CHAPTER 14

SHALLOW FOUNDATION III: COMBINED FOOTINGS AND MAT FOUNDATIONS

14.1 INTRODUCTION

Chapter 12 has considered the common methods of transmitting loads to subsoil through spread footings carrying single column loads. This chapter considers the following types of foundations:

1. Cantilever footings
2. Combined footings
3. Mat foundations

When a column is near or right next to a property limit, a square or rectangular footing concentrically loaded under the column would extend into the adjoining property. If the adjoining property is a public side walk or alley, local building codes may permit such footings to project into public property. But when the adjoining property is privately owned, the footings must be constructed within the property. In such cases, there are three alternatives which are illustrated in Fig. 14.1(a). These are

1. *Cantilever footing.* A cantilever or strap footing normally comprises two footings connected by a beam called a strap. A strap footing is a special case of a combined footing.
2. *Combined footing.* A combined footing is a long footing supporting two or more columns in one row.
3. *Mat or raft foundations.* A mat or raft foundation is a large footing, usually supporting several columns in two or more rows.

The choice between these types depends primarily upon the relative cost. In the majority of cases, mat foundations are normally used where the soil has low bearing capacity and where the total area occupied by an individual footing is not less than 50 per cent of the loaded area of the building.

When the distances between the columns and the loads carried by each column are not equal, there will be eccentric loading. The effect of eccentricity is to increase the base pressure on the side of eccentricity and decrease it on the opposite side. The effect of eccentricity on the base pressure of rigid footings is also considered here.

Mat Foundation in Sand

A foundation is generally termed as a mat if the least width is more than 6 meters. Experience indicates that the ultimate bearing capacity of a mat foundation on cohesionless soil is much higher than that of individual footings of lesser width. With the increasing width of the mat, or increasing relative density of the sand, the ultimate bearing capacity increases rapidly. Hence, the danger that a large mat may break into a sand foundation is too remote to require consideration. On account of the large size of mats the stresses in the underlying soil are likely to be relatively high to a considerable depth. Therefore, the influence of local loose pockets distributed at random throughout the sand is likely to be about the same beneath all parts of the mat and differential settlements are likely to be smaller than those of a spread foundation designed for the same soil

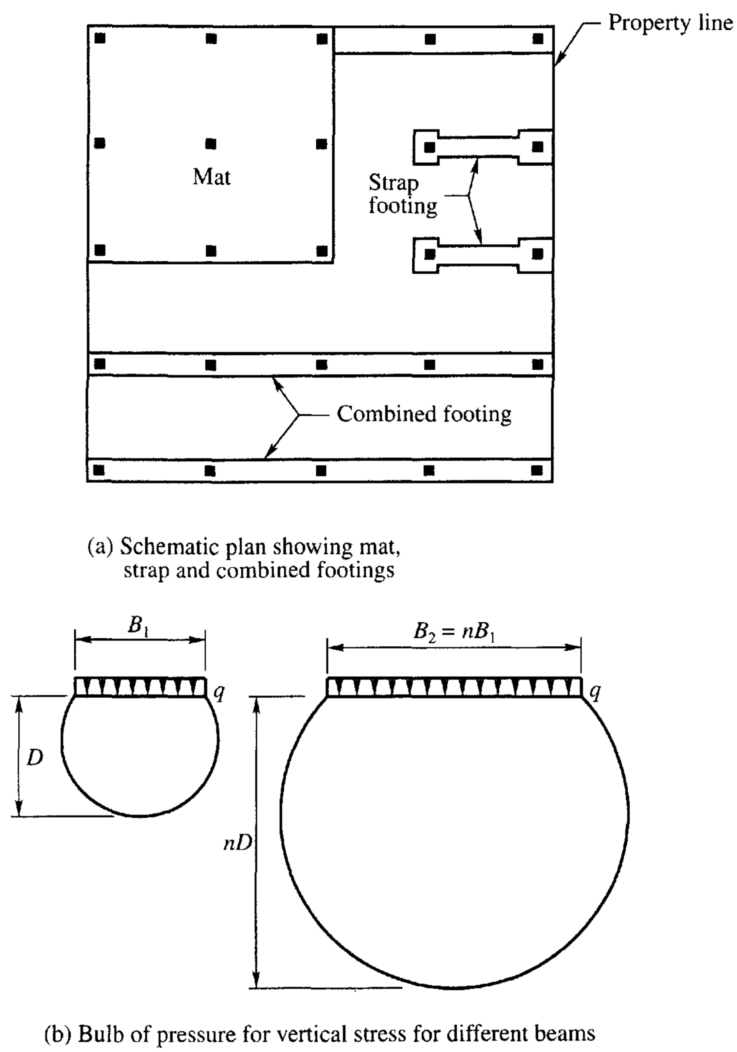

Figure 14.1 (a) Types of footings; (b) beams on compressible subgrade

pressure. The methods of calculating the ultimate bearing capacity dealt with in Chapter 12 are also applicable to mat foundations.

Mat Foundation in Clay

The net ultimate bearing capacity that can be sustained by the soil at the base of a mat on a deep deposit of clay or plastic silt may be obtained in the same manner as for footings on clay discussed in Chapter 12. However, by using the principle of *flotation*, the pressure on the base of the mat that induces settlement can be reduced by increasing the depth of the foundation. A brief discussion on the principle of flotation is dealt with in this chapter.

Rigid and Elastic Foundation

The conventional method of design of combined footings and mat foundations is to assume the foundation as infinitely rigid and the contact pressure is assumed to have a planar distribution. In the case of an elastic foundation, the soil is assumed to be a truly elastic solid obeying Hooke's law in all directions. The design of an elastic foundation requires a knowledge of the subgrade reaction which is briefly discussed here. However, the elastic method does not readily lend itself to engineering applications because it is extremely difficult and solutions are available for only a few extremely simple cases.

