circle which gives that state of stress.

Active State of Stress

From Mohr circle C_a

Major principal stress = $OP_1 = \sigma_1 = \gamma z$
Minor principal stress = $OP_2 = \sigma_3$

$$OO_1 = \frac{\sigma_1 + \sigma_3}{2}, \quad O_1C_1 = \frac{\sigma_1 - \sigma_3}{2}$$

From triangle OO_1C_1, $\dfrac{\sigma_1 - \sigma_3}{2} = \dfrac{\sigma_1 + \sigma_3}{2} \sin \phi$

$$\text{or} \quad \sigma_1 = \sigma_3 \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) = \sigma_3 \tan^2 (45^\circ + \phi/2) = \sigma_3 N_\phi \tag{11.10}$$

$$\text{Therefore,} \quad p_a = \sigma_3 = \frac{\sigma_1}{N_\phi} = \gamma z K_A \tag{11.11}$$

where $\sigma_1 = \gamma z$, K_A = *coefficient of earth pressure for the active state* = $\tan^2 (45^\circ - \phi/2)$.

From point P_1, draw a line parallel to the base AD on which σ_1 acts. Since this line coincides with the σ-axis, point P_2 is the origin of planes. Lines P_2C_1 and $P_2C'_1$ give the orientations of the failure planes. They make an angle of 45° + ϕ/2 with the σ-axis. The lines drawn parallel to the lines P_2C_1 and $P_2C'_1$ in Fig. 11.6(a) give the shear lines along which the soil slips in the plastic state. The angle between a pair of conjugate shear lines is (90° – ϕ).

Passive State of Stress

C_p is the Mohr circle in Fig. (11.6e) for the passive state and P_3 is the origin of planes.

Major principal stress = $\sigma_1 = p_p = OP_3$
Minor principal stress = $\sigma_3 = OP_1 = \gamma z$
From triangle OO_2C_2, $\sigma_1 = \gamma z N_\phi$

Since $\sigma_1 = p_p$ and $\sigma_3 = \gamma z$, we have

$$p_p = \gamma z N_\phi = \gamma z K_p \tag{11.12}$$

where K_p = *coefficient of earth pressure for the passive state* = $\tan^2(45° + \phi/2)$.

The shear failure lines are P_3C_2 and $P_3C'_2$ and they make an angle of 45° – ϕ/2 with the horizontal. The shear failure lines are drawn parallel to P_3C_2 and $P_3C'_2$ in Fig. 11.6(b). The angle between any pair of conjugate shear lines is (90° + ϕ).

11.5 RANKINE'S EARTH PRESSURE AGAINST SMOOTH VERTICAL WALL WITH COHESIONLESS BACKFILL

Backfill Horizontal-Active Earth Pressure

Section AB in Fig. 11.6(a) in a semi-infinite mass is replaced by a smooth wall AB in Fig. 11.7(a).

The lateral pressure acting against smooth wall AB is due to the mass of soil ABC above failure line AC which makes an angle of 45° + ϕ/2 with the horizontal. The lateral pressure distribution on wall AB of height H increases in simple proportion to depth. The pressure acts normal to the wall AB [Fig. 11.7(b)].

The lateral active pressure at A is

$$p_a = \gamma H K_A \tag{11.13}$$

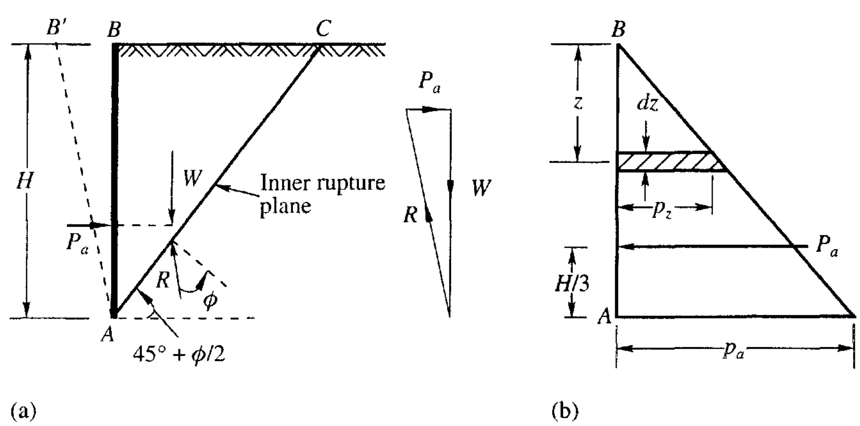

Figure 11.7 Rankine's active earth pressure in cohesionless soil

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The total pressure on AB is therefore

$$P_a = \int_0^H p_z\, dz = K_A \int_0^H \gamma z\, dz = \frac{1}{2} K_A \gamma H^2 \tag{11.14}$$

where, $$K_A = \tan^2(45° - \phi/2) = \frac{1-\sin\phi}{1+\sin\phi} \tag{11.14a}$$

P_a acts at a height $H/3$ above the base of the wall.

Backfill Horizontal-Passive Earth Pressure

If wall AB is pushed into the mass to such an extent as to impart uniform compression throughout the mass, soil wedge ABC in Fig. 11.8(a) will be in Rankine's passive state of plastic equilibrium. The inner rupture plane AC makes an angle $45° + \phi/2$ with the vertical AB. The pressure distribution on wall AB is linear as shown in Fig. 11.8(b).

The passive pressure p_p at A is

$$p_p = \gamma H K_P$$

the total pressure against the wall is

$$P_p = \int_0^H p_z\, dz = K_P \int_0^H \gamma z\, dz = \frac{1}{2} K_P \gamma H^2 \tag{11.15}$$

where, $$K_P = \tan^2(45° + \phi/2) = \frac{1+\sin\phi}{1-\sin\phi} \tag{11.15a}$$

Relationship between K_P and K_A

The ratio of K_P and K_A may be written as

$$\frac{K_P}{K_A} = \frac{\tan^2(45° + \phi/2)}{\tan^2(45° - \phi/2)} = \tan^4(45° + \phi/2) \quad \text{or} \quad K_p = \frac{1}{K_A} \tag{11.16}$$

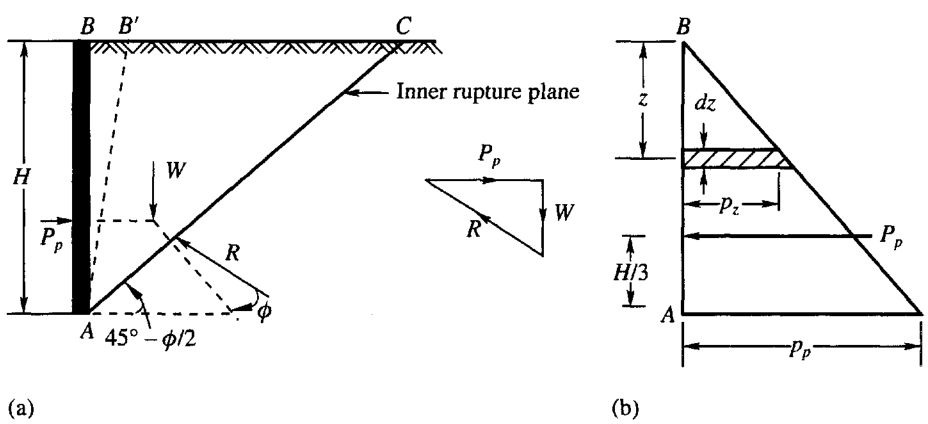

Figure 11.8 Rankine's passive earth pressure in cohesionless soil

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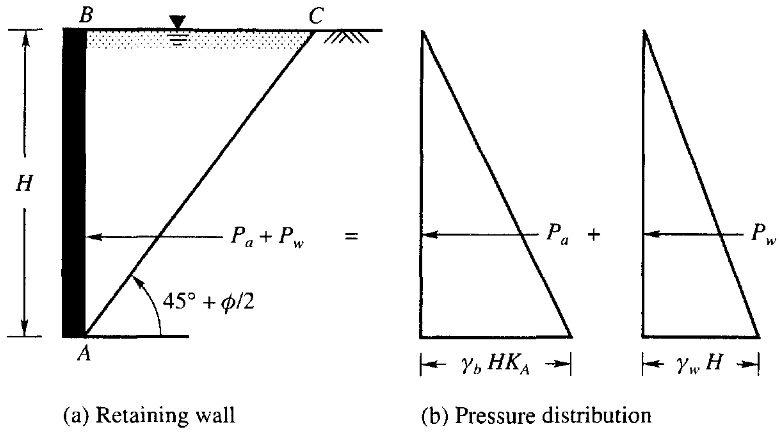

Figure 11.9 Rankine's active pressure under submerged condition in cohesionless soil

For example, if $\phi = 30°$, we have,

$$\frac{K_P}{K_A} = \tan^4 60° = 9, \quad \text{or} \quad K_P = 9K_A$$

This simple demonstration indicates that the value of K_P is quite large compared to K_A.

Active Earth Pressure-Backfill Soil Submerged with the Surface Horizontal

When the backfill is fully submerged, two types of pressures act on wall *AB*. (Fig. 11.9) They are

1. The active earth pressure due to the submerged weight of soil
2. The lateral pressure due to water

At any depth z the total unit pressure on the wall is

$$\overline{p_a} = p_a + p_w = \gamma_b z K_A + \gamma_w z$$

At depth $z = H$, we have

$$\overline{p_a} = \gamma_b H K_A + \gamma_w H$$

where γ_b is the submerged unit weight of soil and γ_w the unit weight of water. The total pressure acting on the wall at a height $H/3$ above the base is

$$\overline{P_a} = P_a + P_w = \frac{1}{2}\gamma_b H^2 K_A + \frac{1}{2}\gamma_w H^2 \tag{11.17}$$

Active Earth Pressure-Backfill Partly Submerged with a Uniform Surcharge Load

The ground water table is at a depth of H_1 below the surface and the soil above this level has an effective moist unit weight of γ. The soil below the water table is submerged with a submerged unit weight γ_b. In this case, the total unit pressure may be expressed as given below.

At depth H_1 at the level of the water table

$$\overline{p_a} = qK_A + \gamma H_1 K_A$$

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At depth H we have

$$\overline{p_a} = qK_A + \gamma H_1 K_A + \gamma_b H_2 K_A + \gamma_w H_2$$

or $$\overline{p_a} = qK_A + (\gamma H_1 + \gamma_b H_2)K_A + \gamma_w H_2 \tag{11.18}$$

The pressure distribution is given in Fig. 11.10(b). It is assumed that the value of ϕ remains the same throughout the depth H.

From Fig. 11.10(b), we may say that the total pressure $\overline{P_a}$ acting per unit length of the wall may be written as equal to

$$\overline{P_a} = qHK_A + \frac{1}{2}\gamma H_1^2 K_A + \gamma H_1 H_2 K_A + \frac{1}{2}H_2^2(\gamma_b K_A + \gamma_w) \tag{11.19}$$

The point of application of $\overline{P_a}$ above the base of the wall can be found by taking moments of all the forces acting on the wall about A.

Sloping Surface-Active Earth Pressure

Figure 11.11(a) shows a smooth vertical wall with a sloping backfill of cohesionless soil. As in the case of a horizontal backfill, the active state of plastic equilibrium can be developed in the backfill by rotating the wall about A away from the backfill. Let AC be the rupture line and the soil within the wedge ABC be in an active state of plastic equilibrium.

Consider a rhombic element E within the plastic zone ABC which is shown to a larger scale outside. The base of the element is parallel to the backfill surface which is inclined at an angle β to the horizontal. The horizontal width of the element is taken as unity.

Let σ_v = the vertical stress acting on an elemental length $ab = \gamma z \cos\beta$

σ_l = the lateral pressure acting on vertical surface bc of the element

The vertical stress σ_v can be resolved into components σ_n the normal stress and τ the shear stress on surface ab of element E. We may now write

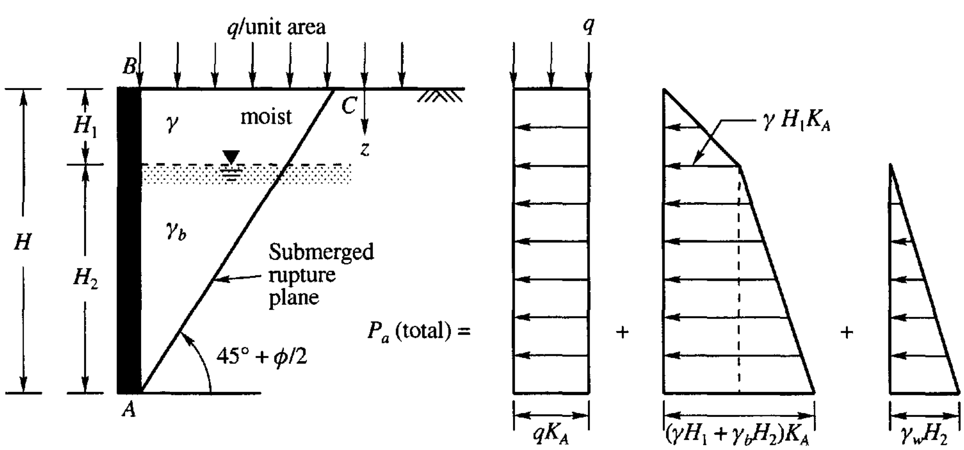

Figure 11.10 Rankine's active pressure in cohesionless backfill under partly submerged condition with surcharge load

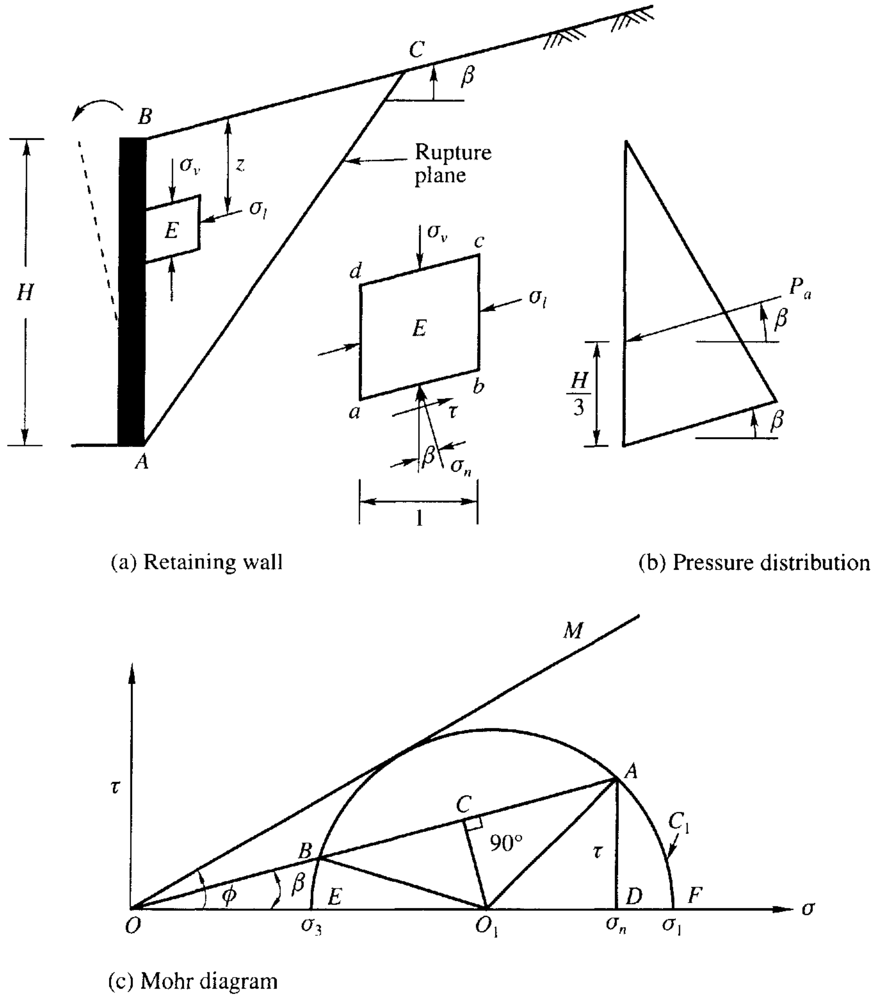

Figure 11.11 Rankine's active pressure for a sloping cohesionless backfill

$$\sigma_n = \sigma_v \cos\beta = \gamma z \cos\beta \cos\beta = \gamma z \cos^2 \beta \quad (11.20)$$

$$\tau = \sigma_v \sin\beta = \gamma z \cos\beta \sin\beta \quad (11.21)$$

A Mohr diagram can be drawn as shown in Fig. 11.11(c). Here, length $OA = \gamma z \cos\beta$ makes an angle β with the σ-axis. $OD = \sigma_n = \gamma z \cos^2\beta$ and $AD = \tau = \gamma z \cos\beta \sin\beta$. OM is the Mohr envelope making an angle ϕ with the σ-axis. Now Mohr circle C_1 can be drawn passing through point A and at the same time tangential to envelope OM. This circle cuts line OA at point B and the σ-axis at E and F.

Now OB = the lateral pressure $\sigma_l = p_a$ in the active state.

The principal stresses are

$OF = \sigma_1$ and $OE = \sigma_3$

The following relationships can be expressed with reference to the Mohr diagram.

$$BC = CA = \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2 \phi - \sin^2 \beta}$$

$$\sigma_v = OA = OC + CA = \frac{\sigma_1 + \sigma_3}{2}\cos\beta + \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\phi - \sin^2\beta}$$

$$\sigma_l = p_a = OC - BC = \frac{\sigma_1 + \sigma_3}{2}\cos\beta - \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\phi - \sin^2\beta} \tag{11.22}$$

Now we have (after simplification)

$$\frac{\sigma_l}{\sigma_v} = \frac{p_a}{\gamma z \cos\beta} = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

or $$p_a = \gamma z \cos\beta \times \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} = \gamma z K_A \tag{11.23}$$

where, $$K_A = \cos\beta \times \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \tag{11.24}$$

is called as *the coefficient of earth pressure* for the active state or the active earth pressure coefficient.

The pressure distribution on the wall is shown in Fig. 11.11(b). The active pressure at depth H is

$$p_a = \gamma H K_A$$

which acts parallel to the surface. The total pressure P_a per unit length of the wall is

$$P_a = \frac{1}{2}\gamma H^2 K_A \tag{11.25}$$

which acts at a height $H/3$ from the base of the wall and parallel to the sloping surface of the backfill.

