CHAPTER 15

DEEP FOUNDATION I:
PILE FOUNDATION

15.1 INTRODUCTION

Shallow foundations are normally used where the soil close to the ground surface and up to the zone of significant stress possesses sufficient bearing strength to carry the superstructure load without causing distress to the superstructure due to settlement. However, where the top soil is either loose or soft or of a swelling type the load from the structure has to be transferred to deeper firm strata.

The structural loads may be transferred to deeper firm strata by means of piles. Piles are long slender columns either driven, bored or cast-in-situ. Driven piles are made of a variety of materials such as concrete, steel, timber etc., whereas cast-in-situ piles are concrete piles. They may be subjected to vertical or lateral loads or a combination of vertical and lateral loads. If the diameter of a bored-cast-in-situ pile is greater than about 0.75 m, it is sometimes called a drilled pier, drilled caisson or drilled shaft. The distinction made between a small diameter bored cast-in-situ pile (less than 0.75 m) and a larger one is just for the sake of design considerations. The design of drilled piers is dealt with in Chapter 17. This chapter is concerned with driven piles and small diameter bored cast-in-situ piles only.

15.2 CLASSIFICATION OF PILES

Piles may be classified as long or short in accordance with the $L/d$ ratio of the pile (where $L =$ length, $d =$ diameter of pile). A short pile behaves as a rigid body and rotates as a unit under lateral loads. The load transferred to the tip of the pile bears a significant proportion of the total vertical load on the top. In the case of a long pile, the length beyond a particular depth loses its significance under lateral loads, but when subjected to vertical load, the frictional load on the sides of the pile bears a significant part to the total load.
Piles may further be classified as vertical piles or inclined piles. Vertical piles are normally used to carry mainly vertical loads and very little lateral load. When piles are inclined at an angle to the vertical, they are called batter piles or raker piles. Batter piles are quite effective for taking lateral loads, but when used in groups, they also can take vertical loads. The behavior of vertical and batter piles subjected to lateral loads is dealt with in Chapter 16.

### Types of Piles According to Their Composition

Piles may be classified according to their composition as

1. Timber Piles,
2. Concrete Piles,
3. Steel Piles.

**Timber Piles:** Timber piles are made of tree trunks with the branches trimmed off. Such piles shall be of sound quality and free of defects. The length of the pile may be 15 m or more. If greater lengths are required, they may be spliced. The diameter of the piles at the butt end may vary from 30 to 40 cm. The diameter at the tip end should not be less than 15 cm.

Piles entirely submerged in water last long without decay provided marine borers are not present. When a pile is subjected to alternate wetting and drying the useful life is relatively short unless treated with a wood preservative, usually creosote at 250 kg per m$^3$ for piles in fresh water and 350 kg/m$^3$ in sea water.

After being driven to final depth, all pile heads, treated or untreated, should be sawed square to sound undamaged wood to receive the pile cap. But before concrete for the pile cap is poured, the head of the treated piles should be protected by a zinc coat, lead paint or by wrapping the pile heads with fabric upon which hot pitch is applied.

Driving of timber piles usually results in the crushing of the fibers on the head (or brooming) which can be somewhat controlled by using a driving cap, or ring around the butt.

The usual maximum design load per pile does not exceed 250 kN. Timber piles are usually less expensive in places where timber is plentiful.

**Concrete Piles.** Concrete piles are either precast or cast-in-situ piles. Precast concrete piles are cast and cured in a casting yard and then transported to the site of work for driving. If the work is of a very big nature, they may be cast at the site also.

Precast piles may be made of uniform sections with pointed tips. Tapered piles may be manufactured when greater bearing resistance is required. Normally piles of square or octagonal sections are manufactured since these shapes are easy to cast in horizontal position. Necessary reinforcement is provided to take care of handling stresses. Piles may also be prestressed.

Maximum load on a prestressed concrete pile is approximately 2000 kN and on precast piles 1000 kN. The optimum load range is 400 to 600 kN.

**Steel Piles.** Steel piles are usually rolled H shapes or pipe piles. H-piles are proportioned to withstand large impact stresses during hard driving. Pipe piles are either welded or seamless steel pipes which may be driven either open-end or closed-end. Pipe piles are often filled with concrete after driving, although in some cases this is not necessary. The optimum load range on steel piles is 400 to 1200 kN.