14.2 SAFE BEARING PRESSURES FOR MAT FOUNDATIONS ON SAND AND CLAY

Mats on Sand

Because the differential settlements of a mat foundation are less than those of a spread foundation designed for the same soil pressure, it is reasonable to permit larger safe soil pressures on a raft foundation. Experience has shown that a pressure approximately twice as great as that allowed for individual footings may be used because it does not lead to detrimental differential settlements. The maximum settlement of a mat may be about 50 mm (2 in) instead of 25 mm as for a spread foundation.

The shape of the curve in Fig. 13.3(a) shows that the net soil pressure corresponding to a given settlement is practically independent of the width of the footing or mat when the width becomes large. The safe soil pressure for design may with sufficient accuracy be taken as twice the pressure indicated in Fig. 13.5. Peck et al., (1974) recommend the following equation for computing net safe pressure,

$$q_s = 21\, N_{cor} \text{ kPa} \tag{14.1}$$

for $5 < N_{cor} < 50$

where N_{cor} is the *SPT* value corrected for energy, overburden pressure and field procedures.

Eq. 14.1 gives q_s values above the water table. A correction factor should be used for the presence of a water table as explained in Chapter 12.

Peck et al., (1974) also recommend that the q_s values as given by Eq. 14.1 may be increased somewhat if bedrock is encountered at a depth less than about one half the width of the raft.

The value of N to be considered is the average of the values obtained up to a depth equal to the least width of the raft. If the average value of N after correction for the influence of overburden pressure and dilatancy is less than about 5, Peck et al., say that the sand is generally considered to be too loose for the successful use of a raft foundation. Either the sand should be compacted or else the foundation should be established on piles or piers.

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The minimum depth of foundation recommended for a raft is about 2.5 m below the surrounding ground surface. Experience has shown that if the surcharge is less than this amount, the edges of the raft settle appreciably more than the interior because of a lack of confinement of the sand.

Safe Bearing Pressures of Mats on Clay

The quantity in Eq. 12.25(b) is the net bearing capacity q_{nu} at the elevation of the base of the raft in excess of that exerted by the surrounding surcharge. Likewise, in Eq. 12.25(c), q_{na} is the net allowable soil pressure. By increasing the depth of excavation, the pressure that can safely be exerted by the building is correspondingly increased. This aspect of the problem is considered further in Section 14.10 in *floating foundation.*

As for footings on clay, the factor of safety against failure of the soil beneath a mat on clay should not be less than 3 under normal loads, or less than 2 under the most extreme loads.

The settlement of the mat under the given loading condition should be calculated as per the procedures explained in Chapter 13. The net safe pressure should be decided on the basis of the permissible settlement.

14.3 ECCENTRIC LOADING

When the resultant of loads on a footing does not pass through the center of the footing, the footing is subjected to what is called *eccentric loading.* The loads on the footing may be vertical or inclined. If the loads are inclined it may be assumed that the horizontal component is resisted by the frictional resistance offered by the base of the footing. The vertical component in such a case is the only factor for the design of the footing. The effects of eccentricity on bearing pressure of the footings have been discussed in Chapter 12.

14.4 THE COEFFICIENT OF SUBGRADE REACTION

The coefficient of subgrade reaction is defined as the ratio between the pressure against the footing or mat and the settlement at a given point expressed as

$$k_s = \frac{q}{S}, \tag{14.2}$$

where k_s = coefficient of subgrade reaction expressed as force/length^3 (FL^{-3}),
q = pressure on the footing or mat at a given point expressed as force/length^2 (FL^{-2}),
S = settlement of the same point of the footing or mat in the corresponding unit of length.

In other words the coefficient of subgrade reaction is the unit pressure required to produce a unit settlement. In clayey soils, settlement under the load takes place over a long period of time and the coefficient should be determined on the basis of the final settlement. On purely granular soils, settlement takes place shortly after load application. Eq. (14.2) is based on two simplifying assumptions:

1. The value of k_s is independent of the magnitude of pressure.
2. The value of k_s has the same value for every point on the surface of the footing.

Both the assumptions are strictly not accurate. The value of k_s decreases with the increase of the magnitude of the pressure and it is not the same for every point of the surface of the footing as the settlement of a flexible footing varies from point to point. However the method is supposed to

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give realistic values for contact pressures and is suitable for beam or mat design when only a low order of settlement is required.

Factors Affecting the Value of k_s

Terzaghi (1955) discussed the various factors that affect the value of k_s. A brief description of his arguments is given below.

Consider two foundation beams of widths B_1 and B_2 such that $B_2 = nB_1$ resting on a compressible subgrade and each loaded so that the pressure against the footing is uniform and equal to q for both the beams (Fig. 14.1b). Consider the same points on each beam and, let

y_1 = settlement of beam of width B_1
y_2 = settlement of beam of width B_2

Hence $k_{s1} = \dfrac{q}{y_1}$ and $k_{s2} = \dfrac{q}{y_2}$

If the beams are resting on a subgrade whose deformation properties are more or less independent of depth (such as a stiff clay) then it can be assumed that the settlement increases in simple proportion to the depth of the pressure bulb.

Then $y_2 = ny_1$

and
$$k_{s2} = \frac{q}{ny_1} = \frac{q}{y_1}\frac{B_1}{B_2} = k_{s1}\frac{B_1}{B_2}. \tag{14.3}$$

A general expression for k_s can now be obtained if we consider B_1 as being of unit width (Terzaghi used a unit width of one foot which converted to metric units may be taken as equal to 0.30 m).