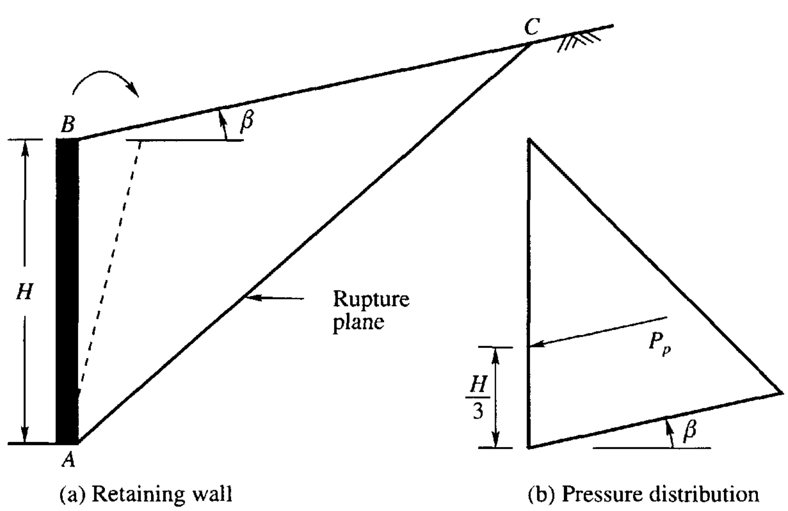

Figure 11.12 Rankine's passive pressure in sloping cohesionless backfill

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Sloping Surface-Passive Earth Pressure (Fig. 11.12)

An equation for P_p for a sloping backfill surface can be developed in the same way as for an active case. The equation for P_p may be expressed as

$$P_p = \frac{1}{2}\gamma H^2 K_p \tag{11.26}$$

where, $$K_p = \cos\beta \times \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \tag{11.27}$$

P_p acts at a height $H/3$ above point A and parallel to the sloping surface.

Example 11.2

A cantilever retaining wall of 7 meter height (Fig. Ex. 11.2) retains sand. The properties of the sand are: $e = 0.5$, $\phi = 30°$ and $G_s = 2.7$. Using Rankine's theory determine the active earth pressure at the base when the backfill is (i) dry, (ii) saturated and (iii) submerged, and also the resultant active force in each case. In addition determine the total water pressure under the submerged condition.

Solution

$$e = 0.5 \text{ and } G_s = 2.7, \quad \gamma_d = \frac{G_s\gamma_w}{1+e} = \frac{2.7}{1+0.5} \times 9.81 = 17.66 \text{ kN/m}^3$$

Saturated unit weight

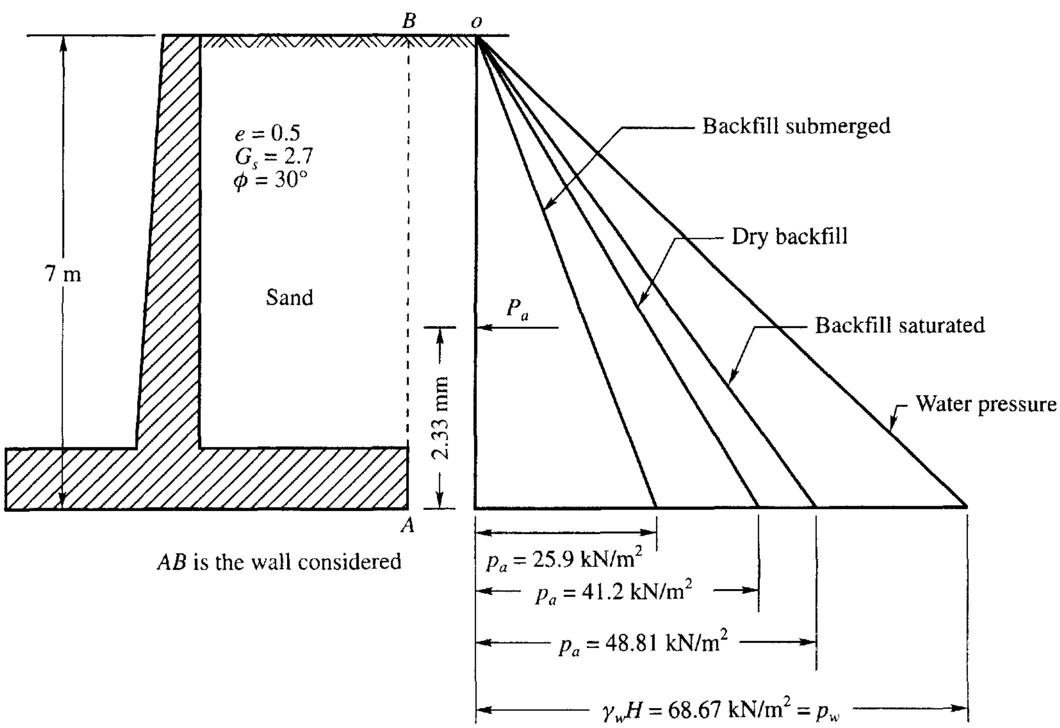

Figure Ex. 11.2

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$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{2.7+0.5}{1+0.5} \times 9.81 = 20.92 \text{ kN/m}^3$$

Submerged unit weight

$$\gamma_b = \gamma_{sat} - \gamma_w = 20.92 - 9.81 = 11.1 \text{ kN/m}^3$$

For $\phi = 30$, $K_A = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$

Active earth pressure at the base is

(i) for dry backfill

$$p_a = K_A \gamma_d H = \frac{1}{3} \times 17.66 \times 7 = 41.2 \text{ kN/m}^2$$

$$P_a = \frac{1}{2} K_A \gamma_d H^2 = \frac{1}{2} \times 41.2 \times 7 = 144.2 \text{ kN / m of wall}$$

(ii) for saturated backfill

$$p_a = K_A \gamma_{sat} H = \frac{1}{3} \times 20.92 \times 7 = 48.81 \text{ kN/m}^2$$

$$P_a = \frac{1}{2} \times 48.81 \times 7 = 170.85 \text{ kN / m of wall}$$

(iii) for submerged backfill
Submerged soil pressure

$$p_a = K_A \gamma_b H = \frac{1}{3} \times 11.1 \times 7 = 25.9 \text{ kN/m}^2$$

$$P_a = \frac{1}{2} \times 25.9 \times 7 = 90.65 \text{ kN / m of wall}$$

Water pressure

$$p_w = \gamma_w H = 9.81 \times 7 = 68.67 \text{ kN/m}^2$$

$$P_w = \frac{1}{2} \gamma_w H^2 = \frac{1}{2} \times 9.81 \times 7^2 = 240.35 \text{ kN/m of wall}$$

Example 11.3

For the earth retaining structure shown in Fig. Ex. 11.3, construct the earth pressure diagram for the active state and determine the total thrust per unit length of the wall.

Solution

For $\phi = 30°$, $K_A = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$

Dry unit weight $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.65}{1+0.65} \times 62.4 = 100.22 \text{ lb/ ft}^3$

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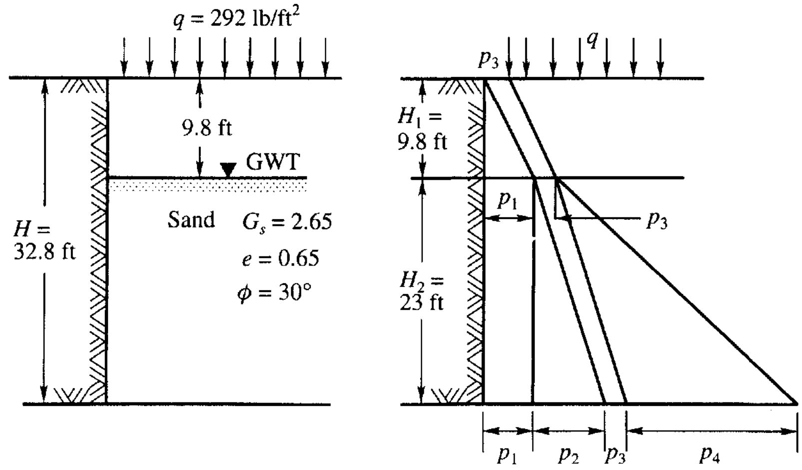

(a) Given system (b) Pressure diagram

Figure Ex. 11.3

$$\gamma_b = \frac{(G_s - 1)\gamma_w}{1+e} = \frac{2.65-1}{1+0.65} \times 62.4 = 62.4 \text{ lb/ft}^3$$

Assuming the soil above the water table is dry, [Refer to Fig. Ex. 11.3(b)].

$$p_1 = K_A \gamma_d H_1 = \frac{1}{3} \times 100.22 \times 9.8 = 327.39 \text{ lb/ft}^2$$

$$p_2 = K_A \gamma_b H_2 = \frac{1}{3} \times 62.4 \times 23 = 478.4 \text{ lb/ft}^2$$

$$p_3 = K_A \times q = \frac{1}{3} \times 292 = 97.33 \text{ lb/ft}^2$$

$$p_4 = (K_A)_w \gamma_w H_2 = 1 \times 62.4 \times 23 = 1435.2 \text{ lb/ft}^2$$

Total thrust = summation of the areas of the different parts of the pressure diagram

$$= \frac{1}{2} p_1 H_1 + p_1 H_2 + \frac{1}{2} p_2 H_2 + p_3 (H_1 + H_2) + \frac{1}{2} p_4 H_2$$

$$= \frac{1}{2} \times 327.39 \times 9.8 + 327.39 \times 23 + \frac{1}{2} \times 478.4 \times 23 + 97.33(32.8) + \frac{1}{2} \times 1435.2 \times 23$$

$$= 34{,}333 \text{ lb/ft} = 34.3 \text{ kips/ft of wall}$$

Example 11.4

A retaining wall with a vertical back of height 7.32 m supports a cohesionless soil of unit weight 17.3 kN/m³ and an angle of shearing resistance $\phi = 30°$. The surface of the soil is horizontal. Determine the magnitude and direction of the active thrust per meter of wall using Rankine theory.

Solution

For the condition given here, Rankine's theory disregards the friction between the soil and the back of the wall.

The coefficient of active earth pressure K_A is

$$K_A = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

The lateral active thrust P_a is

$$P_a = \frac{1}{2}K_A\gamma H^2 = \frac{1}{2}\times\frac{1}{3}\times 17.3(7.32)^2 = 154.5 \text{ kN/m}$$

Example 11.5

A rigid retaining wall 5 m high supports a backfill of cohesionless soil with $\phi = 30°$. The water table is below the base of the wall. The backfill is dry and has a unit weight of 18 kN/m^3. Determine Rankine's passive earth pressure per meter length of the wall (Fig. Ex. 11.5).

Solution

From Eq. (11.15a)

$$K_P = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+\sin 30°}{1-\sin 30°} = \frac{1+0.5}{1-0.5} = 3$$

At the base level, the passive earth pressure is

$$p_p = K_P\gamma H = 3\times 18\times 5 = 270 \text{ kN/m}^2$$

From Eq. (11.15)

$$P_p = \frac{1}{2}K_P\gamma H^2 = \frac{1}{2}\times 3\times 18\times 5^2 = 675 \text{ kN/m length of wall}$$

The pressure distribution is given in Fig. Ex. 11.5.

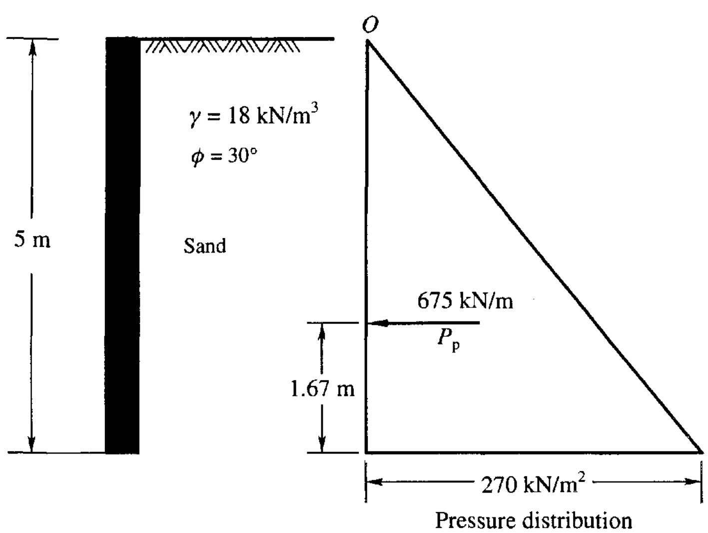

Figure Ex. 11.5

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Example 11.6

A counterfort wall of 10 m height retains a non-cohesive backfill. The void ratio and angle of internal friction of the backfill respectively are 0.70 and 30° in the loose state and they are 0.40 and 40° in the dense state. Calculate and compare active and passive earth pressures for both the cases. Take the specific gravity of solids as 2.7.

Solution

(i) In the loose state, $e = 0.70$ which gives

$$\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.7}{1+0.7} \times 9.81 = 15.6 \text{ kN/m}^3$$

$$\text{For } \phi = 30°, \quad K_A = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}, \text{and } K_P = \frac{1}{K_A} = 3$$

$$\text{Max. } p_a = K_A \gamma_d H = \frac{1}{3} \times 15.6 \times 10 = 52 \text{ kN/m}^2$$

$$\text{Max. } p_p = K_P \gamma_d H = 3 \times 15.6 \times 10 = 468 \text{ kN/m}^2$$

(ii) In the dense state, $e = 0.40$, which gives,

$$\gamma_d = \frac{2.7}{1+0.4} \times 9.81 = 18.92 \text{ kN/m}^3$$

$$\text{For } \phi = 40°, \quad K_A = \frac{1-\sin 40°}{1+\sin 40°} = 0.217, \ K_P = \frac{1}{K_A} = 4.6$$

$$\text{Max. } p_a = K_A \gamma_d H = 0.217 \times 18.92 \times 10 = 41.1 \text{ kN/m}^2$$

$$\text{and Max. } p_p = 4.6 \times 18.92 \times 10 = 870.3 \text{ kN/m}^2$$

Comment: The comparison of the results indicates that densification of soil decreases the active earth pressure and increases the passive earth pressure. This is advantageous in the sense that active earth pressure is a disturbing force and passive earth pressure is a resisting force.

Example 11.7

A wall of 8 m height retains sand having a density of 1.936 Mg/m^3 and an angle of internal friction of 34°. If the surface of the backfill slopes upwards at 15° to the horizontal, find the active thrust per unit length of the wall. Use Rankine's conditions.

Solution

There can be two solutions: analytical and graphical. The analytical solution can be obtained from Eqs. (11.25) and (11.24) viz.,

$$P_a = \frac{1}{2} K_A \gamma H^2$$

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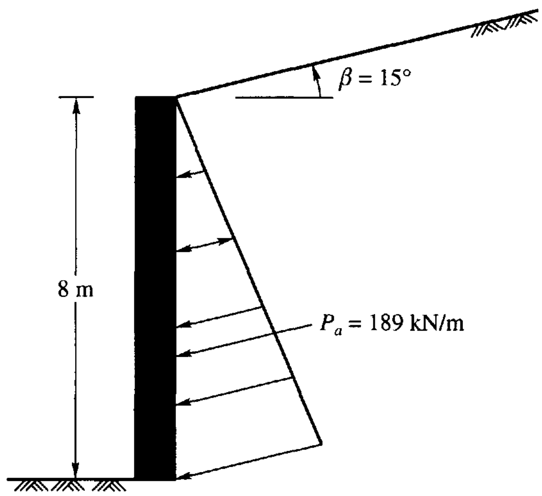

Figure Ex. 11.7a

$$\text{where} \quad K_A = \cos\beta \times \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

where $\beta = 15°$, $\cos\beta = 0.9659$ and $\cos^2\beta = 0.933$

and $\phi = 34°$ gives $\cos^2\phi = 0.688$

$$\text{Hence} \quad K_A = 0.966 \times \frac{0.966 - \sqrt{0.933 - 0.688}}{0.966 + \sqrt{0.933 - 0.688}} = 0.311$$

$$\gamma = 1.936 \times 9.81 = 19.0 \text{ kN/m}^3$$

$$\text{Hence} \quad P_a = \frac{1}{2} \times 0.311 \times 19(8)^2 = 189 \text{ kN/m wall}$$

Graphical Solution

Vertical stress at a depth $z = 8$ m is

$$\gamma H \cos\beta = 19 \times 8 \times \cos 15° = 147 \text{ kN/m}^2$$

Now draw the Mohr envelope at an angle of 34° and the ground line at an angle of 15° with the horizontal axis as shown in Fig. Ex. 11.7b.

Using a suitable scale plot $OP_1 = 147$ kN/m^2.

(i) the center of circle C lies on the horizontal axis,
(ii) the circle passes through point P_1, and
(iii) the circle is tangent to the Mohr envelope

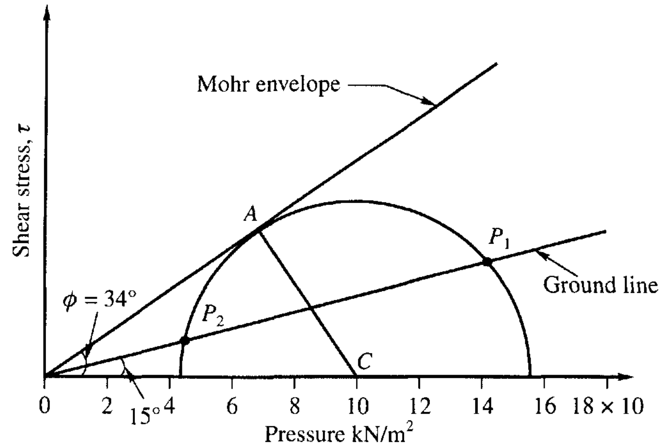

Figure Ex. 11.7b

The point P_2 at which the circle cuts the ground line represents the lateral earth pressure. The length OP_2 measures 47.5 kN/m².

Hence the active thrust per unit length, $P_a = \frac{1}{2} \times 47.5 \times 8 = 190$ kN/m

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11.6 RANKINE'S ACTIVE EARTH PRESSURE WITH COHESIVE BACKFILL

In Fig. 11.13(a) is shown a prismatic element in a semi-infinite mass with a horizontal surface. The vertical pressure on the base AD of the element at depth z is

$$\sigma_v = \gamma z$$

The horizontal pressure on the element when the mass is in a state of plastic equilibrium may be determined by making use of Mohr's stress diagram [Fig. 11.13(b)].