### 15.3 Types of Piles According to the Method of Installation

According to the method of construction, there are three types of piles. They are

1. Driven piles,
2. Cast-in-situ piles and
3. Driven and cast-in-situ piles.
Driven Piles

Piles may be of timber, steel or concrete. When the piles are of concrete, they are to be precast. They may be driven either vertically or at an angle to the vertical. Piles are driven using a pile hammer. When a pile is driven into granular soil, the soil so displaced, equal to the volume of the driven pile, compacts the soil around the sides since the displaced soil particles enter the soil spaces of the adjacent mass which leads to densification of the mass. The pile that compacts the soil adjacent to it is sometimes called a compaction pile. The compaction of the soil mass around a pile increases its bearing capacity.

If a pile is driven into saturated silty or cohesive soil, the soil around the pile cannot be densified because of its poor drainage qualities. The displaced soil particles cannot enter the void space unless the water in the pores is pushed out. The stresses developed in the soil mass adjacent to the pile due to the driving of the pile have to be borne by the pore water only. This results in the development of pore water pressure and a consequent decrease in the bearing capacity of the soil. The soil adjacent to the piles is remolded and loses to a certain extent its structural strength. The immediate effect of driving a pile in a soil with poor drainage qualities is, therefore, to decrease its bearing strength. However, with the passage of time, the remolded soil regains part of its lost strength due to the reorientation of the disturbed particles (which is termed thixotrophy) and due to consolidation of the mass. The advantages and disadvantages of driven piles are:

Advantages

1. Piles can be precast to the required specifications.
2. Piles of any size, length and shape can be made in advance and used at the site. As a result, the progress of the work will be rapid.
3. A pile driven into granular soil compacts the adjacent soil mass and as a result the bearing capacity of the pile is increased.
4. The work is neat and clean. The supervision of work at the site can be reduced to a minimum. The storage space required is very much less.
5. Driven piles may conveniently be used in places where it is advisable not to drill holes for fear of meeting ground water under pressure.
6. Driven pile are the most favored for works over water such as piles in wharf structures or jetties.

Disadvantages

1. Precast or prestressed concrete piles must be properly reinforced to withstand handling stresses during transportation and driving.
2. Advance planning is required for handling and driving.
3. Requires heavy equipment for handling and driving.
4. Since the exact length required at the site cannot be determined in advance, the method involves cutting off extra lengths or adding more lengths. This increases the cost of the project.
5. Driven piles are not suitable in soils of poor drainage qualities. If the driving of piles is not properly phased and arranged, there is every possibility of heaving of the soil or the lifting of the driven piles during the driving of a new pile.
6. Where the foundations of adjacent structures are likely to be affected due to the vibrations generated by the driving of piles, driven piles should not be used.
**Cast-in-situ Piles**

*Cast-in-situ* piles are concrete piles. These piles are distinguished from drilled piers as small diameter piles. They are constructed by making holes in the ground to the required depth and then filling the hole with concrete. Straight bored piles or piles with one or more bulbs at intervals may be cast at the site. The latter type are called *under-reamed piles*. Reinforcement may be used as per the requirements. *Cast-in-situ* piles have advantages as well as disadvantages.

**Advantages**

1. Piles of any size and length may be constructed at the site.
2. Damage due to driving and handling that is common in precast piles is eliminated in this case.
3. These piles are ideally suited in places where vibrations of any type are required to be avoided to preserve the safety of the adjoining structure.
4. They are suitable in soils of poor drainage qualities since cast-in-situ piles do not significantly disturb the surrounding soil.

**Disadvantages**

1. Installation of *cast-in-situ* piles requires careful supervision and quality control of all the materials used in the construction.
2. The method is quite cumbersome. It needs sufficient storage space for all the materials used in the construction.
3. The advantage of increased bearing capacity due to compaction in granular soil that could be obtained by a driven pile is not produced by a *cast-in-situ* pile.
4. Construction of piles in holes where there is heavy current of ground water flow or artesian pressure is very difficult.

A straight bored pile is shown in Fig. 15.1(a).

**Driven and Cast-in-situ Piles**

This type has the advantages and disadvantages of both the driven and the *cast-in-situ* piles. The procedure of installing a driven and *cast-in-situ* pile is as follows:

A steel shell is driven into the ground with the aid of a mandrel inserted into the shell. The mandrel is withdrawn and concrete is placed in the shell. The shell is made of corrugated and reinforced thin sheet steel (mono-tube piles) or pipes (Armco welded pipes or common seamless pipes). The piles of this type are called a shell type. The shell-less type is formed by withdrawing the shell while the concrete is being placed. In both the types of piles the bottom of the shell is closed with a conical tip which can be separated from the shell. By driving the concrete out of the shell an enlarged bulb may be formed in both the types of piles. Franki piles are of this type. The common types of driven and *cast-in-situ* piles are given in Fig. 15.1. In some cases the shell will be left in place and the tube is concreted. This type of pile is very much used in piling over water.