Hence by putting $B_1 = 0.30$ m, $k_s = k_{s2}$, $B = B_2$, we obtain

$$k_s = 0.3\frac{k_{s1}}{B} \tag{14.4}$$

where k_s is the coefficient of subgrade reaction of a long footing of width B meters and resting on stiff clay; k_{s1} is the coefficient of subgrade reaction of a long footing of width 0.30 m (approximately), resting on the same clay. It is to be noted here that the value of k_{s1} is derived from ultimate settlement values, that is, after consolidation settlement is completed.

If the beams are resting on clean sand, the final settlement values are obtained almost instantaneously. Since the modulus of elasticity of sand increases with depth, the deformation characteristics of the sand change and become less compressible with depth. Because of this characteristic of sand, the lower portion of the bulb of pressure for beam B_2 is less compressible than that of the sand enclosed in the bulb of pressure of beam B_1.

The settlement value y_2 lies somewhere between y_1 and ny_1. It has been shown experimentally (Terzaghi and Peck, 1948) that the settlement, y, of a beam of width B resting on sand is given by the expression

$$y = y_1\left[\frac{2B}{B+0.3}\right]^2 \tag{14.5}$$

where y_1 = settlement of a beam of width 0.30 m and subjected to the same reactive pressure as the beam of width B meters.

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Hence, the coefficient of subgrade reaction k_s of a beam of width B meters can be obtained from the following equation

$$k_s = \frac{q}{y} = \frac{q}{y_1}\left[\frac{B+0.30}{2B}\right]^2 = k_{s1}\left[\frac{B+0.30}{2B}\right]^2 \tag{14.6}$$

where k_{s1} = coefficient of subgrade reaction of a beam of width 0.30 m resting on the same sand.

Measurement of k_{s1}

A value for k_{s1} for a particular subgrade can be obtained by carrying out plate load tests. The standard size of plate used for this purpose is 0.30 × 0.30 m size. Let k_1 be the subgrade reaction for a plate of size 0.30 × 0.30 m size.

From experiments it has been found that $k_{s1} \approx k_1$ for sand subgrades, but for clays k_{s1} varies with the length of the beam. Terzaghi (1955) gives the following formula for clays

$$k_{s1} = k_1\left[\frac{L+0.152}{1.5L}\right] \tag{14.7a}$$

where L = length of the beam in meters and the width of the beam = 0.30 m. For a very long beam on clay subgrade we may write

$$k_{s1} = \frac{k_1}{1.5} \tag{14.7b}$$

Procedure to Find k_s

For sand

1. Determine k_1 from plate load test or from estimation.
2. Since $ks_{s1} \approx k_1$, use Eq. (14.6) to determine k_s for sand for any given width B meter.

For clay

1. Determine k_1 from plate load test or from estimation
2. Determine k_{s1} from Eq. (14.7a) as the length of beam is known.
3. Determine k_s from Eq. (14.4) for the given width B meters.

When plate load tests are used, k_1 may be found by one of the two ways,

1. A bearing pressure equal to not more than the ultimate pressure and the corresponding settlement is used for computing k_1
2. Consider the bearing pressure corresponding to a settlement of 1.3 mm for computing k_1.

Estimation of k_1 Values

Plate load tests are both costly and time consuming. Generally a designer requires only the values of the bending moments and shear forces within the foundation. With even a relatively large error in the estimation of k_1, moments and shear forces can be calculated with little error (Terzaghi, 1955); an error of 100 per cent in the estimation of k_s may change the structural behavior of the foundation by up to 15 per cent only.

Table 14.1a k_1 values for foundations on sand (MN/m^3)

Relative density SPT Values (Uncorrected)	Loose < 10	Medium 10–30	Dense > 30
Soil, dry or moist	15	45	175
Soil submerged	10	30	100

Table 14.1b k_1 values for foundation on clay

Consistency	Stiff	Very stiff	Hard
c_u (kN/m^2)	50-100	100-200	>200
k_1 (MN/m^3)	25	50	100

Source: Terzaghi (1955)

In the absence of plate load tests, estimated values of k_1 and hence k_s are used. The values suggested by Terzaghi for k_1 (converted into S.I. units) are given in Table 14.1.

14.5 PROPORTIONING OF CANTILEVER FOOTING

Strap or cantilever footings are designed on the basis of the following assumptions:

1. The strap is infinitely stiff. It serves to transfer the column loads to the soil with equal and uniform soil pressure under both the footings.
2. The strap is a pure flexural member and does not take soil reaction. To avoid bearing on the bottom of the strap a few centimeters of the underlying soil may be loosened prior to the placement of concrete.

A strap footing is used to connect an eccentrically loaded column footing close to the property line to an interior column as shown in Fig. 14.2.

With the above assumptions, the design of a strap footing is a simple procedure. It starts with a trial value of e, Fig. 14.2. Then the reactions R_1 and R_2 are computed by the principle of statics. The tentative footing areas are equal to the reactions R_1 and R_2 divided by the safe bearing pressure q_s. With tentative footing sizes, the value of e is computed. These steps are repeated until the trial value of e is identical with the final one. The shears and moments in the strap are determined, and the straps designed to withstand the shear and moments. The footings are assumed to be subjected to uniform soil pressure and designed as simple spread footings. Under the assumptions given above the resultants of the column loads Q_1 and Q_2 would coincide with the center of gravity of the two footing areas. Theoretically, the bearing pressure would be uniform under both the footings. However, it is possible that sometimes the full design live load acts upon one of the columns while the other may be subjected to little live load. In such a case, the full reduction of column load from Q_2 to R_2 may not be realized. It seems justified then that in designing the footing under column Q_2, only the dead load or dead load plus reduced live load should be used on column Q_1.