Mohr envelopes $O'A$ and $O'B$ for cohesive soils are expressed by Coulomb's equation

$$s = c + \tan\phi \tag{11.28}$$

Point P_1 on the σ-axis represents the state of stress on the base of the prismatic element. When the mass is in the active state σ_v is the major principal stress σ_1. The horizontal stress σ_h is the minor principal stress σ_3. The Mohr circle of stress C_a passing through P_1 and tangential to the Mohr envelopes $O'A$ and $O'B$ represents the stress conditions in the active state. The relation between the two principal stresses may be expressed by the expression

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \tag{11.29}$$

Substituting $\sigma_1 = \gamma z$, $\sigma_3 = p_a$ and transposing we have

$$p_a = \frac{\gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} = \gamma z K_A - 2c\sqrt{K_A} \tag{11.30}$$

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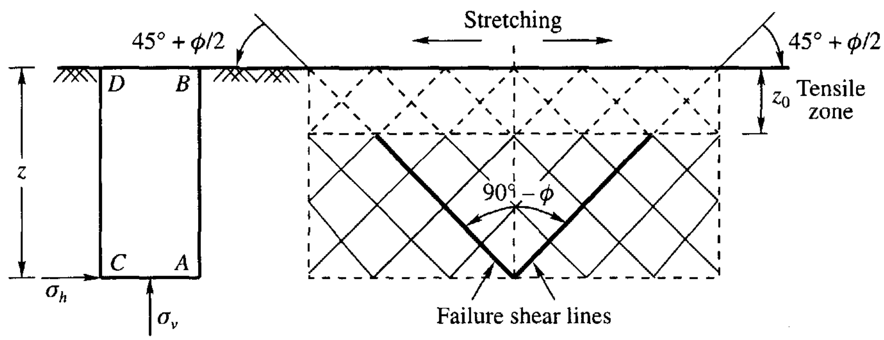
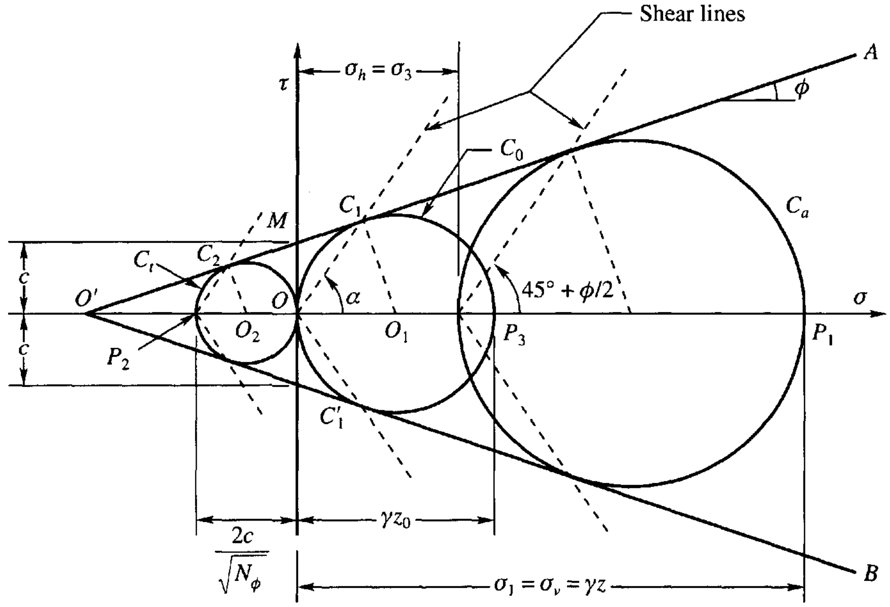


(a) Semi-infinite mass



(b) Mohr diagram

Figure 11.13 Active earth pressure of cohesive soil with horizontal backfill on a vertical wall

The active pressure $p_a = 0$ when

$$\frac{\gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} = 0 \tag{11.31}$$

that is, p_a is zero at depth z, such that

$$z = z_0 = \frac{2c}{\gamma}\sqrt{N_\phi} \tag{11.32}$$

At depth $z = 0$, the pressure p_a is

$$p_a = -\frac{2c}{\sqrt{N_\phi}} \tag{11.33}$$

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Equations (11.32) and (11.33) indicate that the active pressure p_a is tensile between depth 0 and z_0. The Eqs. (11.32) and (11.33) can also be obtained from Mohr circles C_0 and C_t respectively.

Shear Lines Pattern

The shear lines are shown in Fig. 11.13(a). Up to depth z_0 they are shown dotted to indicate that this zone is in tension.

Total Active Earth Pressure on a Vertical Section

If *AB* is the vertical section [11.14(a)], the active pressure distribution against this section of height *H* is shown in Fig. 11.14(b) as per Eq. (11.30). The total pressure against the section is

$$p_a = \int_0^H pz\,dz = \int_0^H \frac{\gamma z}{N_\phi}\,dz - \int_0^H \frac{2c}{\sqrt{N_\phi}}\,dz$$

$$= \frac{1}{2}\gamma H^2 \frac{1}{N_\phi} - 2c\frac{H}{\sqrt{N_\phi}} \tag{11.34}$$

The shaded area in Fig. 11.14(b) gives the total pressure P_a. If the wall has a height

$$H = H_c = \frac{4c}{\gamma}\sqrt{N_\phi} = 2z_0 \tag{11.35}$$

the total earth pressure is equal to zero. This indicates that a vertical bank of height smaller than H_c can stand without lateral support. H_c is called the *critical depth.* However, the pressure against the wall increases from $-2c/\sqrt{N_\phi}$ at the top to $+2c/\sqrt{N_\phi}$ at depth H_c, whereas on the vertical face of an unsupported bank the normal stress is zero at every point. Because of this difference, the greatest depth of which a cut can be excavated without lateral support for its vertical sides is slightly smaller than H_c.

For soft clay, $\phi = 0$, and $N_\phi = 1$

therefore, $$P_a = \frac{1}{2}\gamma H^2 - 2cH \tag{11.36}$$

and $$H_c = \frac{4c}{\gamma} \tag{11.37}$$

Soil does not resist any tension and as such it is quite unlikely that the soil would adhere to the wall within the tension zone of depth z_0 producing cracks in the soil. It is commonly assumed that the active earth pressure is represented by the shaded area in Fig. 11.14(c).

The total pressure on wall *AB* is equal to the area of the triangle in Fig. 11.14(c) which is equal to

$$P_a = \frac{1}{2}\left(\frac{\gamma H}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}\right)(H - z_0) \tag{11.38a}$$

or $$P_a = \frac{1}{2}\left(\frac{\gamma H}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}\right)\left(H - \frac{2c}{\gamma}\sqrt{N_\phi}\right) \tag{11.38b}$$

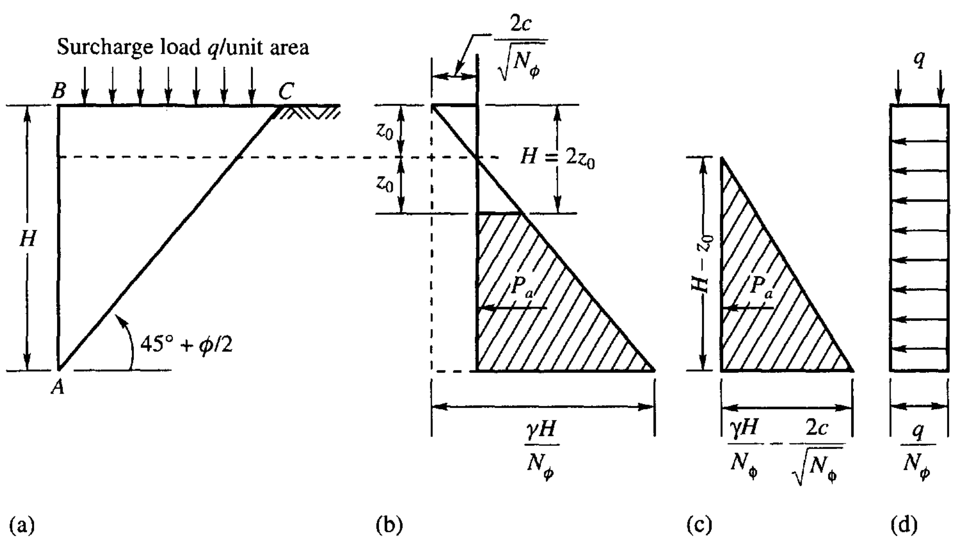

Figure 11.14 Active earth pressure on vertical sections in cohesive soils

Simplifying, we have

$$P_a = \frac{1}{2}\gamma H^2 \frac{1}{N_\phi} - 2cH\frac{1}{\sqrt{N_\phi}} + \frac{2c^2}{\gamma} \tag{11.38c}$$

For soft clay, $\phi = 0$

$$P_a = \frac{1}{2}\gamma H^2 - 2cH + \frac{2c^2}{\gamma} \tag{11.39}$$

It may be noted that $K_A = 1/N_\phi$

Effect of Surcharge and Water Table

Effect of Surcharge

When a surcharge load q per unit area acts on the surface, the lateral pressure on the wall due to surcharge remains constant with depth as shown in Fig. 11.14(d) for the active condition. The lateral pressure due to a surcharge under the active state may be written as

$$p_{aq} = \frac{q}{N_\phi}$$

The total active pressure due to a surcharge load is,

$$P_{aq} = \frac{qH}{N_\phi} \tag{11.40}$$

Effect of Water Table

If the soil is partly submerged, the submerged unit weight below the water table will have to be taken into account in both the active and passive states.

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Figure 11.15(a) shows the case of a wall in the active state with cohesive material as backfill. The water table is at a depth of H_1 below the top of the wall. The depth of water is H_2.

The lateral pressure on the wall due to partial submergence is due to soil and water as shown in Fig. 11.15(b). The pressure due to soil = area of the figure *ocebo*.

The total pressure due to soil

$$P_a = oab + acdb + bde$$

$$\text{or} \quad P_a = \frac{1}{2}(H_1 - z_0)\left(\frac{\gamma_t H_1}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}\right) + \left(\frac{\gamma_t H_1}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}\right)H_2 + \frac{1}{2}\frac{\gamma_b H_2^2}{N_\phi} \tag{11.41}$$

After substituting for $z_0 = \frac{2c}{\gamma_t}\sqrt{N_\phi}$

and simplifying we have

$$P_a = \frac{1}{2N_\phi}(\gamma_t H_1^2 + \gamma_b H_2^2) - \frac{2c}{\sqrt{N_\phi}}(H_1 + H_2) + \frac{\gamma_t H_1 H_2}{N_\phi} + \frac{2c^2}{\gamma_t} \tag{11.42}$$

The total pressure on the wall due to water is

$$P_w = \frac{1}{2}\gamma_w H_2^2 \tag{11.43}$$

The point of application of P_a can be determined without any difficulty. The point of application P_w is at a height of $H_2/3$ from the base of the wall.

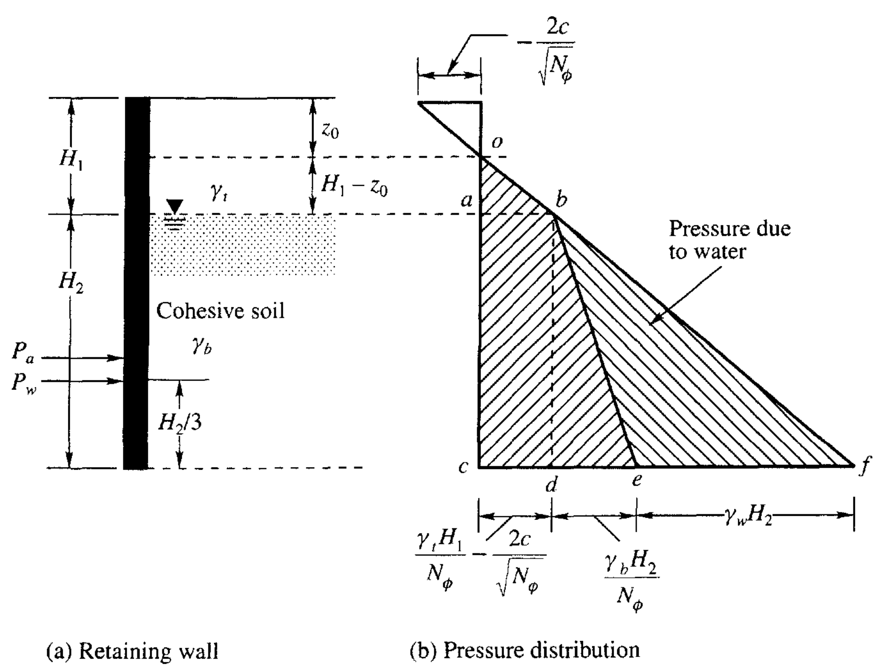

Figure 11.15 Effect of water table on lateral earth pressure

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If the backfill material is cohesionless, the terms containing cohesion c in Eq. (11.42) reduce to zero.

Example 11.8

A retaining wall has a vertical back and is 7.32 m high. The soil is sandy loam of unit weight 17.3 kN/m^3. It has a cohesion of 12 kN/m^2 and $\phi = 20°$. Neglecting wall friction, determine the active thrust on the wall. The upper surface of the fill is horizontal.

Solution

(Refer to Fig. 11.14)

When the material exhibits cohesion, the pressure on the wall at a depth z is given by (Eq. 11.30)

$$p_a = \gamma z K_A - 2c\sqrt{K_A}$$

$$\text{where} \quad K_A = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 20°}{1+\sin 20°} = 0.49, \quad \sqrt{K_A} = 0.7$$

When the depth is small the expression for z is negative because of the effect of cohesion up to a theoretical depth z_0. The soil is in tension and the soil draws away from the wall.

$$z_0 = \frac{2c}{\gamma}\sqrt{N_\phi} = \frac{2c}{\gamma}\sqrt{K_P}$$

$$\text{where} \quad K_P = \frac{1+\sin\phi}{1-\sin\phi} = 2.04, \text{ and } \sqrt{K_P} = 1.43$$

$$\text{Therefore} \quad z_0 = \frac{2\times 12}{17.3}\times 1.43 = 1.98 \text{ m}$$

The lateral pressure at the surface ($z = 0$) is

$$p_a = -2c\sqrt{K_A} = -2\times 12\times 0.7 = -16.8 \text{ kN/m}^2$$

The negative sign indicates tension.

The lateral pressure at the base of the wall ($z = 7.32$ m) is

$$p_a = 17.3\times 7.32\times 0.49 - 16.8 = 45.25 \text{ kN/m}^2$$

Theoretically the area of the upper triangle in Fig. 11.14(b) to the left of the pressure axis represents a tensile force which should be subtracted from the compressive force on the lower part of the wall below the depth z_0. Since tension cannot be applied physically between the soil and the wall, this tensile force is neglected. It is therefore commonly assumed that the active earth pressure is represented by the shaded area in Fig. 11.14(c). The total pressure on the wall is equal to the area of the triangle in Fig. 11.14(c).

$$P_a = \frac{1}{2}(\gamma H K_A - 2c\sqrt{K_A})(H - z_0)$$

$$= \frac{1}{2}(17.3\times 7.32\times 0.49 - 2\times 12\times 0.7)(7.32 - 1.98) = 120.8 \text{ kN/m}$$

Example 11.9

Find the resultant thrust on the wall in Ex. 11.8 if the drains are blocked and water builds up behind the wall until the water table reaches a height of 2.75 m above the bottom of the wall.

Solution

For details refer to Fig. 11.15.

Per this figure,

$$H_1 = 7.32 - 2.75 = 4.57 \text{ m}, \; H_2 = 2.75 \text{ m}, \; H_1 - z_0 = 4.57 - 1.98 = 2.59 \text{ m}$$

The base pressure is detailed in Fig. 11.15(b)

(1) $\gamma_{sat} H_1 K_A - 2c\sqrt{K_A} = 17.3 \times 4.57 \times 0.49 - 2 \times 12 \times 0.7 = 21.94 \text{ kN/m}^2$

(2) $\gamma_b H_2 K_A = (17.3 - 9.81) \times 2.75 \times 0.49 = 10.1 \text{ kN/m}^2$

(3) $\gamma_w H_2 = 9.81 \times 2.75 = 27 \text{ kN/m}^2$

The total pressure = P_a = pressure due to soil + water

From Eqs. (11.41), (11.43), and Fig. 11.15(b)

$$P_a = oab + acdb + bde + bef$$

$$= \frac{1}{2} \times 2.59 \times 21.94 + 2.75 \times 21.94 + \frac{1}{2} \times 2.75 \times 10.1 + \frac{1}{2} \times 2.75 \times 27$$

= 28.41 + 60.34 + 13.89 +37.13 = 139.7 kN/m or say 140 kN/m

The point of application of P_a may be found by taking moments of each area and P_a about the base. Let h be the height of P_a above the base. Now

$$140 \times h = 28.41 \;\; \frac{1}{3} \times 2.59 + 2.75 \;\; + 60.34 \times \frac{2.75}{2} + 13.89 \times \frac{2.75}{3} + \frac{37.13 \times 2.75}{3}$$

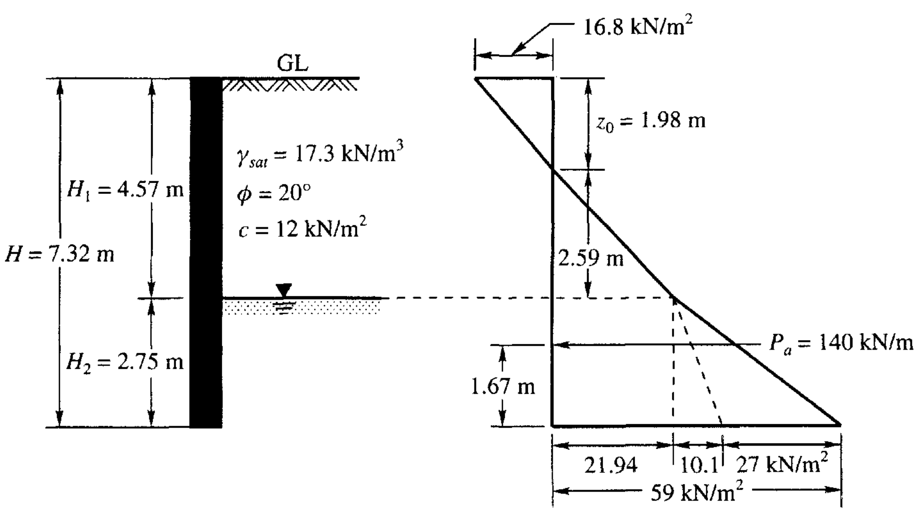

Figure Ex. 11.9

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$$= 102.65 + 83.0 + 12.7 + 34.0 = 232.4$$

or $$h = \frac{232.4}{140} = 1.66 \text{ m}$$

Example 11.10

A rigid retaining wall 19.69 ft high has a saturated backfill of soft clay soil. The properties of the clay soil are $\gamma_{sat} = 111.76$ lb/ft^3, and unit cohesion $c_u = 376$ lb/ft^2. Determine (a) the expected depth of the tensile crack in the soil (b) the active earth pressure before the occurrence of the tensile crack, and (c) the active pressure after the occurrence of the tensile crack. Neglect the effect of water that may collect in the crack.

Solution

At z = 0, $p_a = -2c = -2 \times 376 = -752$ lb/ft^2 since $\phi = 0$

At $z = H$, $p_a = \gamma H - 2c = 111.76 \times 19.69 - 2 \times 376 = 1449$ lb/ft^2

(a) From Eq. (11.32), the depth of the tensile crack z_0 is (for $\phi = 0$)

$$z_0 = \frac{2c}{\gamma} = \frac{2 \times 376}{111.76} = 6.73 \text{ ft}$$

(b) The active earth pressure before the crack occurs.