**15.4 USES OF PILES**

The major uses of piles are:

1. To carry vertical compression load.
2. To resist uplift load.
3. To resist horizontal or inclined loads.
Normally vertical piles are used to carry vertical compression loads coming from superstructures such as buildings, bridges etc. The piles are used in groups joined together by pile caps. The loads carried by the piles are transferred to the adjacent soil. If all the loads coming on the tops of piles are transferred to the tips, such piles are called end-bearing or point-bearing piles. However, if all the load is transferred to the soil along the length of the pile such piles are called friction piles. If, in the course of driving a pile into granular soils, the soil around the pile gets compacted, such piles are called compaction piles. Fig. 15.2(a) shows piles used for the foundation of a multitistoried building to carry loads from the superstructure.

Piles are also used to resist uplift loads. Piles used for this purpose are called tension piles or uplift piles or anchor piles. Uplift loads are developed due to hydrostatic pressure or overturning movement as shown in Fig. 15.2(a).

Piles are also used to resist horizontal or inclined forces. Batter piles are normally used to resist large horizontal loads. Fig. 15.2(b) shows the use of piles to resist lateral loads.

### 15.5 SELECTION OF PILE

The selection of the type, length and capacity is usually made from estimation based on the soil conditions and the magnitude of the load. In large cities, where the soil conditions are well known and where a large number of pile foundations have been constructed, the experience gained in the past is extremely useful. Generally the foundation design is made on the preliminary estimated values. Before the actual construction begins, pile load tests must be conducted to verify the design values. The foundation design must be revised according to the test results. The factors that govern the selection of piles are:

1. Length of pile in relation to the load and type of soil
2. Character of structure
3. Availability of materials
4. Type of loading
5. Factors causing deterioration
6. Ease of maintenance
7. Estimated costs of types of piles, taking into account the initial cost, life expectancy and cost of maintenance
8. Availability of funds

All the above factors have to be largely analyzed before deciding up on a particular type.

### 15.6 INSTALLATION OF PILES

The method of installing a pile at a site depends upon the type of pile. The equipment required for this purpose varies. The following types of piles are normally considered for the purpose of installation

#### 1. Driven piles

The piles that come under this category are,

a. Timber piles,
b. Steel piles, *H*-section and pipe piles,
c. Precast concrete or prestressed concrete piles, either solid or hollow sections.

2. Driven *cast-in-situ* piles
This involves driving of a steel tube to the required depth with the end closed by a detachable conical tip. The tube is next concreted and the shell is simultaneously withdrawn. In some cases the shell will not be withdrawn.

3. Bored *cast-in-situ* piles
Boring is done either by auguring or by percussion drilling. After boring is completed, the bore is concreted with or without reinforcement.

**Pile Driving Equipment for Driven and Driven *Cast-in-situ* Piles**
Pile driving equipment contains three parts. They are

1. A pile frame,
2. Piling winch,
3. Impact hammers.

**Pile Frame**
Pile driving equipment is required for driven piles or driven *cast-in-situ* piles. The driving pile frame must be such that it can be mounted on a standard tracked crane base machine for mobility on land sites or on framed bases for mounting on stagings or pontoons in offshore construction. Fig. 15.3 gives a typical pile frame for both onshore and offshore construction. Both the types must be capable of full rotation and backward or forward raking. All types of frames consist essentially of *leaders*, which are a pair of steel members extending for the full height of the frame and which guide the hammer and pile as it is driven into the ground. Where long piles have to be driven the leaders can be extended at the top by a telescopic boom.

The base frame may be mounted on swivel wheels fitted with self-contained jacking screws for leveling the frame or it may be carried on steel rollers. The rollers run on steel girders or long timbers and the frame is moved along by winching from a deadman set on the roller track, or by turning the rollers by a tommy-bar placed in holes at the ends of the rollers. Movements parallel to the rollers are achieved by winding in a wire rope terminating in hooks on the ends of rollers; the frame then skids in either direction along the rollers. It is important to ensure that the pile frame remains in its correct position throughout the driving of a pile.