The equations for determining the position of the reactions (Fig. 14.2) are

$$R_1 = Q_1\left(1+\frac{e}{L_R}\right), \qquad R_2 = Q_2 - \frac{Q_1 e}{L_R} \tag{14.8}$$

where R_1 and R_2 = reactions for the column loads Q_1 and Q_2 respectively, e = distance of R_1 from Q_1, L_R = distance between R_1 and R_2.

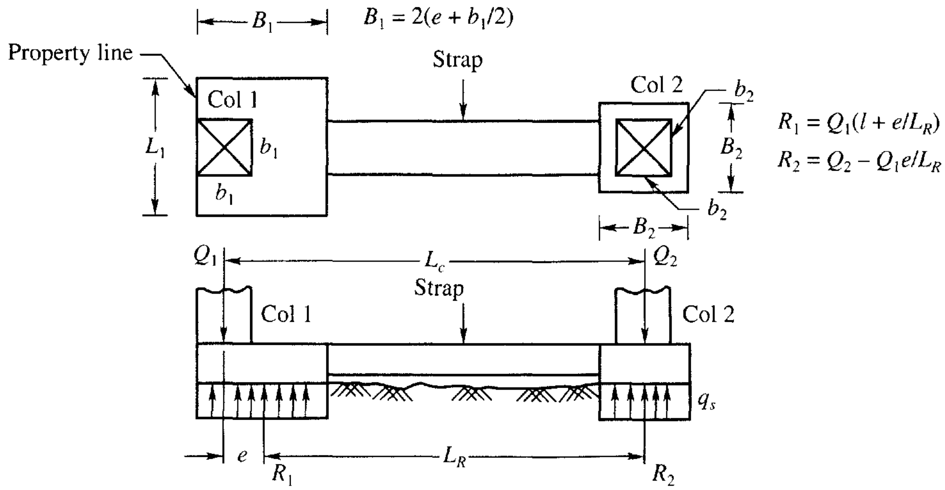

Figure 14.2 Principles of cantilever or strap footing design

14.6 DESIGN OF COMBINED FOOTINGS BY RIGID METHOD (CONVENTIONAL METHOD)

The rigid method of design of combined footings assumes that

1. The footing or mat is infinitely rigid, and therefore, the deflection of the footing or mat does not influence the pressure distribution,
2. The soil pressure is distributed in a straight line or a plane surface such that the centroid of the soil pressure coincides with the line of action of the resultant force of all the loads acting on the foundation.

Design of Combined Footings

Two or more columns in a row joined together by a stiff continuous footing form a combined footing as shown in Fig. 14.3a. The procedure of design for a combined footing is as follows:

1. Determine the total column loads $\Sigma Q = Q_1 + Q_2 + Q_3 + \ldots$ and location of the line of action of the resultant ΣQ. If any column is subjected to bending moment, the effect of the moment should be taken into account.
2. Determine the pressure distribution q per lineal length of footing.
3. Determine the width, B, of the footing.
4. Draw the shear diagram along the length of the footing. By definition, the shear at any section along the beam is equal to the summation of all vertical forces to the left or right of the section. For example, the shear at a section immediately to the left of Q_1 is equal to the area *abcd*, and immediately to the right of Q_1 is equal to (*abcd* − Q_1) as shown in Fig. 14.3a.
5. Draw the moment diagram along the length of the footing. By definition the bending moment at any section is equal to the summation of moment due to all the forces and reaction to the left (or right) of the section. It is also equal to the area under the shear diagram to the left (or right) of the section.
6. Design the footing as a continuous beam to resist the shear and moment.
7. Design the footing for transverse bending in the same manner as for spread footings.

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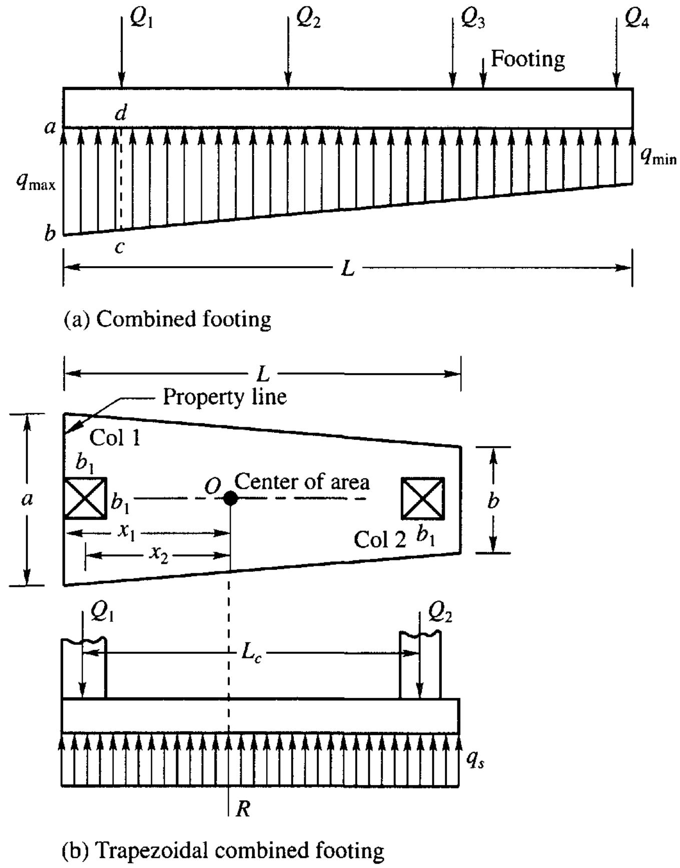

Figure 14.3 Combined or trapezoidal footing design

It should be noted here that the end column along the property line may be connected to the interior column by a rectangular or trapezoidal footing. In such a case no strap is required and both the columns together will be a combined footing as shown in Fig. 14.3b. It is necessary that the center of area of the footing must coincide with the center of loading for the pressure to remain uniform.