Use Eq. (11.36) for computing P_a

$$P_a = \frac{1}{2}\gamma H^2 - 2cH$$

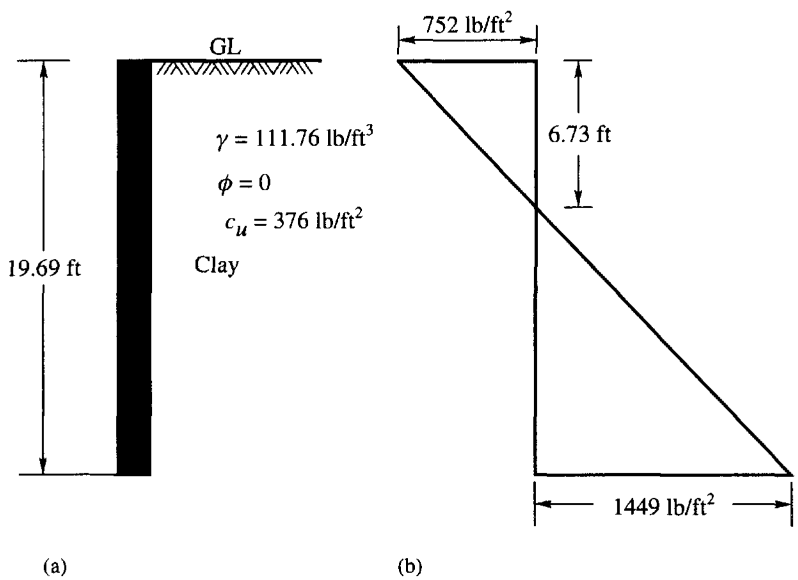

Figure Ex. 11.10

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since $K_A = 1$ for $\phi = 0$. Substituting, we have

$$P_a = \frac{1}{2} \times 111.76 \times (19.69)^2 - 2 \times 376 \times 19.69 = 21{,}664 - 14{,}807 = 6857 \text{ lb/ ft}$$

(c) P_a after the occurrence of a tensile crack.

Use Eq. (11.38a),

$$P_a = \frac{1}{2}(\gamma H - 2c)(H - z_0)$$

Substituting

$$P_a = \frac{1}{2}(111.76 \times 19.69 - 2 \times 376)(19.69 - 6.73) = 9387 \text{ lb/ft}$$

Example 11.11

A rigid retaining wall of 6 m height (Fig. Ex. 11.11) has two layers of backfill. The top layer to a depth of 1.5 m is sandy clay having $\phi = 20°$, $c = 12.15$ kN/m^2 and $\gamma = 16.4$ kN/m^3. The bottom layer is sand having $\phi = 30°$, $c = 0$, and $\gamma = 17.25$ kN/m^3.

Determine the total active earth pressure acting on the wall and draw the pressure distribution diagram.

Solution

For the top layer,

$$K_A = \tan^2 45° - \frac{20}{2} = 0.49, \quad K_P = \frac{1}{0.49} = 2.04$$

The depth of the tensile zone, z_0 is

$$z_0 = \frac{2c}{\gamma}\sqrt{K_p} = \frac{2 \times 12.15\sqrt{2.04}}{16.4} = 2.12 \text{ m}$$

Since the depth of the sandy clay layer is 1.5 m, which is less than z_0, the tensile crack develops only to a depth of 1.5 m.

K_A for the sandy layer is

$$K_A = \tan^2\left(45° - \frac{\phi}{2}\right) = \tan^2\left(45° - \frac{30}{2}\right) = \frac{1}{3}$$

At a depth $z = 1.5$, the vertical pressure σ_v is

$$\sigma_v = \gamma z = 16.4 \times 1.5 = 24.6 \text{ kN/m}^2$$

The active pressure is

$$p_a = K_A \gamma z = \frac{1}{3} \times 24.6 = 8.2 \text{ kN/m}^2$$

At a depth of 6 m, the effective vertical pressure is

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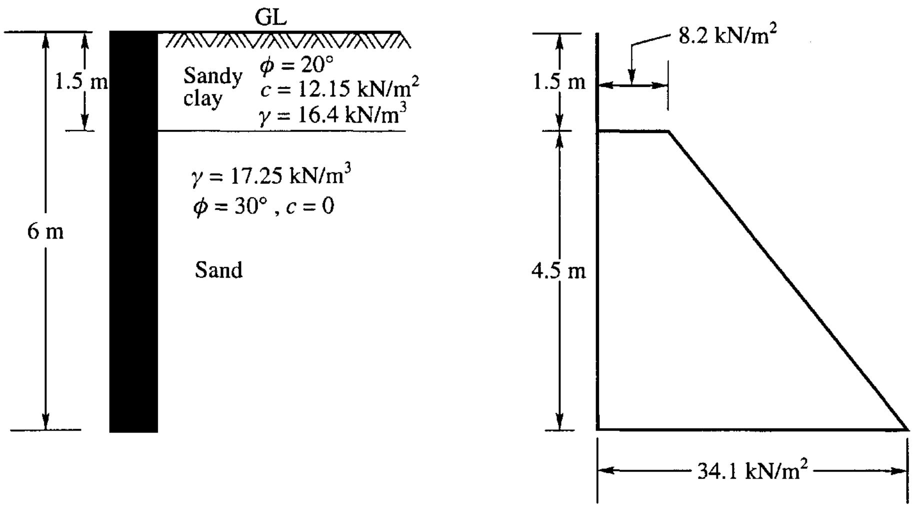

Figure Ex. 11.11

$$\sigma_v = 1.5 \times 16.4 + 4.5 \times 17.25 = 24.6 + 77.63 = 102.23 \text{ kN/m}^2$$

The active pressure p_a is

$$p_a = K_A \sigma_v = \frac{1}{3} \times 102.23 = 34.1 \text{ kN/m}^2$$

The pressure distribution diagram is given in Fig. Ex. 11.11.

11.7 RANKINE'S PASSIVE EARTH PRESSURE WITH COHESIVE BACKFILL

If the wall AB in Fig. 11.16(a) is pushed towards the backfill, the horizontal pressure p_h on the wall increases and becomes greater than the vertical pressure σ_v. When the wall is pushed sufficiently inside, the backfill attains Rankine's state of plastic equilibrium. The pressure distribution on the wall may be expressed by the equation

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

In the passive state, the horizontal stress σ_h is the major principal stress σ_1 and the vertical stress σ_v is the minor principal stress σ_3. Since $\sigma_3 = \gamma z$, the passive pressure at any depth z may be written as

$$\sigma_1 = \sigma_h = p_p = \gamma z N_\phi + 2c\sqrt{N_\phi} = \gamma z K_P + 2c\sqrt{K_P} \quad (11.44a)$$

At depth $z = O$, $p_p = 2c\sqrt{N_\phi} = 2c\sqrt{K_P}$

At depth $z = H$, $p_p = \gamma H N_\phi + 2c\sqrt{N_\phi} = \gamma H K_P + 2c\sqrt{K_P} \quad (11.44b)$

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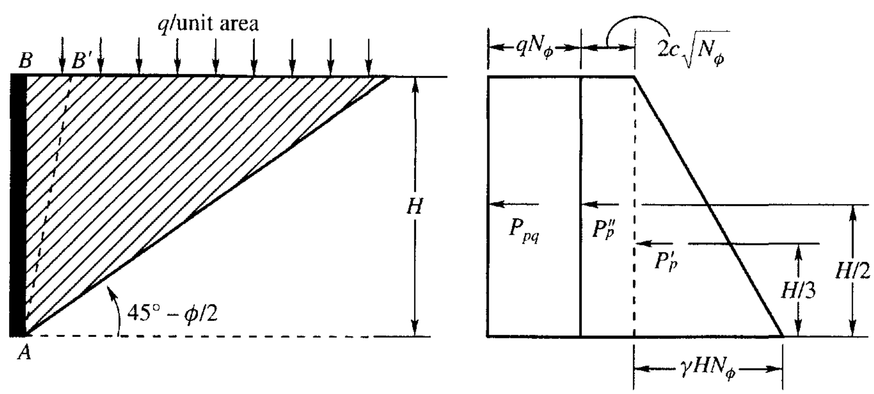

(a) Wall (b) Pressure distribution

Figure 11.16 Passive earth pressure on vertical sections in cohesive soils

The distribution of pressure with respect to depth is shown in Fig. 11.16(b). The pressure increases hydrostatically. The total pressure on the wall may be written as a sum of two pressures P'_p and P''_p

$$P'_p = \int_0^H \gamma z N_\phi \, dz = \frac{1}{2}\gamma H^2 N_\phi = \frac{1}{2}\gamma H^2 K_P \tag{11.45a}$$

This acts at a height $H/3$ from the base.

$$P''_p = \int_0^H 2c\sqrt{N_\phi}\, dz = 2cH\sqrt{N_\phi} = 2cH\sqrt{K_P} \tag{11.45b}$$

This acts at a height of $H/2$ from the base.

$$P_p = P'_p + P''_p = \frac{1}{2}\gamma H^2 K_p + 2cH\sqrt{K_p} \tag{11.45c}$$

The passive pressure due to a surcharge load of q per unit area is

$$p_{pq} = qN_\phi = qK_P$$

The total passive pressure due to a surcharge load is

$$P_{pq} = qHN_\phi = qHK_P \tag{11.46}$$

which acts at mid-height of the wall.

It may be noted here that $N_\phi = K_P$.

Example 11.12

A smooth rigid retaining wall 19.69 ft high carries a uniform surcharge load of 251 lb/ft². The backfill is clayey sand with the following properties:

$\gamma = 102$ lb/ft³, $\phi = 25°$, and $c = 136$ lb/ft².

Determine the passive earth pressure and draw the pressure diagram.

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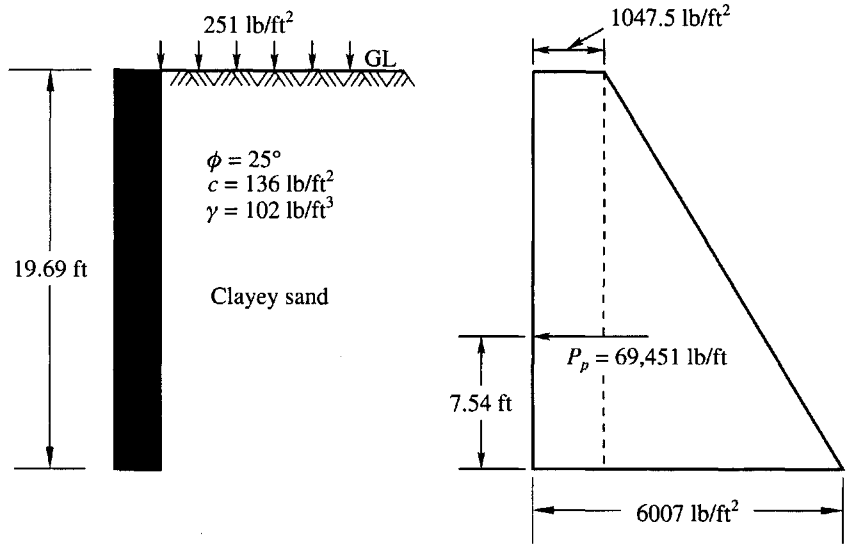


Figure Ex. 11.12

Solution

For $\phi = 25°$, the value of K_P is

$$K_P = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+0.423}{1-0.423} = \frac{1.423}{0.577} = 2.47$$

From Eq. (11.44a), p_p at any depth z is

$$p_p = \gamma z K_P + 2c\sqrt{K_P} = \sigma_v K_P + 2c\sqrt{K_P}$$

At depth $z = 0$, $\sigma_v = 251$ lb/ft²

$$p_p = 251 \times 2.47 + 2 \times 136\sqrt{2.47} = 1047.5 \text{ lb/ ft}^2$$

At $\quad z = 19.69$ ft, $\sigma_v = 251 + 19.69 \times 102 = 2259$ lb/ ft²

$$p_p = 2259 \times 2.47 + 2 \times 136\sqrt{2.47} = 6007 \text{ lb/ ft}^2$$

The pressure distribution is shown in Fig. Ex. 11.12.

The total passive pressure P_P acting on the wall is

$$P_P = 1047.5 \times 19.69 + \frac{1}{2} \times 19.69(6007 - 1047.5) = 69{,}451 \text{ lb/ ft of wall} \approx 69.5 \text{ kips/ft of wall.}$$

Location of resultant

Taking moments about the base

$$P_p \times h = \frac{1}{2} \times (19.69)^2 \times 1047.5 + \frac{1}{6} \times (19.69)^2 \times 4959.5$$
$$= 523{,}518 \text{ lb.ft.}$$

$$\text{or} \quad h = \frac{523{,}518}{P_p} = \frac{523{,}518}{69{,}451} = 7.54 \text{ ft}$$

11.8 COULOMB'S EARTH PRESSURE THEORY FOR SAND FOR ACTIVE STATE

Coulomb made the following assumptions in the development of his theory:

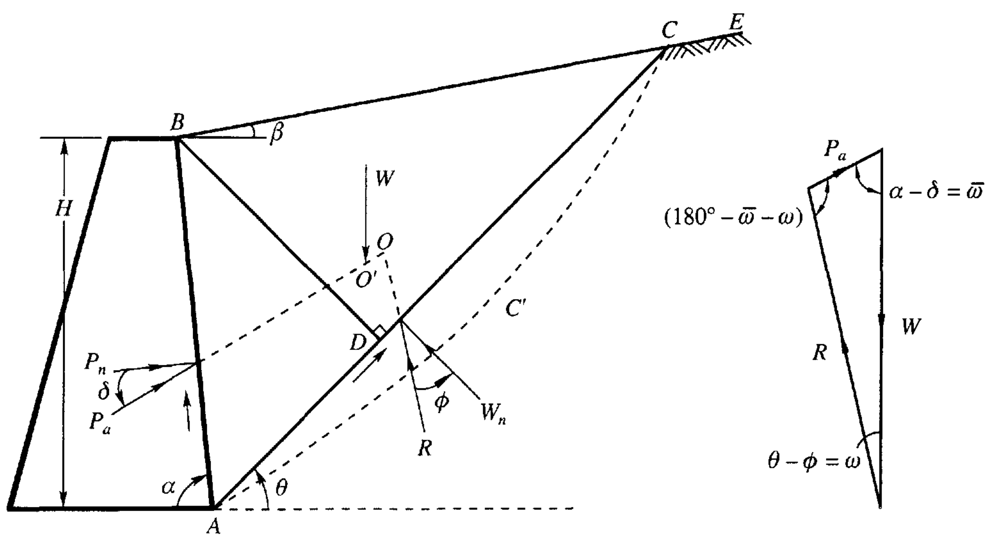
1. The soil is isotropic and homogeneous
2. The rupture surface is a plane surface
3. The failure wedge is a rigid body
4. The pressure surface is a plane surface
5. There is wall friction on the pressure surface
6. Failure is two-dimensional and
7. The soil is cohesionless

Consider Fig. 11.17.

1. AB is the pressure face
2. The backfill surface BE is a plane inclined at an angle β with the horizontal
3. α is the angle made by the pressure face AB with the horizontal
4. H is the height of the wall
5. AC is the assumed rupture plane surface, and
6. θ is the angle made by the surface AC with the horizontal

If AC in Fig. 17(a) is the probable rupture plane, the weight of the wedge W per unit length of the wall may be written as

$W = \gamma A$, where A = area of wedge ABC



(a) Retaining wall (b) Polygon of forces

Figure 11.17 Conditions for failure under active conditions

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Area of wedge $ABC = A = 1/2\ AC \times BD$

where BD is drawn perpendicular to AC.

From the law of sines, we have

$$AC = AB\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}, \quad BD = AB\sin(\alpha+\theta), \quad AB = \frac{H}{\sin\alpha}$$

Making the substitution and simplifying we have,

$$W = \gamma A = \frac{\gamma H^2}{2\sin^2\alpha}\sin(\alpha+\theta)\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)} \tag{11.47}$$

The various forces that are acting on the wedge are shown in Fig. 11.17(a). As the pressure face AB moves away from the backfill, there will be sliding of the soil mass along the wall from B towards A. The sliding of the soil mass is resisted by the friction of the surface. The direction of the shear stress is in the direction from A towards B. If P_n is the total normal reaction of the soil pressure acting on face AB, the resultant of P_n and the shearing stress is the active pressure P_a making an angle δ with the normal. Since the shearing stress acts upwards, the resulting P_a dips below the normal. The angle δ for this condition is considered *positive.*

As the wedge ABC ruptures along plane AC, it slides along this plane. This is resisted by the frictional force acting between the soil at rest below AC, and the sliding wedge. The resisting shearing stress is acting in the direction from A towards C. If W_n is the normal component of the weight of wedge W on plane AC, the resultant of the normal W_n and the shearing stress is the reaction R. This makes an angle ϕ with the normal since the rupture takes place within the soil itself. Statical equilibrium requires that the three forces P_a, W, and R meet at a point. Since AC is not the actual rupture plane, the three forces do not meet at a point. But if the actual surface of failure $AC'C$ is considered, all three forces meet at a point. However, the error due to the nonconcurrence of the forces is very insignificant and as such may be neglected.