**Piling Winches**
Piling winches are mounted on the base. Winches may be powered by steam, diesel or gasoline engines, or electric motors. Steam-powered winches are commonly used where steam is used for the piling hammer. Diesel or gasoline engines, or electric motors (rarely) are used in conjunction with drop hammers or where compressed air is used to operate the hammers.

**Impact Hammers**
The impact energy for driving piles may be obtained by any one of the following types of hammers. They are

1. Drop hammers,
2. Single-acting steam hammers,
3. Double-acting steam hammers,
4. Diesel hammer,
5. Vibratory hammer.

Drop hammers are at present used for small jobs. The weight is raised and allowed to fall freely on the top of the pile. The impact drives the pile into the ground.

In the case of a single-acting steam hammer steam or air raises the moveable part of the hammer which then drops by gravity alone. The blows in this case are much more rapidly delivered than for a drop hammer. The weights of hammers vary from about 1500 to 10,000 kg with the length of stroke being about 90 cm. In general the ratio of ram weight to pile weight may vary from 0.5 to 1.0.

In the case of a double-acting hammer steam or air is used to raise the moveable part of the hammer and also to impart additional energy during the down stroke. The downward acceleration of the ram owing to gravity is increased by the acceleration due to steam pressure. The weights of hammers vary from about 350 to 2500 kg. The length of stroke varies from about 20 to 90 cm. The rate of driving ranges from 300 blows per minute for the light types, to 100 blows per minute for the heaviest types.

Diesel or internal combustion hammers utilize diesel-fuel explosions to provide the impact energy to the pile. Diesel hammers have considerable advantage over steam hammers because they are lighter, more mobile and use a smaller amount of fuel. The weight of the hammer varies from about 1000 to 2500 kg.
The advantage of the power-hammer type of driving is that the blows fall in rapid succession (50 to 150 blows per minute) keeping the pile in continuous motion. Since the pile is continuously moving, the effects of the blows tend to convert to pressure rather than impact, thus reducing damage to the pile.

The vibration method of driving piles is now coming into prominence. Driving is quiet and does not generate local vibrations. Vibration driving utilizes a variable speed oscillator attached to the top of the pile (Fig. 15.3(b)). It consists of two counter rotating eccentric weights which are in phase twice per cycle (180° apart) in the vertical direction. This introduces vibration through the pile which can be made to coincide with the resonant frequency of the pile. As a result, a push-pull effect is created at the pile tip which breaks up the soil structure allowing easy pile penetration into the ground with a relatively small driving effort. Pile driving by the vibration method is quite common in Russia.

Jetting Piles
Water jetting may be used to aid the penetration of a pile into dense sand or dense sandy gravel. Jetting is ineffective in firm to stiff clays or any soil containing much coarse to stiff cobbles or boulders.

Where jetting is required for pile penetration a stream of water is discharged near the pile point or along the sides of the pile through a pipe 5 to 7.5 cm in diameter. An adequate quantity of water is essential for jetting. Suitable quantities of water for jetting a 250 to 350 mm pile are

- Fine sand: 15-25 liters/second,
- Coarse sand: 25-40 liters/second,
- Sandy gravels: 45-600 liters/second.

A pressure of at least 5 kg/cm² or more is required.

PART A—VERTICAL LOAD BEARING CAPACITY OF A SINGLE VERTICAL PILE

15.7 GENERAL CONSIDERATIONS
The bearing capacity of groups of piles subjected to vertical or vertical and lateral loads depends upon the behavior of a single pile. The bearing capacity of a single pile depends upon

1. Type, size and length of pile,
2. Type of soil,
3. The method of installation.

The bearing capacity depends primarily on the method of installation and the type of soil encountered. The bearing capacity of a single pile increases with an increase in the size and length. The position of the water table also affects the bearing capacity.

In order to be able to design a safe and economical pile foundation, we have to analyze the interactions between the pile and the soil, establish the modes of failure and estimate the settlements from soil deformation under dead load, service load etc. The design should comply with the following requirements.

1. It should ensure adequate safety against failure; the factor of safety used depends on the importance of the structure and on the reliability of the soil parameters and the loading systems used in the design.
2. The settlements should be compatible with adequate behavior of the superstructure to avoid impairing its efficiency.

Load Transfer Mechanism

Statement of the Problem

Fig. 15.4(a) gives a single pile of uniform diameter $d$ (circular or any other shape) and length $L$ driven into a homogeneous mass of soil of known physical properties. A static vertical load is applied on the top. It is required to determine the ultimate bearing capacity $Q_u$ of the pile.