14.7 DESIGN OF MAT FOUNDATION BY RIGID METHOD

In the conventional rigid method the mat is assumed to be infinitely rigid and the bearing pressure against the bottom of the mat follows a planar distribution where the centroid of the bearing pressure coincides with the line of action of the resultant force of all loads acting on the mat. The procedure of design is as follows:

1. The column loads of all the columns coming from the superstructure are calculated as per standard practice. The loads include live and dead loads.
2. Determine the line of action of the resultant of all the loads. However, the weight of the mat is not included in the structural design of the mat because every point of the mat is supported by the soil under it, causing no flexural stresses.
3. Calculate the soil pressure at desired locations by the use of Eq. (12.73a)

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$$q = \frac{Q_t}{A} \pm \frac{Q_t e_x}{I_y} x \pm \frac{Q_t e_y}{I_x} y$$

where $Q_t = \Sigma Q$ = total load on the mat,

A = total area of the mat,

x, y = coordinates of any given point on the mat with respect to the x and y axes passing through the centroid of the area of the mat,

e_x, e_y = eccentricities of the resultant force,

I_x, I_y = moments of inertia of the mat with respect to the x and y axes respectively.

4. The mat is treated as a whole in each of two perpendicular directions. Thus the total shear force acting on any section cutting across the entire mat is equal to the arithmetic sum of all forces and reactions (bearing pressure) to the left (or right) of the section. The total bending moment acting on such a section is equal to the sum of all the moments to the left (or right) of the section.

14.8 DESIGN OF COMBINED FOOTINGS BY ELASTIC LINE METHOD

The relationship between deflection, y, at any point on an elastic beam and the corresponding bending moment M may be expressed by the equation

$$EI\frac{d^2y}{dx^2} = M \tag{14.10}$$

The equations for shear V and reaction q at the same point may be expressed as

$$EI\frac{d^3y}{dx^3} = V \tag{14.11}$$

$$EI\frac{d^4y}{dx^4} = q \tag{14.12}$$

where x is the coordinate along the length of the beam.

From the basic assumption of an elastic foundation

$$q = -yBk_s$$

where, B = width of footing, k_s = coefficient of subgrade reaction.

Substituting for q, Eq. (14.12) may be written as

$$EI\frac{d^4y}{dx^4} = -yBk_s \tag{14.13}$$

The classical solutions of Eq. (14.13) being of closed form, are not general in their application. Hetenyi (1946) developed equations for a load at any point along a beam. The development of solutions is based on the concept that the beam lies on a bed of elastic springs which is based on Winkler's hypothesis. As per this hypothesis, the reaction at any point on the beam depends only on the deflection at that point.

Methods are also available for solving the beam-problem on an elastic foundation by the method of finite differences (Malter, 1958). The finite element method has been found to be the most efficient of the methods for solving beam-elastic foundation problem. Computer programs are available for solving the problem.

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Since all the methods mentioned above are quite involved, they are not dealt with here. Interested readers may refer to Bowles (1996).

14.9 DESIGN OF MAT FOUNDATIONS BY ELASTIC PLATE METHOD

Many methods are available for the design of mat-foundations. The one that is very much in use is the finite difference method. This method is based on the assumption that the subgrade can be substituted by a bed of uniformly distributed coil springs with a spring constant k_s which is called the coefficient of subgrade reaction. The finite difference method uses the fourth order differential equation

$$\nabla^4 w = \frac{q - k_s w}{D}$$

where $$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \quad (14.14)$$

q = subgrade reaction per unit area,
k_s = coefficient of subgrade reaction,
w = deflection,

$$D = \text{rigidity of the mat } = \frac{Et^3}{12\left(1-\mu^2\right)}$$

E = modulus of elasticity of the material of the footing,
t = thickness of mat,
μ = Poisson's ratio.

Eq. (14.14) may be solved by dividing the mat into suitable square grid elements, and writing difference equations for each of the grid points. By solving the simultaneous equations so obtained the deflections at all the grid points are obtained. The equations can be solved rapidly with an electronic computer. After the deflections are known, the bending moments are calculated using the relevant difference equations.

Interested readers may refer to Teng (1969) or Bowles (1996) for a detailed discussion of the method.

14.10 FLOATING FOUNDATION

General Consideration

A *floating foundation* for a building is defined as a foundation in which the weight of the building is approximately equal to the full weight including water of the soil removed from the site of the building. This principle of flotation may be explained with reference to Fig. 14.4. Fig. 14.4(a) shows a horizontal ground surface with a horizontal water table at a depth d_w below the ground surface. Fig. 14.4(b) shows an excavation made in the ground to a depth D where $D > d_w$, and Fig. 14.4(c) shows a structure built in the excavation and completely filling it.

If the weight of the building is equal to the weight of the soil and water removed from the excavation, then it is evident that the total vertical pressure in the soil below depth D in Fig. 14.4(c) is the same as in Fig. 14.4(a) before excavation.

Since the water level has not changed, the neutral pressure and the effective pressure are therefore unchanged. Since settlements are caused by an increase in effective vertical pressure, if we could move from Fig. 14.4(a) to Fig. 14.4(c) without the intermediate case of 14.4(b), the building in Fig. 14.4(c) would not settle at all.

This is the principle of a floating foundation, an exact balance of weight removed against weight imposed. The result is zero settlement of the building.

However, it may be noted, that we cannot jump from the stage shown in Fig. 14.4(a) to the stage in Fig. 14.4(c) without passing through stage 14.4(b). The excavation stage of the building is the critical stage.