The polygon of forces is shown in Fig. 11.17(b). From the polygon of forces, we may write

$$\frac{P_a}{\sin(\theta-\phi)} = \frac{W}{\sin(180^\circ-\alpha-\theta+\phi+\delta)}$$

$$\text{or} \quad P_a = \frac{W\sin(\theta-\phi)}{\sin(180^\circ-\alpha-\theta+\phi+\delta)} \tag{11.48}$$

In Eq. (11.48), the only variable is θ and all the other terms for a given case are constants. Substituting for W, we have

$$P_a = \frac{\gamma H^2}{2\sin^2\alpha}\,\frac{\sin(\theta-\phi)}{\sin(180^\circ-\alpha-\theta+\phi+\delta)}\left(\sin(\alpha+\phi)\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}\right) \tag{11.49}$$

The maximum value for P_a is obtained by differentiating Eq. (11.49) with respect to θ and equating the derivative to zero, i.e.

$$\frac{dP_a}{d\theta} = 0$$

The maximum value of P_a so obtained may be written as

$$P_a = \frac{1}{2}\gamma H^2 K_A \tag{11.50}$$

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Table 11.2a Active earth pressure coefficients K_A for $\beta = 0$ and $\alpha = 90°$

ϕ^0	15	20	25	30	35	40
$\delta = 0$	0.59	0.49	0.41	0.33	0.27	0.22
$\delta = +\phi/2$	0.55	0.45	0.38	0.32	0.26	0.22
$\delta = +/2/3\phi$	0.54	0.44	0.37	0.31	0.26	0.22
$\delta = +\phi$	0.53	0.44	0.37	0.31	0.26	0.22

Table 11.2b Active earth pressure coefficients K_A for $\delta = 0$, β varies from −30° to +30° and α from 70° to 110°

$\beta =$		−30°	−12°	0°	+12°	+ 30°
$\phi = 20°$	α =70°	–	0.54	0.61	0.76	–
	80°	–	0.49	0.54	0.67	–
	90°	–	0.44	0.49	0.60	–
	100	–	0.37	0.41	0.49	–
	110	–	0.30	0.33	0.38	–
$\phi = 30°$	70°	0.32	0.40	0.47	0.55	1.10
	80°	0.30	0.35	0.40	0.47	0.91
	90°	0.26	0.30	0.33	0.38	0.75
	100	0.22	0.25	0.27	0.31	0.60
	110	0.17	0.19	0.20	0.23	0.47
$\phi = 40°$	70	0.25	0.31	0.36	0.40	0.55
	80	0.22	0.26	0.28	0.32	0.42
	90	0.18	0.20	0.22	0.24	0.32
	100	0.13	0.15	0.16	0.17	0.24
	110	0.10	0.10	0.11	0.12	0.15

where K_A is the active earth pressure coefficient.

$$K_A = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha \sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2} \tag{11.51}$$

The total normal component P_n of the earth pressure on the back of the wall is

$$P_n = P_a \cos\delta = \frac{1}{2}\gamma H^2 K_A \cos\delta \tag{11.52}$$

If the wall is vertical and smooth, and if the backfill is horizontal, we have

$\beta = \delta = 0$ and $\alpha = 90°$

Substituting these values in Eq. (11.51), we have

$$K_A = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45° - \frac{\phi}{2}\right) = \frac{1}{N_\phi} \tag{11.53}$$

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where $N_\phi = \tan^2\left(45° + \frac{\phi}{2}\right)$ (11.54)

The coefficient K_A in Eq. (11.53) is the same as Rankine's. The effect of wall friction is frequently neglected where active pressures are concerned. Table 11.2 makes this clear. It is clear from this table that K_A decreases with an increase of δ and the maximum decrease is not more than 10 percent.

11.9 COULOMB'S EARTH PRESSURE THEORY FOR SAND FOR PASSIVE STATE

In Fig. 11.18, the notations used are the same as in Fig. 11.17. As the wall moves into the backfill, the soil tries to move up on the pressure surface *AB* which is resisted by friction of the surface. Shearing stress on this surface therefore acts downward. The passive earth pressure P_p is the resultant of the normal pressure P_{pn} and the shearing stress. The shearing force is rotated upward with an angle δ which is again the angle of wall friction. In this case δ is *positive.*

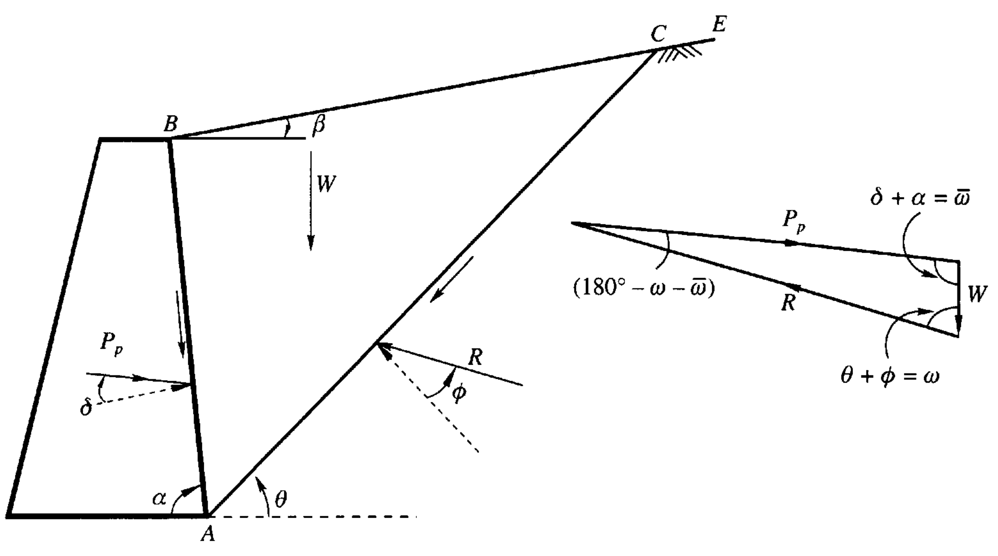
As the rupture takes place along assumed plane surface *AC*, the soil tries to move up the plane which is resisted by the frictional force acting on that line. The shearing stress therefore, acts downward. The reaction *R* makes an angle ϕ with the normal and is rotated upwards as shown in the figure.

The polygon of forces is shown in (b) of the Fig. 11.18. Proceeding in the same way as for active earth pressure, we may write the following equations:

$$W = \frac{\gamma H^2}{2\sin^2\alpha}\sin(\alpha+\theta)\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)} \quad (11.55)$$

$$P_p = \frac{W\sin(\theta+\phi)}{\sin(180° - \theta - \phi - \delta - \alpha)} \quad (11.56)$$

Differentiating Eq. (11.56) with respect to θ and setting the derivative to zero, gives the minimum value of P_p as



(a) Forces on the sliding wedge (b) Polygon of forces

Figure 11.18 Conditions for failure under passive state

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$$P_p = \frac{1}{2}\gamma H^2 K_p \tag{11.57}$$

where K_p is called the *passive earth pressure coefficient.*

$$K_P = \frac{\sin^2(\alpha - \phi)}{\sin^2\alpha \sin(\alpha + \delta)\left[1 - \sqrt{\frac{\sin(\phi + \delta)\sin(\phi + \beta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^2} \tag{11.58}$$

Eq. (11.58) is valid for both positive and negative values of β and δ.

The total normal component of the passive earth pressure P_p on the back of the wall is

$$P_{pn} = \frac{1}{2}\gamma H^2 K_P \cos\delta \tag{11.59}$$

For a smooth vertical wall with a horizontal backfill, we have

$$K_P = \frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2\left(45° + \frac{\phi}{2}\right) = N_\phi \tag{11.60}$$

Eq. (11.60) is Rankine's passive earth pressure coefficient. We can see from Eqs. (11.53) and (11.60) that

$$K_P = \frac{1}{K_A} \tag{11.61}$$

Coulomb sliding wedge theory of plane surfaces of failure is valid with respect to passive pressure, i.e., to the resistance of non-cohesive soils only. If wall friction is zero for a vertical wall and horizontal backfill, the value of K_P may be calculated using Eq. (11.59). If wall friction is considered in conjunction with plane surfaces of failure, much too high, and therefore unsafe values of earth resistance will be obtained, especially in the case of high friction angles ϕ. For example for $\phi = \delta = 40°$, and for plane surfaces of failure, $K_P = 92.3$, whereas for curved surfaces of failure $K_P = 17.5$. However, if δ is smaller than $\phi/2$, the difference between the real surface of sliding and Coulomb's plane surface is very small and we can compute the corresponding passive earth pressure coefficient by means of Eq. (11.57). If δ is greater than $\phi/2$, the values of K_P should be obtained by analyzing curved surfaces of failure.

11.10 ACTIVE PRESSURE BY CULMANN'S METHOD FOR COHESIONLESS SOILS

Without Surcharge Line Load

Culmann's (1875) method is the same as the trial wedge method. In Culmann's method, the force polygons are constructed directly on the ϕ-line AE taking AE as the load line. The procedure is as follows:

In Fig. 11.19(a) AB is the retaining wall drawn to a suitable scale. The various steps in the construction of the pressure locus are:

1. Draw ϕ -line AE at an angle ϕ to the horizontal.
2. Lay off on AE distances, AV, $A1$, A2, A3, etc. to a suitable scale to represent the weights of wedges ABV, AB1, AB2, AB3, etc. respectively.

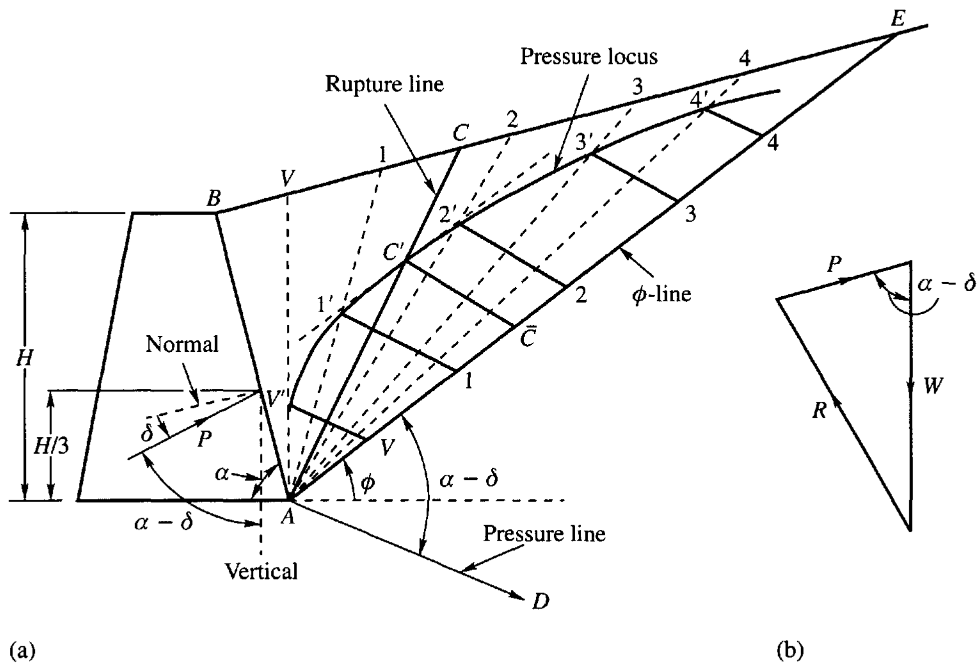

Figure 11.19 Active pressure by Culmann's method for cohesionless soils

3. Draw lines parallel to AD from points V, 1, 2, 3 to intersect assumed rupture lines AV, A1, A2, A3 at points V', $1'$, $2'$, $3'$, etc. respectively.
4. Join points V', $1'$, $2'$, $3'$, etc. by a smooth curve which is the pressure locus.
5. Select point C' on the pressure locus such that the tangent to the curve at this point is parallel to the ϕ-line AE.
6. Draw $C'\bar{C}$ parallel to the pressure line AD. The magnitude of $C'\bar{C}$ in its natural units gives the active pressure P_a.
7. Join AC' and produce to meet the surface of the backfill at C. AC is the rupture line.

For the plane backfill surface, the point of application of P_a is at a height of $H/3$ from the base of the wall.

Example 11.13

For a retaining wall system, the following data were available: (i) Height of wall = 7 m, (ii) Properties of backfill: γ_d = 16 kN/m^3, ϕ = 35°, (iii) angle of wall friction, δ = 20°, (iv) back of wall is inclined at 20° to the vertical (positive batter), and (v) backfill surface is sloping at 1 : 10.

Determine the magnitude of the active earth pressure by Culmann's method.

Solution

(a) Fig. Ex. 11.13 shows the ϕ line and pressure lines drawn to a suitable scale.
(b) The trial rupture lines Bc_1, Bc_2, Bc_3, etc. are drawn by making $Ac_1 = c_1c_2 = c_2c_3$, etc.
(c) The length of a vertical line from B to the backfill surface is measured.
(d) The areas of wedges BAc_1, BAc_2, BAc_3, etc. are respectively equal to 1/2(base lengths Ac_1, Ac_2, Ac_3, etc.) × perpendicular length.

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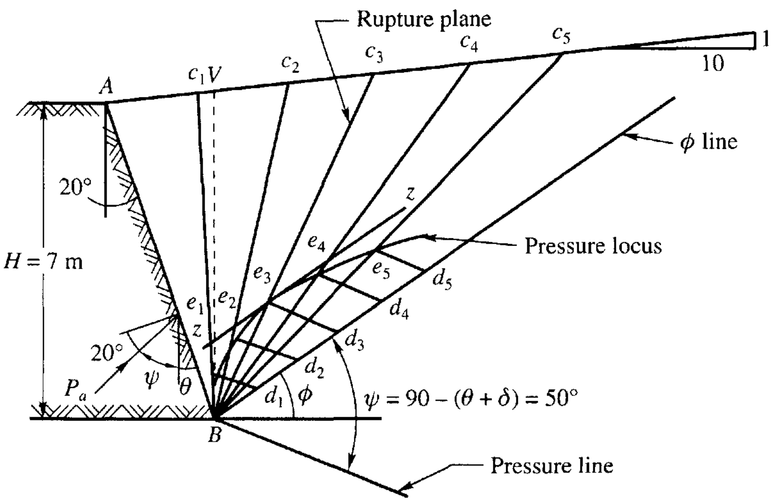

Figure Ex. 11.13

(e) The weights of the wedges in (*d*) above per meter length of wall may be determined by multiplying the areas by the unit weight of the soil. The results are tabulated below:

Wedge	Weight, kN	Wedge	Weight, kN
BAc_1	115	BAc_4	460
BAc_2	230	BAc_5	575
BAc_3	345		

(f) The weights of the wedges BAc_1, BAc_2, etc. are respectively plotted are Bd_1, Bd_2, etc. on the ϕ-line.

(g) Lines are drawn parallel to the pressure line from points d_1, d_2, d_3 etc. to meet respectively the trial rupture lines Bc_1, Bc_2, Bc_3 etc. at points e_1, e_2, e_3, etc.

(h) The pressure locus is drawn passing through points e_1, e_2, e_3, etc.

(i) Line *zz* is drawn tangential to the pressure locus at a point at which *zz* is parallel to the ϕ line. This point coincides with the point e_3.

(j) e_3d_3 gives the active earth pressure when converted to force units.
P_a = 180 kN per meter length of wall.

(k) Bc_3 is the critical rupture plane.

11.11 LATERAL PRESSURES BY THEORY OF ELASTICITY FOR SURCHARGE LOADS ON THE SURFACE OF BACKFILL

The surcharges on the surface of a backfill parallel to a retaining wall may be any one of the following

1. A concentrated load
2. A line load
3. A strip load

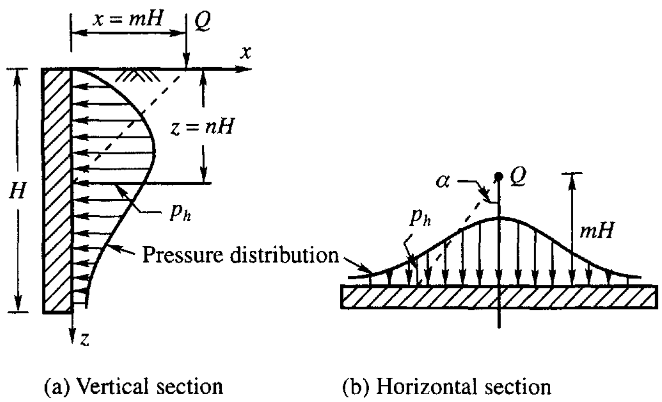

Figure 11.20 Lateral pressure against a rigid wall due to a point load

Lateral Pressure at a Point in a Semi-Infinite Mass due to a Concentrated Load on the Surface

Tests by Spangler (1938), and others indicate that lateral pressures on the surface of rigid walls can be computed for various types of surcharges by using modified forms of the theory of elasticity equations. Lateral pressure on an element in a semi-infinite mass at depth z from the surface may be calculated by Boussinesq theory for a concentrated load Q acting at a point on the surface. The equation may be expressed as (refer to Section 6.2 for notation)

$$p_h = \frac{Q}{2\pi z^2}\left[3\sin^2\beta\cos^2\beta - \frac{(1-2\mu)\cos^2\beta}{1+\cos\beta}\right] \tag{11.62}$$

If we write r = x in Fig. 6.1 and redefine the terms as
$x = mH$ and, $z = nH$

where H = height of the rigid wall and take Poisson's ratio $\mu = 0.5$, we may write Eq. (11.62) as

$$p_h = \frac{3Q}{2\pi H^2}\frac{m^2 n}{(m^2+n^2)^{5/2}} \tag{11.63}$$

Eq. (11.63) is strictly applicable for computing lateral pressures at a point in a semi-infinite mass. However, this equation has to be modified if a rigid wall intervenes and breaks the continuity of the soil mass. The modified forms are given below for various types of surcharge loads.

Lateral Pressure on a Rigid Wall Due to a Concentrated Load on the Surface

Let Q be a point load acting on the surface as shown in Fig. 11.20. The various equations are

(a) For $m > 0.4$

$$p_h = \frac{1.77Q}{H^2}\frac{n^2}{(m^2+n^2)^3} \tag{11.64}$$

(b) For $m \leq 0.4$

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$$p_h = \frac{0.28Q}{H^2} \frac{n^2}{(0.16+n^2)^3} \tag{11.65}$$

(c) Lateral pressure at points along the wall on each side of a perpendicular from the concentrated load Q to the wall (Fig. 11.20b)

$$p_h' = p_h \cos^2(1.1\alpha) \tag{11.66}$$

Lateral Pressure on a Rigid Wall due to Line Load

A concrete block wall conduit laid on the surface, or wide strip loads may be considered as a series of parallel line loads as shown in Fig. 11.21. The modified equations for computing p_h are as follows:

(a) For $m > 0.4$

$$p_h = \frac{4}{\pi}\frac{q}{H}\left[\frac{m^2 n}{(m^2+n^2)^2}\right] \tag{11.67}$$

(a) For $m \le 0.4$

$$p_h = \frac{q}{H}\left[\frac{0.203n}{(0.16+n^2)^2}\right] \tag{11.68}$$

Lateral Pressure on a Rigid Wall due to Strip Load

A strip load is a load intensity with a finite width, such as a highway, railway line or earth embankment which is parallel to the retaining structure. The application of load is as given in Fig. 11.22.