When the ultimate load applied on the top of the pile is $Q_u$, a part of the load is transmitted to the soil along the length of the pile and the balance is transmitted to the pile base. The load transmitted to the soil along the length of the pile is called the ultimate friction load or skin load $Q_f$, and that transmitted to the base is called the base or point load $Q_b$. The total ultimate load $Q_u$ is expressed as the sum of these two, that is,

$$Q_u = Q_b + Q_f = q_b A_b + f_s A_s \quad (15.1)$$

where

- $Q_u$ = ultimate load applied on the top of the pile
- $q_b$ = ultimate unit bearing capacity of the pile at the base
- $A_b$ = bearing area of the base of the pile
- $A_s$ = total surface area of pile embedded below ground surface
- $f_s$ = unit skin friction (ultimate)

Load Transfer Mechanism

Consider the pile shown in Fig. 15.4(b) is loaded to failure by gradually increasing the load on the top. If settlement of the top of the pile is measured at every stage of loading after an equilibrium condition is attained, a load settlement curve as shown in Fig. 15.4(c) can be obtained.

If the pile is instrumented, the load distribution along the pile can be determined at different stages of loading and plotted as shown in Fig. 15.4(b).

When a load $Q_1$ acts on the pile head, the axial load at ground level is also $Q_1$, but at level $A_1$ (Fig. 15.4(b)), the axial load is zero. The total load $Q_1$ is distributed as friction load within a length of pile $L_1$. The lower section $A_1 B$ of pile will not be affected by this load. As the load at the top is increased to $Q_2$, the axial load at the bottom of the pile is just zero. The total load $Q_2$ is distributed as friction load along the whole length of pile $L$. The friction load distribution curves along the pile shaft may be as shown in the figure. If the load put on the pile is greater than $Q_2$, a part of this load is transferred to the soil at the base as point load and the rest is transferred to the soil surrounding the pile. With the increase of load $Q$ on the top, both the friction and point loads continue to increase. The friction load attains an ultimate value $Q_f$ at a particular load level, say $Q_m$, at the top, and any further increment of load added to $Q_m$ will not increase the value of $Q_f$. However, the point load, $Q_p$, still goes on increasing till the soil fails by punching shear failure. It has been determined by Van Wheele (1957) that the point load $Q_p$ increases linearly with the elastic compression of the soil at the base.

The relative proportions of the loads carried by skin friction and base resistance depend on the shear strength and elasticity of the soil. Generally the vertical movement of the pile which is required to mobilize full end resistance is much greater than that required to mobilize full skin friction. Experience indicates that in bored cast-in-situ piles full frictional load is normally mobilized at a settlement equal to 0.5 to 1 percent of pile diameter and the full base load $Q_b$ at 10 to 20 percent of the diameter. But, if this ultimate load criterion is applied to piles of large diameter in clay, the settlement at the working load (with a factor of safety of 2 on the ultimate load) may be excessive. A typical load-settlement relationship of friction load and base load is shown in
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Figure 15.4 Load transfer mechanism

Fig. 15.4(d) (Tomlinson, 1986) for a large diameter bored and *cast-in-situ* pile in clay. It may be seen from this figure that the full shaft resistance is mobilized at a settlement of only 15 mm whereas the full base resistance, and the ultimate resistance of the entire pile, is mobilized at a settlement of 120 mm. The shaft load at a settlement of 15 mm is only 1000 kN which is about 25 percent of the base resistance. If a working load of 2000 kN at a settlement of 15 mm is used for the design, at this working load, the full shaft resistance will have been mobilized whereas only about 50 percent of the base resistance has been mobilized. This means if piles are designed to carry a working load equal to 1/3 to 1/2 the total failure load, there is every likelihood of the shaft resistance being fully mobilized at the working load. This has an important bearing on the design.

The type of load-settlement curve for a pile depends on the relative strength values of the surrounding and underlying soil. Fig. 15.5 gives the types of failure (Kézdi, 1975). They are as follows:
Figure 15.5  Types of failure of piles. Figures (a) to (e) indicate how strength of soil determines the type of failure: (a) buckling in very weak surrounding soil; (b) general shear failure in the strong lower soil; (c) soil of uniform strength; (d) low strength soil in the lower layer, skin friction predominant; (e) skin friction in tension (Kézdi, 1975)

Fig 15.5(a) represents a driven-pile (wooden or reinforced concrete), whose tip bears on a very hard stratum (rock). The soil around the shaft is too weak to exert any confining pressure or lateral resistance. In such cases, the pile fails like a compressed, slender column of the same material; after a more or less elastic compression buckling occurs. The curve shows a definite failure load.