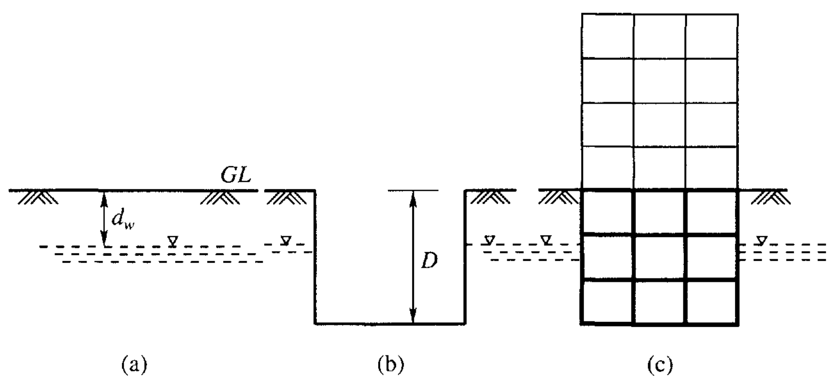
Cases may arise where we cannot have a fully floating foundation. The foundations of this type are sometimes called *partly compensated foundations* (as against *fully compensated* or *fully floating* foundations).

While dealing with floating foundations, we have to consider the following two types of soils. They are:

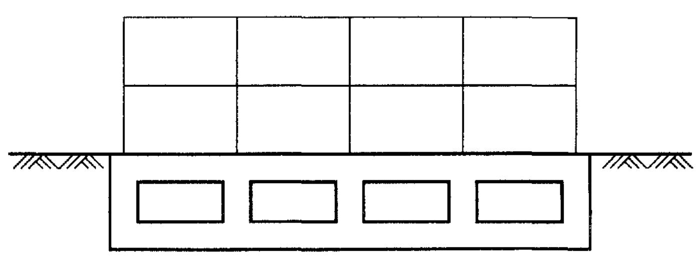
Type 1: The foundation soils are of such a strength that shear failure of soil will not occur under the building load but the settlements and particularly differential settlements, will be too large and will constitute *failure* of the structure. A floating foundation is used to reduce settlements to an acceptable value.

Type 2: The shear strength of the foundation soil is so low that rupture of the soil would occur if the building were to be founded at ground level. In the absence of a strong layer at a reasonable depth, the building can only be built on a floating foundation which reduces the shear stresses to an acceptable value. Solving this problem solves the settlement problem.

In both the cases, a rigid raft or box type of foundation is required for the floating foundation [Fig. 14.4(d)]



Balance of stresses in foundation excavation



(d) Rigid raft foundation

Figure 14.4 Principles of floating foundation; and a typical rigid raft foundation

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Problems to be Considered in the Design of a Floating Foundation

The following problems are to be considered during the design and construction stage of a floating foundation.

1. Excavation

The excavation for the foundation has to be done with care. The sides of the excavation should suitably be supported by sheet piling, soldier piles and timber or some other standard method.

2. Dewatering

Dewatering will be necessary when excavation has to be taken below the water table level. Care has to be taken to see that the adjoining structures are not affected due to the lowering of the water table.

3. Critical depth

In Type 2 foundations the shear strength of the soil is low and there is a theoretical limit to the depth to which an excavation can be made. Terzaghi (1943) has proposed the following equation for computing the critical depth D_c,

$$D_c = \frac{5.7s}{\gamma - (s / B)\sqrt{2}} \tag{14.15}$$

for an excavation which is long compared to its width

where γ = unit weight of soil,
s = shear strength of soil = $q_u/2$,
B = width of foundation,
L = length of foundation.

Skempton (1951) proposes the following equation for D_c, which is based on actual failures in excavations

$$D_c = N_c \frac{s}{\gamma} \tag{14.16}$$

or the factor of safety F_s against bottom failure for an excavation of depth D is

$$F_s = N_c \frac{s}{\gamma D + p}$$

where N_c is the bearing capacity factor as given by Skempton, and p is the surcharge load. The values of N_c may be obtained from Fig 12.13(a). The above equations may be used to determine the maximum depth of excavation.

4. Bottom heave

Excavation for foundations reduces the pressure in the soil below the founding depth which results in the heaving of the bottom of the excavation. Any heave which occurs will be reversed and appear as settlement during the construction of the foundation and the building. Though heaving of the bottom of the excavation cannot be avoided it can be minimized to a certain extent. There are three possible causes of heave:

1. Elastic movement of the soil as the existing overburden pressure is removed.
2. A gradual swelling of soil due to the intake of water if there is some delay for placing the foundation on the excavated bottom of the foundation.

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3. Plastic inward movement of the surrounding soil.

The last movement of the soil can be avoided by providing proper lateral support to the excavated sides of the trench.

Heaving can be minimized by phasing out excavation in narrow trenches and placing the foundation soon after excavation. It can be minimized by lowering the water table during the excavation process. Friction piles can also be used to minimize the heave. The piles are driven either before excavation commences or when the excavation is at half depth and the pile tops are pushed down to below foundation level. As excavation proceeds, the soil starts to expand but this movement is resisted by the upper part of the piles which go into tension. This heave is prevented or very much reduced.

It is only a *practical and pragmatic approach* that would lead to a safe and sound settlement free floating (or partly floating) foundation.

Example 14.1

A beam of length 4 m and width 0.75 m rests on stiff clay. A plate load test carried out at the site with the use of a square plate of size 0.30 m gives a coefficient of subgrade reaction k_1 equal to 25 MN/m^3. Determine the coefficient of subgrade reaction k_s for the beam.

Solution

First determine k_{s1} from Eq. (14.7a) for a beam of 0.30 m. wide and length 4 m. Next determine k_s from Eq. (14.4) for the same beam of width 0.75 m.

$$k_{s1} = k_1 \frac{L+0.152}{1.5L} = 25 \frac{4+0.152}{1.5\times 4} = 17.3 \text{ MN/m}^3$$

$$k_s = 0.3\frac{k_{s1}}{B} = \frac{0.3\times 17.3}{0.75} \approx 7 \text{ MN/m}^3$$

Example 14.2

A beam of length 4 m and width 0.75 m rests in dry medium dense sand. A plate load test carried out at the same site and at the same level gave a coefficient of subgrade reaction k_1 equal to 47 MN/m^3. Determine the coefficient of subgrade reaction for the beam.