The equation for computing p_h is

$$p_h = \frac{2q}{\pi}(\beta - \sin\beta\cos 2\alpha) \tag{11.69a}$$

The total lateral pressure per unit length of wall due to strip loading may be expressed as (Jarquio, 1981)

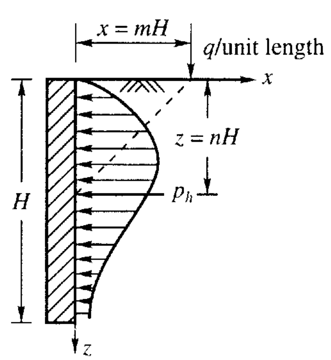

Figure 11.21 Lateral pressure against a rigid wall due to a line load

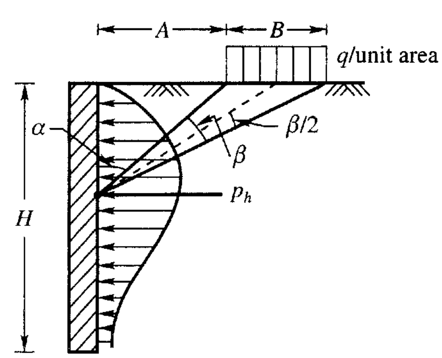

Figure 11.22 Lateral pressure against a rigid wall due to a strip load

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$$P_h = \frac{q}{90}\left[H(\alpha_2 - \alpha_1)\right] \tag{11.69b}$$

where $\alpha_1 = \tan^{-1}\dfrac{A}{H}$ and $\alpha_2 = \tan^{-1}\dfrac{A+B}{H}$

Example 11.14

A railway line is laid parallel to a rigid retaining wall as shown in Fig. Ex. 11.14. The width of the railway track and its distance from the wall is shown in the figure. The height of the wall is 10 m. Determine

(a) The unit pressure at a depth of 4m from the top of the wall due to the surcharge load
(b) The total pressure acting on the wall due to the surcharge load

Solution

(a) From Eq (11.69a)

The lateral earth pressure p_h at depth 4 m is

$$p_h = \frac{2q}{\pi}(\beta - \sin\beta\cos 2\alpha)$$

$$= \frac{2\times 60}{3.14}\ \frac{18.44}{180}\times 3.14 - \sin 18.44°\cos 2\times 36.9\ = 8.92\ \text{kN/m}^2$$

(b) From Eq. (11.69b)

$$P_h = \frac{q}{90}\left[H(\alpha_2 - \alpha_1)\right]$$

where, $q = 60$ kN/m^2, H = 10 m

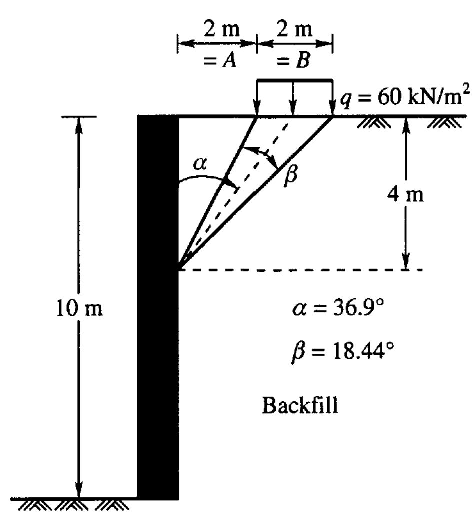

Figure Ex. 11.14`

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$$\alpha_1 = \tan^{-1}\frac{A}{H} = \tan^{-1}\frac{2}{10} = 11.31°$$

$$\alpha_1 = \tan^{-1}\frac{A+B}{H} = \tan^{-1}\frac{2+2}{10} = 21.80°$$

$$P_h = \frac{60}{90}[10(21.80 - 11.31)] \approx 70 \text{ kN/m}$$

11.12 CURVED SURFACES OF FAILURE FOR COMPUTING PASSIVE EARTH PRESSURE

It is customary practice to use curved surfaces of failure for determining the passive earth pressure P_p on a retaining wall with granular backfill if δ is greater than $\phi/3$. If tables or graphs are available for determining K_p for curved surfaces of failure the passive earth pressure P_p can be calculated. If tables or graphs are not available for this purpose, P_p can be calculated graphically by any one of the following methods.

1. Logarithmic spiral method
2. Friction circle method

In both these methods, the failure surface close to the wall is assumed as the part of a logarithmic spiral or a part of a circular arc with the top portion of the failure surface assumed as planar. This statement is valid for both cohesive and cohesionless materials. The methods are applicable for both horizontal and inclined backfill surfaces. However, in the following investigations it will be assumed that the surface of the backfill is horizontal.

Logarithmic Spiral Method of Determining Passive Earth Pressure of Ideal Sand

Property of a Logarithmic Spiral

The equation of a logarithmic spiral may be expressed as

$$r = r_0 e^{\theta \tan\phi} \tag{11.70}$$

where

r_0 = arbitrarily selected radius vector for reference
r = radius vector of any chosen point on the spiral making an angle θ with r_0.
ϕ = angle of internal friction of the material.

In Fig. 11.23a O is the origin of the spiral. The property of the spiral is that every radius vector such as Oa makes an angle of $90°-\phi$ to the tangent of the spiral at a or in other words, the vector Oa makes an angle ϕ with the normal to the tangent of the spiral at a.

Analysis of Forces for the Determination of Passive Pressure P_p

Fig. 11.23b gives a section through the plane contact face AB of a rigid retaining wall which rotates about point A into the backfill of cohesionless soil with a horizontal surface. BD is drawn at an angle $45°-\phi/2$ to the surface. Let O_1 be an arbitrary point selected on the line BD as the center of a logarithmic spiral, and let O_1A be the reference vector r_0. Assume a trial sliding surface Ae_1c_1 which consists of two parts. The first part is the curved part Ae_1 which is the part of the logarithmic

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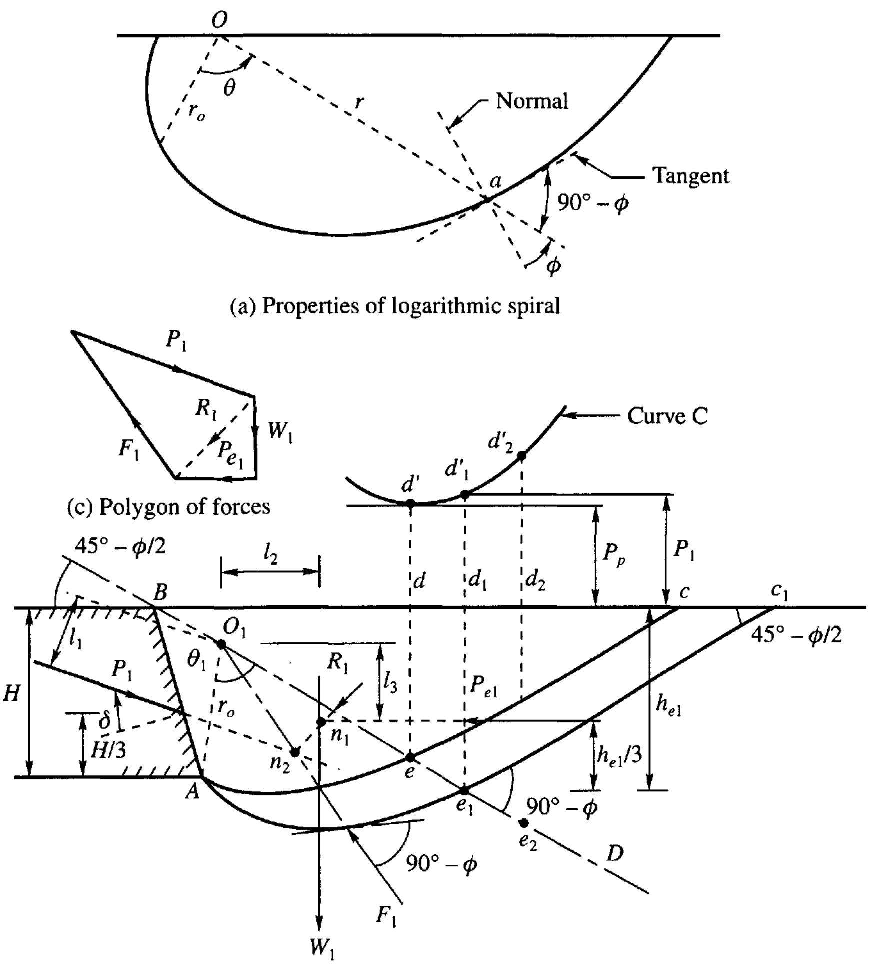

(b) Methods of analysis

Figure 11.23 Logarithmic spiral method of obtaining passive earth pressure of sand (After Terzaghi, 1943)

spiral with center at O_1 and the second a straight portion e_1c_1 which is tangential to the spiral at point e_1 on the line *BD*.

e_1c_1 meets the horizontal surface at c_1 at an angle 45°− $\phi/2$. O_1e_1 is the end vector r_1 of the spiral which makes an angle θ_1 with the reference vector r_0. Line *BD* makes an angle 90°− ϕ with line e_1c_1 which satisfies the property of the spiral.

It is now necessary to analyze the forces acting on the soil mass lying above the assumed sliding surface Ae_1c_1.

Within the mass of soil represented by triangle Be_1c_1 the state of stress is the same as that in a semi-infinite mass in a passive Rankine state. The shearing stresses along vertical sections are zero in this triangular zone. Therefore, we can replace the soil mass lying in the zone $e_1d_1c_1$ by a passive earth pressure P_{el} acting on vertical section e_1d_1 at a height $h_{el}/3$ where h_{el} is the height of the vertical section e_1d_1. This pressure is equal to

$$P_{e1} = \frac{1}{2}\gamma h_{e1}^2 N_\phi \tag{11.71}$$

where $N_\phi = \tan^2 (45° + \phi/2)$

The body of soil mass BAe_1d_1 (Fig. 11.23b) is acted on by the following forces:

1. The weight W_1 of the soil mass acting through the center of gravity of the mass having a lever arm l_2 with respect to O_1, the center of the spiral.
2. The passive earth pressure P_{el} acting on the vertical section e_1d_1 having a lever arm l_3.
3. The passive earth pressure P_1 acting on the surface AB at an angle δ to the normal and at a height $H/3$ above A having a lever arm l_1.
4. The resultant reaction force F_1 on the curved surface Ae_1 and passing through the center O_1.

Determination of the Force P_1 Graphically

The directions of all the forces mentioned above except that of F_1 are known. In order to determine the direction of F_1 combine the weight W_1 and the force P_{el} which gives the resultant R_1 (Fig. 11.23c). This resultant passes through the point of intersection n_1 of W_1 and P_{el} in Fig. 11.23b and intersects force P_1 at point n_2. Equilibrium requires that force F_1 pass through the same point. According to the property of the spiral, it must pass through the same point. According to the property of the spiral, it must pass through the center O_1 of the spiral also. Hence, the direction of F_1 is known and the polygon of forces shown in Fig. 11.23c can be completed. Thus we obtain the intensity of the force P_1 required to produce a slip along surface Ae_1c_1.

Determination of P_1 by Moments

Force P_1 can be calculated by taking moments of all the forces about the center O_1 of the spiral. Equilibrium of the system requires that the sum of the moments of all the forces must be equal to zero. Since the direction of F_1 is now known and since it passes through O_1, it has no moment. The sum of the moments of all the other forces may be written as

$$P_1l_1 + W_1l_2 + P_{e1}l_3 = 0 \tag{11.72}$$

Therefore, $$P_1 = -\frac{1}{l_1}\left(W_1l_2 + P_{e1}l_3\right) \tag{11.73}$$

P_1 is thus obtained for an assumed failure surface Ae_1c_1. The next step consists in repeating the investigation for more trial surfaces passing through A which intersect line BD at points e_2, e_3 etc. The values of P_1, P_2, P_3 etc so obtained may be plotted as ordinates $d_1\,d_1'$, $d_2\,d_2'$ etc., as shown in Fig. 11.23b and a smooth curve C is obtained by joining points d_1', d_2' etc. Slip occurs along the surface corresponding to the minimum value P_p which is represented by the ordinate dd'. The corresponding failure surface is shown as Aec in Fig. 11.23b.

11.13 COEFFICIENTS OF PASSIVE EARTH PRESSURE TABLES AND GRAPHS

Concept of Coulomb's Formula

Coulomb (1776) computed the passive earth pressure of ideal sand on the simplifying assumption that the entire surface of sliding consists of a plane through the lower edge A of contact face AB as shown in Fig. 11.24a. Line AC represents an arbitrary plane section through this lower edge. The forces acting on this wedge and the polygon of forces are shown in the figure. The basic equation for computing the passive earth pressure coefficient may be developed as follows:

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Consider a point on pressure surface AB at a depth z from point B (Fig 11.24a). The normal component of the earth pressure per unit area of surface AB may be expressed by the equation,

$$p_{pn} = \gamma z K_p \tag{11.74}$$

where K_p is the coefficient of passive earth pressure. The total passive earth pressure normal to surface AB, P_{pn}, is obtained from Eq. (11.74) as follows,

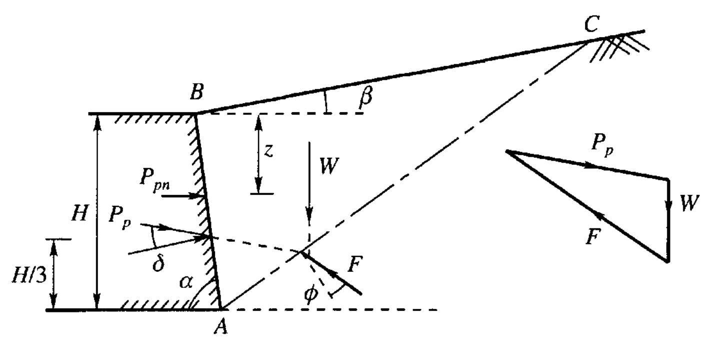
$$P_{pn} = \int_0^H \frac{p_{pn}}{\sin\alpha}\,dz = \frac{\gamma K_p}{\sin\alpha}\int_0^H z\,dz$$

$$P_{pn} = \frac{1}{2}\gamma H^2 \frac{K_P}{\sin\alpha} \tag{11.75}$$

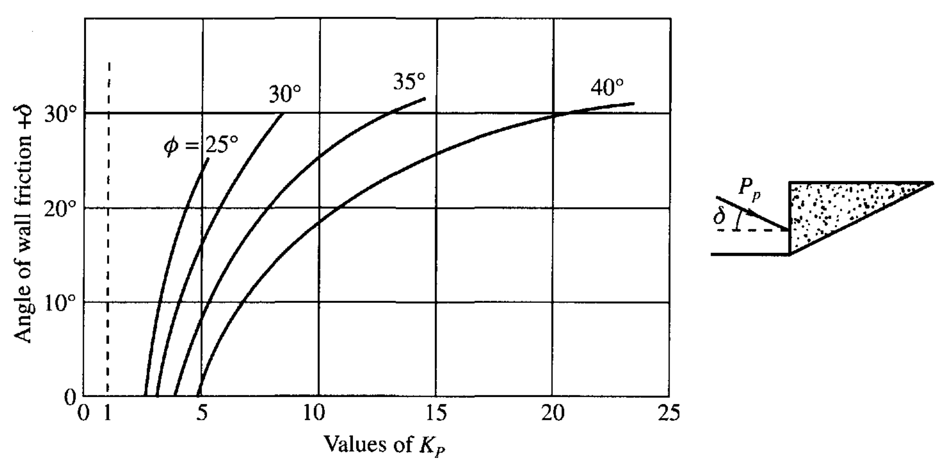
where α is the angle made by pressure surface AB with the horizontal.

Since the resultant passive earth pressure P_p acts at an angle δ to the normal,

$$P_p = \frac{P_{pn}}{\cos\delta} = \frac{1}{2}\gamma H^2 \frac{K_P}{\sin\alpha\cos\delta} \tag{11.76}$$



(a) Principles of Coulomb's Theory of passive earth pressure of sand



(b) Coefficient of passive earth pressure K_P

Figure 11.24 Diagram illustrating passive earth pressure theory of sand and relation between φ, δ and K_p (After Terzaghi, 1943)

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Table 11.3 Passive earth pressure coefficient K'_p for curved surfaces of failure (After Caquot and Kerisel 1948).

ϕ =	10°	15°	20°	25°	30°	35°	40°
$\delta = 0$	1.42	1.70	2.04	2.56	3.0	3.70	4.6
$\delta = \phi/2$	1.56	1.98	2.59	3.46	4.78	6.88	10.38
$\delta = \phi$	1.65	2.19	3.01	4.29	6.42	10.20	17.50
$\delta = -\phi/2$	0.73	0.64	0.58	0.55	0.53	0.53	0.53

Eq. (11.76) may also be expressed as

$$P_p = \frac{1}{2}\gamma H^2 K'_p \tag{11.77}$$

$$\text{where } K'_p = \frac{K_p}{\sin\alpha\cos\delta} \tag{11.78}$$

Passive Earth Pressure Coefficient

Coulomb developed an analytical solution for determining K_p based on a plane surface of failure and this is given in Eq. (11.57). Figure 11.24(b) gives curves for obtaining Coulomb's values of K_p for various values of δ and ϕ for plane surfaces of failure with a horizontal backfill. They indicate that for a given value of ϕ the value of K_p increases rapidly with increasing values of δ. The limitations of plane surfaces of failure are given in Section 11.9. Curved surfaces of failure are normally used for computing P_p or K_p when the angle of wall friction δ exceeds $\phi/3$. Experience indicates that the curved surface of failure may be taken either as a part of a logarithmic spiral or a circular arc. Caquot and Kerisel (1948) computed K'_p by making use of curved surfaces of failure for various values of ϕ, δ, θ and β. Caquot and Kerisel's calculations for determining K'_p for curved surfaces of failure are available in the form of graphs.

Table 11.3 gives the values of K'_p for various values of ϕ and δ for a vertical wall with a horizontal backfill (after Caquot and Kerisel, 1948).

In the vast majority of practical cases the angle of wall friction has a positive sign, that is, the wall transmits to a soil a downward shearing force. The negative angle of wall friction might develop in the case of positive batter piles subjected to lateral loads, and also in the case of pier foundations for bridges subjected to lateral loads.

Example 11.15

A gravity retaining wall is 10 ft high with sand backfill. The backface of the wall is vertical. Given $\delta = 20°$, and $\phi = 40°$, determine the total passive thrust using Eq. (11.76) and Fig. 11.24 for a plane failure. What is the passive thrust for a curved surface of failure? Assume $\gamma = 18.5$ kN/m^3.

Solution

From Eq. (11.76)

$$P_p = \frac{1}{2}\gamma H^2 \frac{K_p}{\sin\alpha\cos\delta} \quad \text{where } \alpha = 90°$$

From Fig. 11.24 (b) for $\delta = 20°$, and $\phi = 40°$, we have $K_p = 11$

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$$P_p = \frac{1}{2} \times 18.5 \times 10^2 \frac{11}{\sin 90 \cos 20^\circ} = 10{,}828 \text{ kN/m}$$

From Table 11.3 K'_p for a curved surface of failure (Caquot and Kerisel. 1948) for $\phi = 40°$ and $\delta = 20°$ is 10.38.

From Eq. (11.77)

$$P_p = \frac{1}{2} \gamma H^2 K'_p = \frac{1}{2} \times 18.5 \times 10^2 \times 10.38$$

$$= 9602 \text{kN/m}$$

Comments

For $\delta = \phi/2$, the reduction in the passive earth pressure due to a curved surface of failure is

$$\text{Reduction} = \frac{10{,}828 - 9602}{10{,}828} \times 100 = 11.32\%$$

Example 11.16

For the data given in Example 11.15, determine the reduction in passive earth pressure for a curved surface of failure if $\delta = 30°$.