Solution

For sand the coefficient of subgrade reaction, k_{s1}, for a long beam of width 0.3 m is the same as that for a square plate of size 0.3 × 0.3 m that is $k_{s1} = k_s$. k_s now can be found from Eq. (14.6) as

$$k_s = k_1 \left(\frac{B+0.3}{2B}\right)^2 = 47 \left(\frac{0.75+0.30}{1.5}\right)^2 = 23 \text{ MN/m}^3$$

Example 14.3

The following information is given for proportioning a cantilever footing with reference to Fig. 14.2.

Column Loads: Q_1 = 1455 kN, Q_2 = 1500 kN.

Size of column: 0.5 × 0.5 m.

L_c = 6.2 m, q_s = 384 kN/m^2

It is required to determine the size of the footings for columns 1 and 2.

Solution

Assume the width of the footing for column 1 = B_1 = 2 m.

First trial

Try e = 0.5 m. Now, L_R = 6.2 – 0.5 = 5.7 m.

Reactions

$$R_1 = Q_1 \; 1+\frac{e}{L_R} \; = 1455 \; 1+\frac{0.5}{5.7} \; = 1583 \text{ kN}$$

$$R_2 = \; Q_2 - \frac{Q_1 e}{L_R} \; = \; 1500 - \frac{1455 \times 0.5}{5.7} \; = 1372 \text{ kN}$$

Size of footings - First trial

Col. 1. Area of footing $\quad A_1 = \dfrac{1583}{384} = 4.122 \text{ sq.m}$

Col. 2. Area of footing $\quad A_2 = \dfrac{1372}{384} = 3.57 \text{ sq.m}$

Try 1.9 × 1.9 m

Second trial

New value of $\; e = \dfrac{B_1}{2} - \dfrac{b_1}{2} = \dfrac{2}{2} - \dfrac{0.5}{2} = 0.75 \text{ m}$

New $\; L_R = 6.20 - 0.75 = 5.45 \text{ m}$

$$R_1 = 1455 \; 1+\frac{0.75}{5.45} \; = 1655 \text{ kN}$$

$$R_2 = \; 1500 - \frac{1455 \times 0.75}{5.45} \; = 1300 \text{ kN}$$

$$A_1 = \frac{1655}{384} = 4.31 \text{ sq.m or } 2.08 \times 2.08 \text{ m}$$

$$A_2 = \frac{1300}{384} = 3.38 \text{ sq.m or } 1.84 \times 1.84 \text{ m}$$

Check $\quad e = \dfrac{B_1}{2} - \dfrac{b_1}{2} = 1.04 - 0.25 = 0.79 \approx 0.75 \text{ m}$

Use 2.08 × 2.08 m for Col. 1 and 1.90 × 1.90 m for Col. 2.
Note: Rectangular footings may be used for both the columns.

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Example 14.4

Figure Ex. 14.4 gives a foundation beam with the vertical loads and moment acting thereon. The width of the beam is 0.70 m and depth 0.50 m. A uniform load of 16 kN/m (including the weight of the beam) is imposed on the beam. Draw (a) the base pressure distribution, (b) the shear force diagram, and (c) the bending moment diagram. The length of the beam is 8 m.

Solution

The steps to be followed are:

1. Determine the resultant vertical force R of the applied loadings and its eccentricity with respect to the centers of the beam.
2. Determine the maximum and minimum base pressures.
3. Draw the shear and bending moment diagrams.

$R = 320 + 400 + 16 \times 8 = 848$ kN.

Taking the moment about the right hand edge of the beam, we have,

$$Rx = 848x = 320 \times 7 + 400 \times 1 + 16 \times \frac{8^2}{2} - 160 = 2992$$

$$\text{or} \qquad x = \frac{2992}{848} = 3.528 \text{ m}$$

$e = 4.0 - 3.528 = 0.472$ m to the right of center of the beam. Now from Eqs 12.39(a) and (b), using $e_y = 0$,

$$q_{\min}^{\max} = \frac{\Sigma Q}{A}\left(1 \pm 6\frac{e_x}{L}\right) = \frac{848}{8 \times 0.7}\left(1 \pm \frac{6 \times 0.472}{8}\right) = 205.02 \text{ or } 97.83 \text{ kN/m}^2$$

Convert the base pressures per unit area to load per unit length of beam.

The maximum vertical load = $0.7 \times 205.02 = 143.52$ kN/m.

The minimum vertical load = $0.7 \times 97.83 = 68.48$ kN/m.

The reactive loading distribution is given in Fig. Ex. 14.4(b).

Shear force diagram

Calculation of shear for a typical point such as the reaction point R_1 (Fig. Ex. 14.4(a)) is explained below.

Consider forces to the left of R_1 (without 320 kN).

Shear force V = upward shear force equal to the area *abcd* – downward force due to distributed load on beam *ab*

$$= \frac{68.48 + 77.9}{2} - 16 \times 1 = 57.2 \text{ kN}$$

Consider to the right of reaction point R_1 (with 320 kN).

$V = -320 + 57.2 = -262.8$ kN.

In the same away the shear at other points can be calculated. Fig. Ex. 14.4(c) gives the complete shear force diagram.

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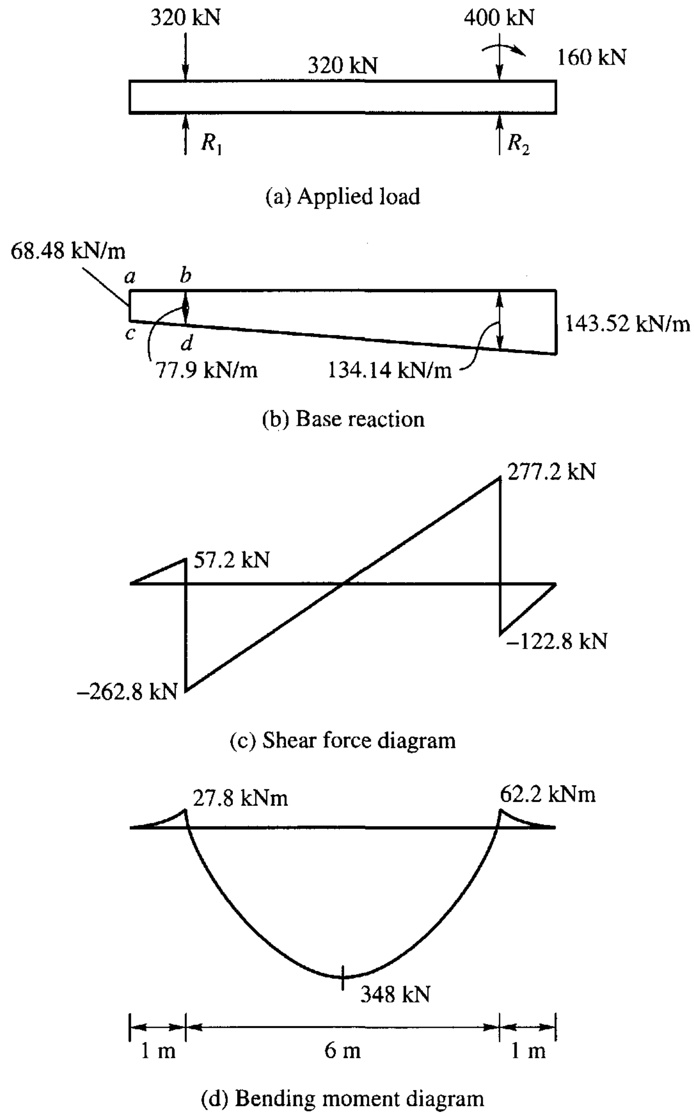

Figure Ex. 14.4

Bending Moment diagram

Bending moment at the reaction point R_1 = moment due to force equal to the area *abcd* + moment due to distributed load on beam *ab*

$$= 68.48 \times \frac{1}{2} + \frac{9.42}{2} \times \frac{1}{3} - 16 \times \frac{1}{2}$$

= 27.8 kN–m

The moments at other points can be calculated in the same way. The complete moment diagram is given in Fig. Ex. 14.4(d)

Example 14.5

The end column along a property line is connected to an interior column by a trapezoidal footing. The following data are given with reference to Fig. 14.3(b):

Column Loads: Q_1 = 2016 kN, Q_2 = 1560 kN.

Size of columns: 0.46 × 0.46 m.

L_c = 5.48 m.

Determine the dimensions a and b of the trapezoidal footing. The net allowable bearing pressure q_{na} = 190 kPa.

Solution

Determine the center of bearing pressure x_2 from the center of Column 1. Taking moments of all the loads about the center of Column 1, we have

$$(2016 + 1560)x_2 = 1560 \times 5.48$$

$$x_2 = \frac{1560 \times 5.48}{3576} = 2.39 \text{ m}$$

Now $$x_1 = 2.39 + \frac{0.46}{2} = 2.62 \text{ m}$$

Point O in Fig. 14.3(b) is the center of the area coinciding with the center of pressure. From the allowable pressure q_a = 190 kPa, the area of the combined footing required is

$$A = \frac{3576}{190} = 18.82 \text{ sq. m}$$

From geometry, the area of the trapezoidal footing (Fig. 14.3(b)) is

$$A = \frac{(a+b)L}{2} = \frac{(a+b)}{2}(5.94) = 18.82$$

or $$(a+b) = 6.34 \text{ m}$$

where, $$L = L_c + b_1 = 5.48 + 0.46 = 5.94 \text{ m}$$

From the geometry of the Fig. (14.3b), the distance of the center of area x_1 can be written in terms of a, b and L as

$$x_1 = \frac{L}{3}\frac{2a+b}{a+b}$$

or $$\frac{2a+b}{a+b} = \frac{3x_1}{L} = \frac{3 \times 2.62}{5.94} = 1.32 \text{ m}$$

but $a + b$ = 6.32 m or b = 6.32 – a. Now substituting for b we have,

$$\frac{2a + 6.34 - a}{6.34} = 1.32$$

and solving, a = 2.03 m, from which, b = 6.34 – 2.03 = 4.31 m.

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14.11 PROBLEMS

14.1 A beam of length 6 m and width 0.80 m is founded on dense sand under submerged conditions. A plate load test with a plate of 0.30 × 0.30 m conducted at the site gave a value for the coefficient of subgrade reaction for the plate equal to 95 MN/m^3. Determine the coefficient of subgrade reaction for the beam.

14.2 If the beam in Prob 14.1 is founded in very stiff clay with the value for k_1 equal to 45 MN/m^3, what is the coefficient of subgrade reaction for the beam?

14.3 Proportion a strap footing given the following data with reference to Fig. 14.2:
Q_1 = 580 kN, Q_2 = 900 kN
L_c = 6.2 m, b_1 = 0.40 m, q_s = 120 kPa.

14.4 Proportion a rectangular combined footing given the following data with reference to Fig. 14.3 (the footing is rectangular instead of trapezoidal):
Q_1 = 535 kN, Q_2 = 900 kN, b_1 = 0.40 m,
L_c = 4.75 m, q_s = 100 kPa.

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