Solution

For a plane surface of failure P_p from Eq. (11.76) is

$$P_p = \frac{1}{2} \times 18.5 \times 10^2 \times \frac{21}{\sin 90^\circ \cos 30^\circ} = 22{,}431 \text{ kN/m}$$

where, $K_p = 21$ from Fig. 11.24 for $\delta = 30°$ and $\phi = 40°$

From Table 11.3 for $\delta = 30°$ and $\phi = 40°$

$$K'_p = \frac{10.38 + 17.50}{2} = 13.94$$

From Eq (11.77)

$$P_p = \frac{1}{2} \times 18.5 \times 10^2 \times 13.94 = 12{,}895 \text{ kN/m}$$

$$\text{Reduction in passive pressure} = \frac{22{,}431 - 12{,}895}{22{,}431} = 42.5\%$$

It is clear from the above calculations, that the soil resistance under a passive state gives highly erroneous values for plane surfaces of failure with an increase in the value of δ. This error could lead to an unsafe condition because the computed values of P_p would become higher than the actual soil resistance.

11.14 LATERAL EARTH PRESSURE ON RETAINING WALLS DURING EARTHQUAKES

Ground motions during an earthquake tend to increase the earth pressure above the static earth pressure. Retaining walls with horizontal backfills designed with a factor of safety of 1.5 for static

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loading are expected to withstand horizontal accelerations up to 0.2g. For larger accelerations, and for walls with sloping backfill, additional allowances should be made for the earthquake forces. Murphy (1960) shows that when subjected to a horizontal acceleration at the base, failure occurs in the soil mass along a plane inclined at 35° from the horizontal . The analysis of Mononobe (1929) considers a soil wedge subjected to vertical and horizontal accelerations to behave as a rigid body sliding over a plane slip surface.

The current practice for earthquake design of retaining walls is generally based on design rules suggested by Seed and Whitman (1970). Richards et al. (1979) discuss the design and behavior of gravity retaining walls with unsaturated cohesionless backfill. Most of the papers make use of the popular Mononobe-Okabe equations as a starting point for their own analysis. They follow generally the pseudoplastic approach for solving the problem. Solutions are available for both the active and passive cases with as granular backfill materials. Though solutions for (c-ϕ) soils have been presented by some investigators (Prakash and Saran, 1966, Saran and Prakash, 1968), their findings have not yet been confirmed, and as such the solutions for (c-ϕ) soils have not been taken up in this chapter.

Earthquake Effect on Active Pressure with Granular Backfill

The Mononobe-Okabe method (1929, 1926) for dynamic lateral pressure on retaining walls is a straight forward extension of the Coulomb sliding wedge theory. The forces that act on a wedge under the active state are shown in Fig. 11.25

In Fig. 11.25 *AC* in the sliding surface of failure of wedge *ABC* having a weight *W* with inertial components $k_v W$ and $k_h W$. The equation for the total active thrust P_{ae} acting on the wall *AB* under dynamic force conditions as per the analysis of Mononobe-Okabe is

$$P_{ae} = \frac{1}{2}\gamma H^2 (1 - k_v) K_{Ae} \tag{11.79}$$

in which

$$K_{Ae} = \frac{\cos^2(\phi - \eta - \theta)}{\cos\eta \cos^2\theta \cos(\delta + \theta + \eta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \eta - \beta)}{\cos(\delta + \theta + \eta)\cos(\beta - \theta)}}\right]^2} \tag{11.80}$$

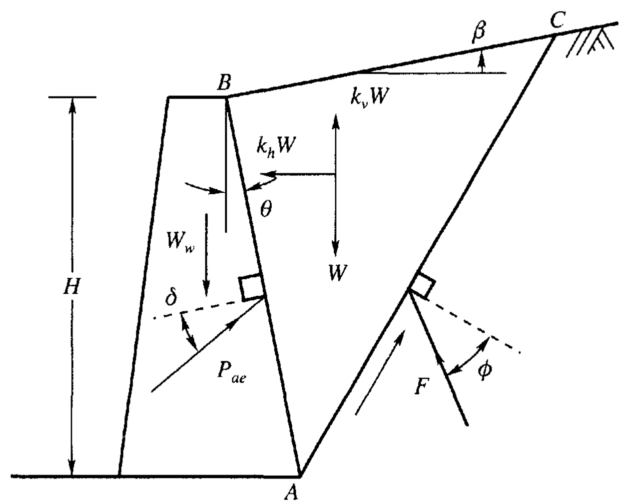

Figure 11.25 Active force on a retaining wall with earthquake forces

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where $\overline{P}_{ae}$ =dynamic component of the total earth pressure P_{ae} or $P_{ae} = P_a + \overline{P}_{ae}$

K_{Ae} = the dynamic earth pressure coefficient

$$\eta = \tan^{-1}\left[\frac{k_h}{1-k_v}\right] \tag{11.81}$$

P_a = active earth pressure [Eq. (11.50)]

k_h = (horizontal acceleration)/g

k_v =(vertical acceleration)/g

g = acceleration due to gravity

γ = unit weight of soil

ϕ = angle of friction of soil

δ = angle of wall friction

β = slope of backfill

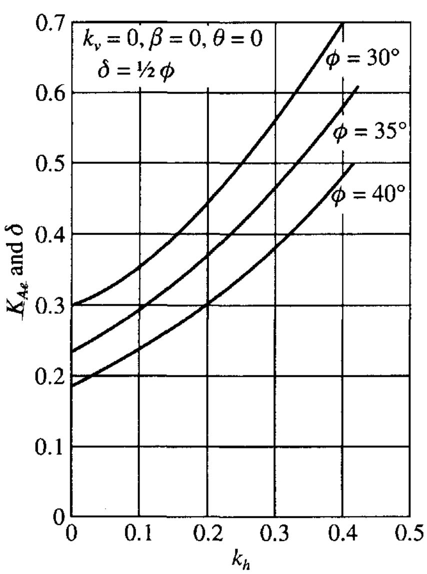
θ = slope of pressure surface of retaining wall with respect to vertical at point B (Fig. 11.25)

H = height of wall

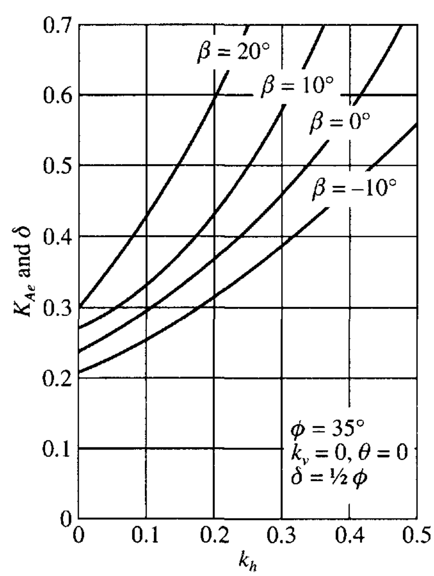
The total resultant active earth pressure P_{ae} due to an earthquake is expressed as

$$P_{ae} = P_a + \overline{P}_{ae} \tag{11.82}$$

The dynamic component $\overline{P}_{ae}$ is expected to act at a height $0.6H$ above the base whereas the static earth pressure acts at a height $H/3$. For all practical purposes it would be sufficient to assume that the resultant force P_{ae} acts at a height $H/2$ above the base with a uniformly distributed pressure.



(a) Influence of soil friction on soil dynamic pressure



(b) Influence of backfill slope on dynamic lateral pressure

Figure 11.26 Dynamic lateral active pressure (after Richards et al., 1979)

It has been shown that the active pressure is highly sensitive to both the backfill slope β, and the friction angle ϕ of the soil (Fig. 11.26).

It is necessary to recognize the significance of the expression

$$\sin(\phi - \eta - \beta) \quad (11.83)$$

given under the root sign in Eq. (11.80).

a. When Eq. (11.83) is negative no real solution is possible. Hence for stability, the limiting slope of the backfill must fulfill the condition

$$\beta \le (\varphi - \eta) \quad (11.84a)$$

b. For no earthquake condition, $\eta = 0$. Therefore for stability we have

$$\beta \le \varphi \quad (11.85)$$

c. When the backfill is horizontal $\beta = 0$. For stability we have

$$\eta \le \varphi \quad (11.86)$$

d. By combining Eqs. (11.81) and (11.86), we have

$$k_h \le (1 - k_v)\tan\phi \quad (11.87a)$$

From Eq. (11.87a), we can define a critical value for horizontal acceleration k_h^* as

$$k_h^* = (1 - k_v)\tan\phi \quad (11.87b)$$

Values of critical accelerations are given in Fig 11.27 which demonstrates the sensitivity of the various quantities involved.

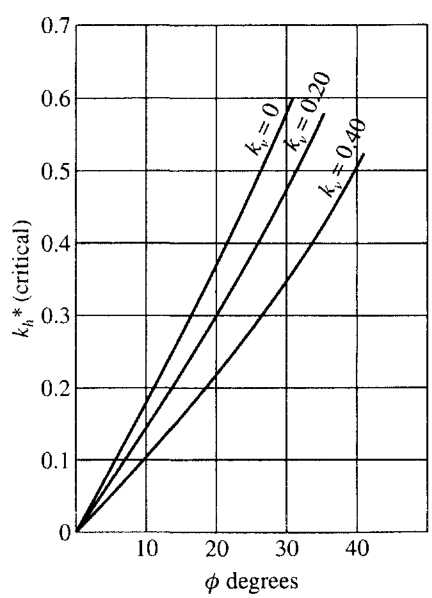


Figure 11.27 Critical values of horizontal accelerations

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Effect of Wall Lateral Displacement on the Design of Retaining Wall

It is the usual practice of some designers to ignore the inertia forces of the mass of the gravity retaining wall in seismic design. Richards and Elms (1979) have shown that this approach is unconservative since it is the weight of the wall which provides most of the resistance to lateral movement. Taking into account all the seismic forces acting on the wall and at the base they have developed an expression for the weight of the wall W_w under the equilibrium condition as (for failing by sliding)

$$W_w = \frac{1}{2}\gamma H^2(1-k_v)K_{Ae}C_{IE} \tag{11.88}$$

in which,

$$C_{IE} = \frac{\cos(\delta+\theta)-\sin(\delta+\theta)\tan\delta}{(1-k_v)(\tan\delta-\tan\eta)} \tag{11.89}$$

where W_w = weight of retaining wall (Fig. 11.25)
δ = angle of friction between the wall and soil

Eq. (11.89) is considerably affected by δ. If the wall inertia factor is neglected, a designer will have to go to an exorbitant expense to design gravity walls.

It is clear that tolerable displacement of gravity walls has to be considered in the design. The weight of the retaining wall is therefore required to be determined to limit the displacement to the tolerable limit. The procedure is as follows

1. Set the tolerable displacement Δd
2. Determine the design value of k_h by making use of the following equation (Richards et al., 1979)

$$k_h = A_a\left[\frac{0.2A_v^2}{A_a(\Delta d)}\right]^{1/4} \tag{11.90}$$

where A_a, A_v = acceleration coefficients used in the Applied Technology Council (ATC) Building Code (1978) for various regions of the United States. Δd is in inches.

3. Using the values of k_h calculated above, and assuming $k_v = 0$, calculate K_{Ae} from Eq (11.80)
4. Using the value of K_{Ae}, calculate the weight, W_w, of the retaining wall by making use of Eqs. (11.88) and (11.89)
5. Apply a suitable factor of safety, say, 1.5 to W_w.

Passive Pressure During Earthquakes

Eq. (11.79) gives an expression for computing seismic active thrust which is based on the well known Mononobe-Okabe analysis for a plane surface of failure. The corresponding expression for passive resistance is

$$P_{pe} = \frac{1}{2}\gamma H^2(1-k_v)K_{Pe} \tag{11.91}$$

$$K_{Pe} = \frac{\cos^2(\phi-\eta+\theta)}{\cos\eta\cos^2\theta\cos(\delta-\theta+\eta)\left[1-\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\eta+\beta)}{\cos(\delta-\theta+\eta)\cos(\beta-\theta)}}\right]^2}$$

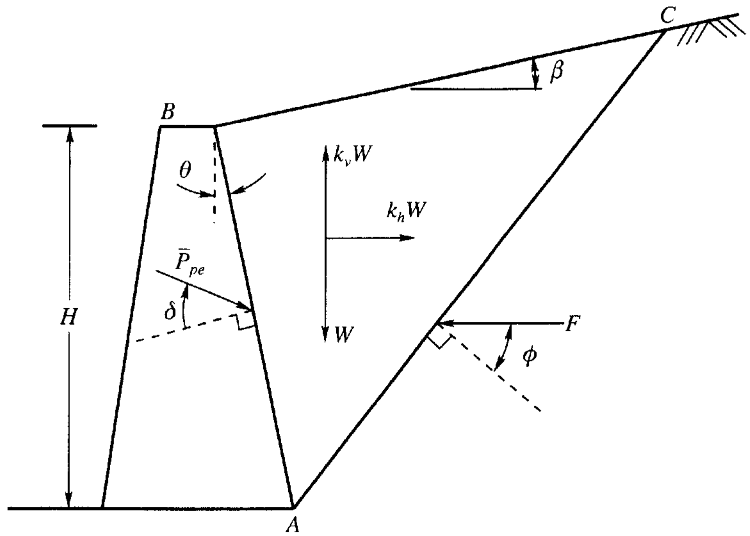

Figure 11.28 Passive pressure on a retaining wall during earthquake

Fig. 11.28 gives the various forces acting on the wall under seismic conditions. All the other notations in Fig. 11.28 are the same as those in Fig. 11.25. The effect of increasing the slope angle β is to increase the passive resistance (Fig. 11.29). The influence of the friction angle of the soil (ϕ) on the passive resistance is illustrated the Fig. 11.30.

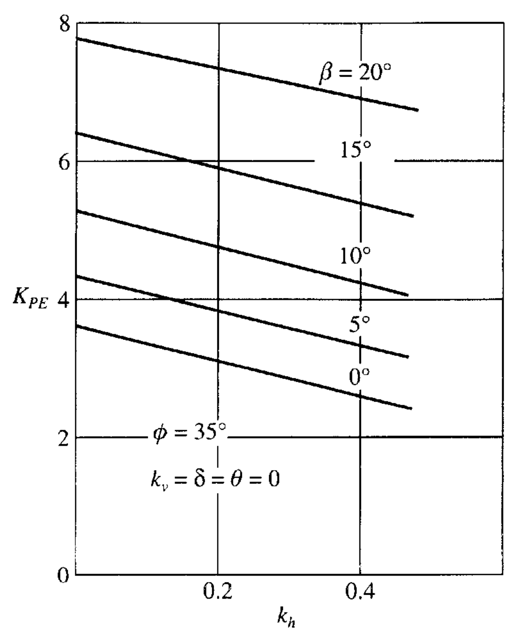

Figure 11.29 Influence of backfill slope angle on passive pressure

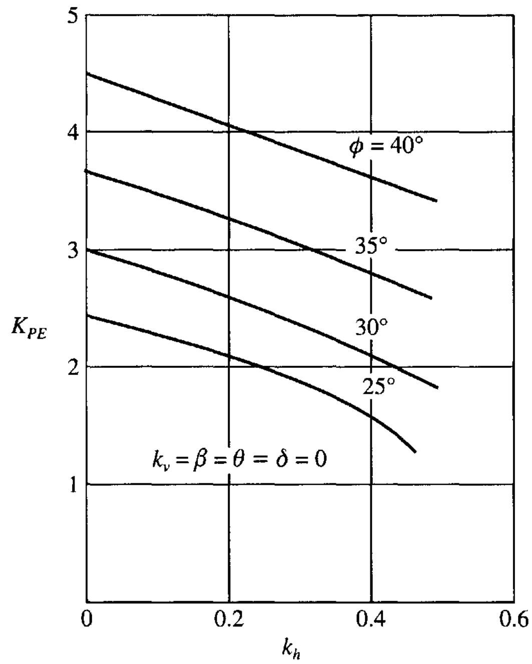

Figure 11.30 Influence of soil friction angle on passive pressure

It has been explained in earlier sections of this chapter that the passive earth pressures calculated on the basis of a plane surface of failure give unsafe results if the magnitude of δ exceeds $\phi/2$. The error occurs because the actual failure plane is curved, with the degree of curvature increasing with an increase in the wall friction angle. The dynamic Mononobe-Okabe solution assumes a linear failure surface, as does the static Coulomb formulation.

In order to set right this anomaly Morrison and Ebelling (1995) assumed the failure surface as an arc of a logarithmic spiral (Fig. 11.31) and calculated the magnitude of the passive pressure under seismic conditions.

It is assumed here that the pressure surface is vertical (θ = 0) and the backfill surface horizontal (β = 0). The following charts have been presented by Morrison and Ebelling on the basis of their analysis.

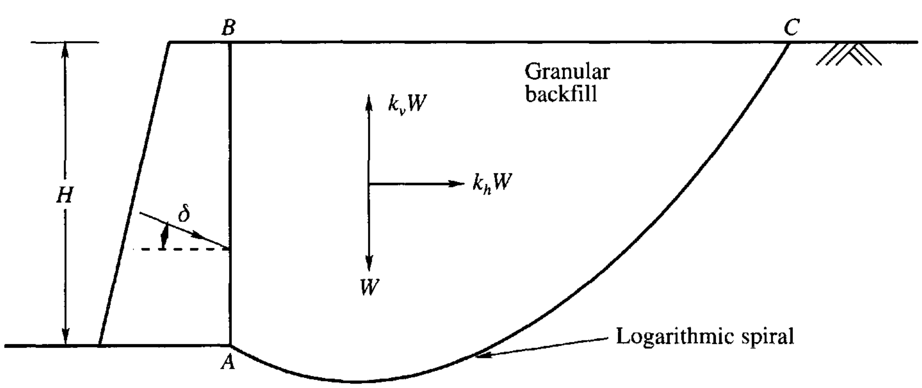

Figure 11.31 Passive pressure from log spiral failure surface during earthquakes

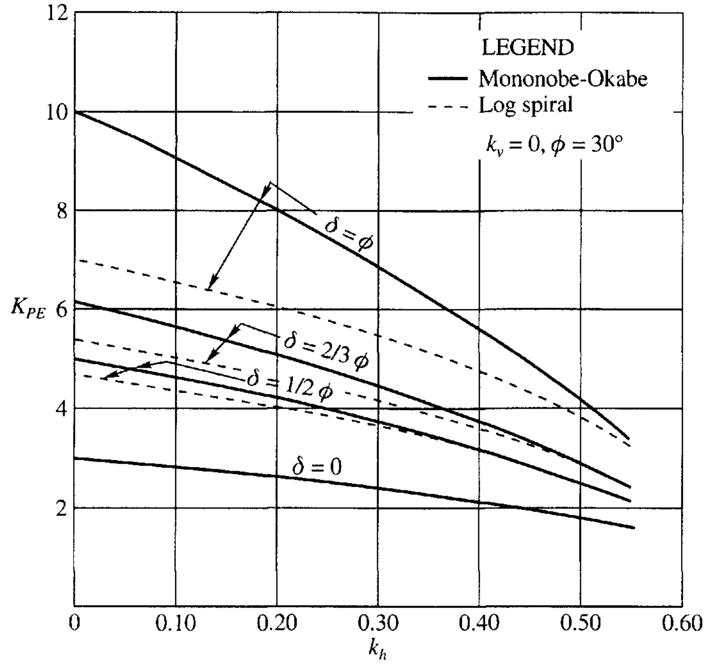

Figure 11.32 K_{Pe} versus k_h, effect of δ

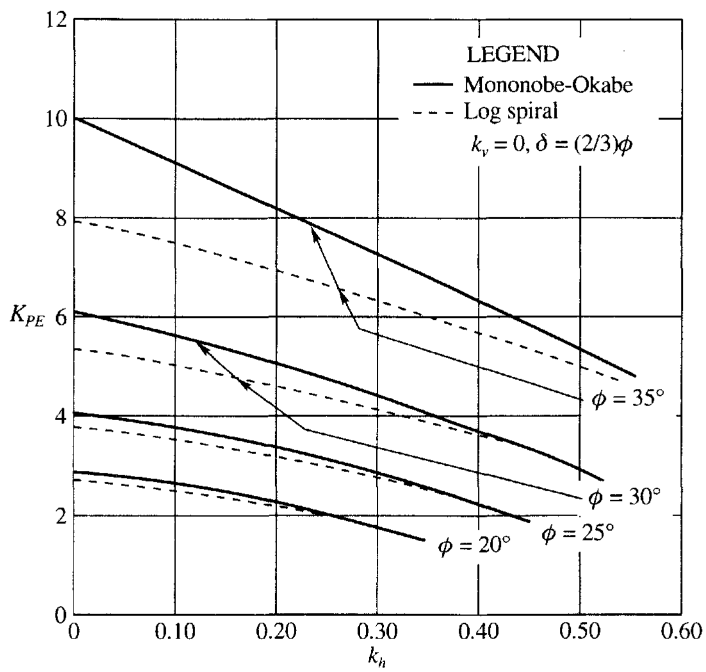

Figure 11.33 K_{Pe} versus k_h, effect of ϕ

1. Fig. 11.32 gives the effect of δ on the plot K_{Pe} versus k_h with $k_v = 0$, for $\phi = 30°$. The values of δ assumed are 0, 1/2 (ϕ) and(2/3ϕ). The plot shows clearly the difference between the Mononobe-Okabe and log spiral values. The difference between the two approaches is greatest at $k_h = 0$

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2. Fig. 11.33 shows the effect of ϕ on K_{Pe}. The figure shows the difference between Mononobe-Okabe and log spiral values of K_{pe} versus k_h with δ=(2/3ϕ) and $k_v = 0$. It is also clear from the figure the difference between the two approaches is greatest for $k_h = 0$ and decreases with an increase in the value of k_h.

Example 11.17

A gravity retaining wall is required to be designed for seismic conditions for the active state. The following data are given:

Height of wall = 8 m $\theta = 0°$, $\beta = 0$, $\phi = 30°$, $\delta = 15°$, $k_v = 0$, $k_h = 0.25$ and $\gamma = 19$ kN/m^3. Determine P_{ae} and the approximate point of application. What is the additional active pressure caused by the earthquake?

Solution

From Eq. (11.79)

$$P_{ae} = \frac{1}{2}\gamma H^2(1-k_v)K_{Ae} = \frac{1}{2}\gamma H^2 K_{Ae}, \text{ since } k_v = 0$$

For $\phi = 30°$, $\delta = 15°$ and $k_h = 0.25$, we have from Fig.11.26 a
$K_{Ae} = 0.5$. Therefore

$$P_{ae} = \frac{1}{2}19 \times 8^2 \times 0.5 = 304 \text{ kN/m}$$

From Eq. (11.14) $P_a = \frac{1}{2}\gamma H^2 K_A$

where $K_A = \tan^2(45° - \phi/2) = \tan^2 30° = 0.33$

Therefore $P_a = \frac{1}{2} \times 19 \times 8^2 \times 0.33 = 202.7$ kN/m

ΔP_{ae} = the additional pressure due to the earthquake = 304 – 202.7 = 101.3 kN/m

For all practical purposes, the point of application of P_{ae} may be taken as equal to H/2 above the base of the wall or 4 m above the base in this case.

Example 11.18

For the wall given in Example 11.17, determine the total passive pressure P_{pe} under seismic conditions. What is the additional pressure due to the earthquake?

Solution

From Eq. (11.91),

$$P_{ae} = \frac{1}{2}\gamma H^2(1-k_v)K_{pe} = \frac{1}{2}\gamma H^2 K_{pe}, \text{ since } k_v = 0$$

From Fig 11.32, (from M-O curves), $K_{Pe} = 4.25$ for $\phi = 30°$, and $\delta = 15°$

Now $P_{pe} = \frac{1}{2}\gamma H^2 K_{pe} = \frac{1}{2} \times 19 \times 8^2 \times 4.25 = 2584$ kN/m

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From Eq. (11.15)

$$P_p = \frac{1}{2}\gamma H^2 K_p = \frac{1}{2} \times 19 \times 8^2 \times 3 = 1824 \text{ kN/m}$$

where $K_p = \tan^2\left(45° + \frac{30}{2}\right) = \tan^2 60° = 3$

$$\Delta P_{pe} = \left(P_{pe} - P_{pe}\right) = 2584 - 1824 = 760 \text{ kN/m}$$

11.15 PROBLEMS

11.1 Fig. Prob. 11.1 shows a rigid retaining wall prevented from lateral movements. Determine for this wall the lateral thrust for the at-rest condition and the point of application of the resultant force.

11.2 For Prob 11.1, determine the active earth pressure distribution for the following cases:

(a) when the water table is below the base and $\gamma = 17$ kN/m^3.

(b) when the water table is at 3m below ground level

(c) when the water table is at ground level

11.3 Fig. Prob. 11.3 gives a cantilever retaining wall with a sand backfill. The properties of the sand are:

$e = 0.56$, $\phi = 38°$, and $G_s = 2.65$.

Using Rankine theory, determine the pressure distribution with respect to depth, the magnitude and the point of application of the resultant active pressure with the surcharge load being considered.

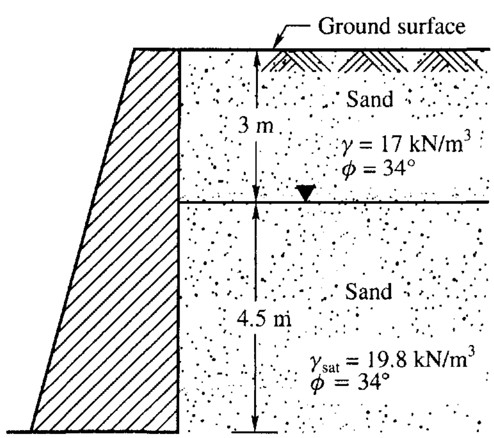

Figure Prob. 11.1

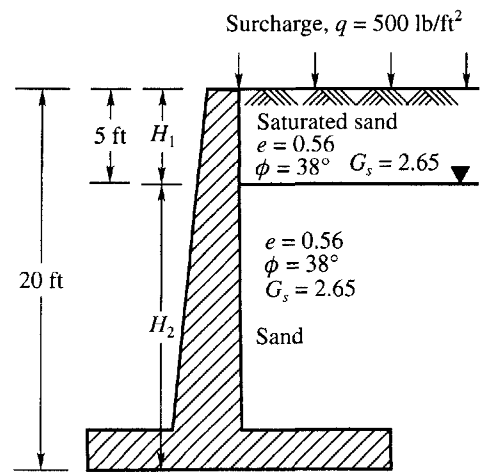

Figure Prob. 11.3

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11.4 A smooth vertical wall 3.5 m high retains a mass of dry loose sand. The dry unit weight of the sand is 15.6 kN/m^3 and an angle of internal friction ϕ is 32°. Estimate the total thrust per meter acting against the wall (a) if the wall is prevented from yielding, and (b) if the wall is allowed to yield.

11.5 A wall of 6 m height retains a non-cohesive backfill of dry unit weight 18 kN/m^3 and an angle of internal friction of 30°. Use Rankine's theory and find the total active thrust per meter length of the wall. Estimate the change in the total pressure in the following circumstances:

(i) The top of the backfill carrying a uniformly distributed load of 6 kN/m^2

(ii) The backfill under a submerged condition with the water table at an elevation of 2 m below the top of the wall. Assume $G_s = 2.65$, and the soil above the water table being saturated.

11.6 For the cantilever retaining wall given in Fig. Prob 11.3 with a sand backfill, determine pressure distribution with respect to depth and the resultant thrust. Given:

$H_1 = 3\text{m}$, $H_2 = 6\text{m}$, $\gamma_{sat} = 19.5$ kN/m^3

$q = 25$ kN/m^2, and $\phi = 36°$

Assume the soil above the GWT is saturated

11.7 A retaining wall of 6 m height having a smooth back retains a backfill made up of two strata shown in Fig. Prob. 11.7. Construct the active earth pressure diagram and find the magnitude and point of application of the resultant thrust. Assume the backfill above WT remains dry.

11.8 (a) Calculate the total active thrust on a vertical wall 5 m high retaining sand of unit weight 17 kN/m^3 for which $\phi = 35°$. The surface is horizontal and the water table is below the bottom of the wall. (b) Determine the thrust on the wall if the water table rises to a level 2 m below the surface of the sand. The saturated unit weight of the sand is 20 kN/m^3.

11.9 Figure Problem 11.9 shows a retaining wall with a sloping backfill. Determine the active earth pressure distribution, the magnitude and the point of application of the resultant by the analytical method.

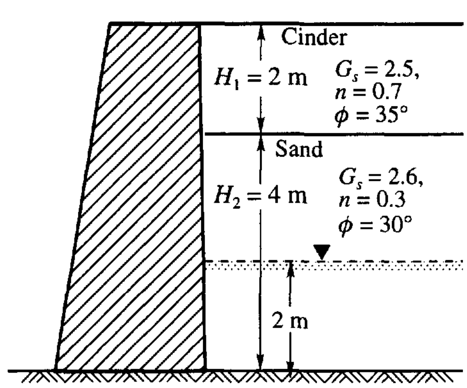

Figure Prob. 11.7

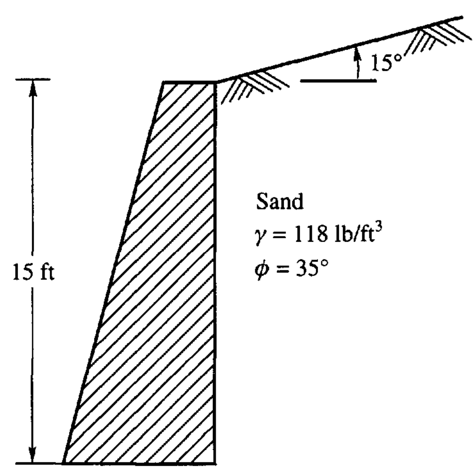

Figure Prob. 11.9

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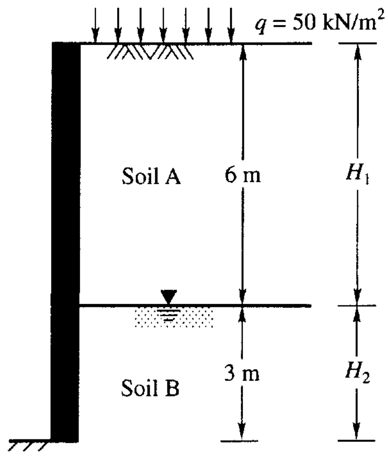

Figure Prob. 11.10

11.10 The soil conditions adjacent to a rigid retaining wall are shown in Fig. Prob. 11.10, A surcharge pressure of 50 kN/m^2 is carried on the surface behind the wall. For soil (A) above the water table, $c' = 0$, $\phi' = 38°$, $\gamma' = 18\ kN/m^3$. For soil (B) below the WT, $c' = 10\ kN/m^2$, $\phi' = 28°$, and $\gamma_{sat} = 20\ kN/m^3$. Calculate the maximum unit active pressure behind the wall, and the resultant thrust per unit length of the wall.

11.11 For the retaining wall given in Fig. Prob. 11.10, assume the following data:

(a) surcharge load = 1000 lb/ft^2, and (b) $H_1 = 10$ ft, $H_2 = 20$ ft,

(c) Soil A: $c' = 500\ lb/ft^2$, $\phi' = 30°$, $\gamma = 110\ lb/ft^3$

(d) Soil B: $c' = 0$, $\phi' = 35°$, $\gamma_{sat} = 120\ lb/ft^3$

Required:

(a) The maximum active pressure at the base of the wall.

(b) The resultant thrust per unit length of wall.

11.12 The depths of soil behind and in front of a rigid retaining wall are 25 ft and 10 ft respectively, both the soil surfaces being horizontal (Fig. Prob 11.12). The appropriate

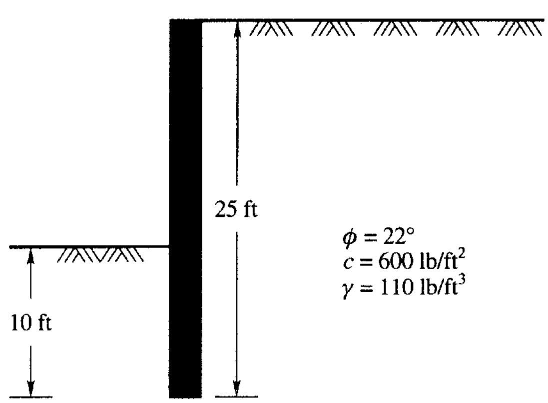

Figure Prob. 11.12

shear strength parameters for the soil are $c = 600$ lb/ft^2, and $\phi = 22°$, and the unit weight is 110 lb/ft^3. Using Rankine theory, determine the total active thrust behind the wall and the total passive resistance in front of the wall. Assume the water table is at a great depth.

11.13 For the retaining wall given in Fig. Prob. 11.12, assume the water table is at a depth of 10 ft below the backfill surface. The saturated unit weight of the soil is 120 lb/ft^3. The soil above the GWT is also saturated. Compute the resultant active and passive thrusts per unit length of the wall.

11.14 A retaining wall has a vertical back face and is 8 m high. The backfill has the following properties:

cohesion c = 15 kN/m^2, $\phi = 25°$, $\gamma = 18.5$ kN/m^3

The water table is at great depth. The backfill surface is horizontal. Draw the pressure distribution diagram and determine the magnitude and the point of application of the resultant active thrust.

11.15 For the retaining wall given in Prob. 11.14, the water table is at a depth of 3 m below the backfill surface. Determine the magnitude of the resultant active thrust.

11.16 For the retaining wall given in Prob. 11.15, compute the magnitude of the resultant active thrust, if the backfill surface carries a surcharge load of 30 kN/m^2.

11.17 A smooth retaining wall is 4 m high and supports a cohesive backfill with a unit weight of 17 kN/m^3. The shear strength parameters of the soil are cohesion = 10 kPa and $\phi = 10°$. Calculate the total active thrust acting against the wall and the depth to the point of zero lateral pressure.

11.18 A rigid retaining wall is subjected to passive earth pressure. Determine the passive earth pressure distribution and the magnitude and point of application of the resultant thrust by Rankine theory.

Given: Height of wall = 10 m; depth of water table from ground surface = 3 m; $c = 20$ kN/m^2, $\phi = 20°$ and $\gamma_{sat} = 19.5$ kN/m^3. The backfill carries a uniform surcharge of 20 kN/m^2.

Assume the soil above the water table is saturated.

11.19 Fig. Prob. 11.19 gives a retaining wall with a vertical back face and a sloping backfill. All the other data are given in the figure. Determine the magnitude and point of application of resultant active thrust by the Culmann method.

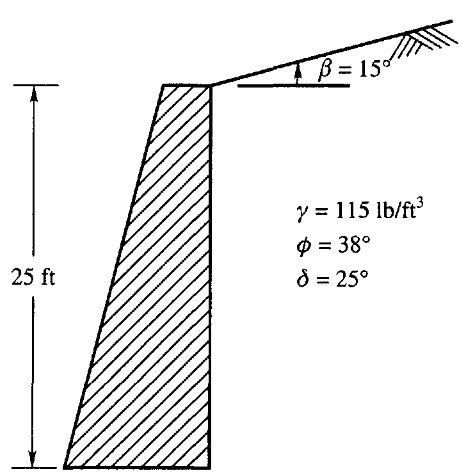

Figure Prob. 11.19

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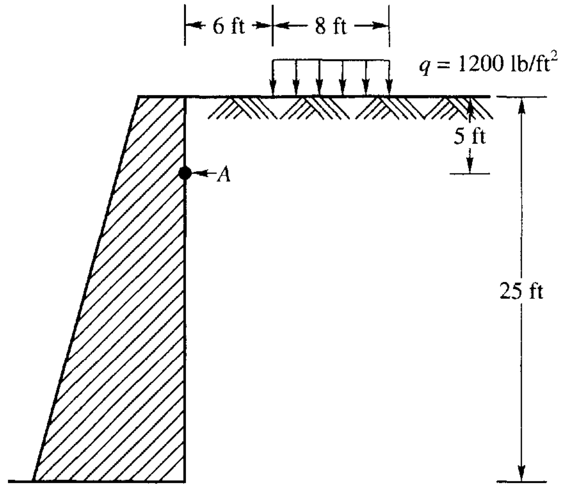

Figure Prob. 11.20

11.20 Fig. Prob. 11.20 gives a rigid retaining wall with a horizontal backfill. The backfill carries a strip load of 1200 lb/ft^2 as shown in the figure. Determine the following:

(a) The unit pressure on the wall at point A at a depth of 5 ft below the surface due to the surcharge load.

(b) The total thrust on the wall due to surcharge load.

11.21 A gravity retaining wall with a vertical back face is 10 m high. The following data are given:

$\phi = 25°$, $\delta = 15°$, and $\gamma = 19$ kN/m^3

Determine the total passive thrust using Eq (11.76). What is the total passive thrust for a curved surface of failure?

11.22 A gravity retaining wall is required to be designed for seismic conditions for the active state. The back face is vertical. The following data are given:

Height of wall = 30 ft, backfill surface is horizontal; $\phi = 40°$, $\delta = 20°$, $k_v = 0$, $k_h = 0.3$, $\gamma =$ 120 lb/ft^3.

Determine the total active thrust on the wall. What is the additional lateral pressure due to the earthquake?

11.23 For the wall given in Prob 11.22, determine the total passive thrust during the earthquake What is the change in passive thrust due to the earthquake? Assume $\phi = 30°$ and $\delta = 15°$.

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