Fig. 15.5(b) is the type normally met in practice. The pile penetrates through layers of soil having low shear strength down to a layer having a high strength and the layer extending sufficiently below the tip of the pile. At ultimate load $Q_u$, there will be a base general shear failure at the tip of the pile, since the upper layer does not prevent the formation of a failure surface. The effect of the shaft friction is rather less, since the lower dense layer prevents the occurrence of excessive settlements. Therefore, the degree of mobilization of shear stresses along the shaft will be low. The load settlement diagram is of the shape typical for a shallow footing on dense soil.

Fig. 15.5(c) shows the case where the shear strength of the surrounding soil is fairly uniform; therefore, a punching failure is likely to occur. The load-settlement diagram does not have a vertical tangent, and there is no definite failure load. The load will be carried by point resistance as well as by skin friction.

Fig 15.5(d) is a rare case where the lower layer is weaker. In such cases, the load will be carried mainly by shaft friction, and the point resistance is almost zero. The load-settlement curve shows a vertical tangent, which represents the load when the shaft friction has been fully mobilized.
Fig. 15.5(e) is a case when a pull, \(-Q\), acts on the pile. Since the point resistance is again zero the same diagram, as in Fig. 15.5(d), will characterize the behavior, but heaving occurs.

**Definition of Failure Load**

The methods of determining failure loads based on load-settlement curves are described in subsequent sections. However, in the absence of a load settlement curve, a failure load may be defined as that which causes a settlement equal to 10 percent of the pile diameter or width (as per the suggestion of Terzaghi) which is widely accepted by engineers. However, if this criterion is applied to piles of large diameter in clay and a nominal factor of safety of 2 is used to obtain the working load, then the settlement at the working load may be excessive.

**Factor of Safety**

In almost all cases where piles are acting as structural foundations, the allowable load is governed solely from considerations of tolerable settlement at the working load.

The working load for all pile types in all types of soil may be taken as equal to the sum of the base resistance and shaft friction divided by a suitable factor of safety. A safety factor of 2.5 is normally used. Therefore we may write

\[
Q_a = \frac{Q_b + Q_f}{2.5} \tag{15.2}
\]

In case where the values of \(Q_b\) and \(Q_f\) can be obtained independently, the allowable load can be written as

\[
Q_a = \frac{Q_b}{3} + \frac{Q_f}{1.5} \tag{15.3}
\]

It is permissible to take a safety factor equal to 1.5 for the skin friction because the peak value of skin friction on a pile occurs at a settlement of only 3–8 mm (relatively independent of shaft diameter and embedded length but may depend on soil parameters) whereas the base resistance requires a greater settlement for full mobilization.

The least of the allowable loads given by Eqs. (15.2) and (15.3) is taken as the design working load.

**15.8 METHODS OF DETERMINING ULTIMATE LOAD BEARING CAPACITY OF A SINGLE VERTICAL PILE**

The ultimate bearing capacity, \(Q_u\), of a single vertical pile may be determined by any of the following methods.

1. By the use of static bearing capacity equations.
2. By the use of SPT and CPT values.
3. By field load tests.
4. By dynamic method.

The determination of the ultimate point bearing capacity, \(q_b\), of a deep foundation on the basis of theory is a very complex one since there are many factors which cannot be accounted for in the theory. The theory assumes that the soil is homogeneous and isotropic which is normally not the case. All the theoretical equations are obtained based on plane strain conditions. Only shape factors are applied to take care of the three-dimensional nature of the problem. Compressibility
characteristics of the soil complicate the problem further. Experience and judgment are therefore very essential in applying any theory to a specific problem. The skin load $Q_s$ depends on the nature of the surface of the pile, the method of installation of the pile and the type of soil. An exact evaluation of $Q_s$ is a difficult job even if the soil is homogeneous over the whole length of the pile. The problem becomes all the more complicated if the pile passes through soils of variable characteristics.

15.9 GENERAL THEORY FOR ULTIMATE BEARING CAPACITY

According to Vesic (1967), only punching shear failure occurs in deep foundations irrespective of the density of the soil so long as the depth-width ratio $L/d$ is greater than 4 where $L =$ length of pile and $d =$ diameter (or width of pile). The types of failure surfaces assumed by different investigators are shown in Fig. 15.6 for the general shear failure condition. The detailed experimental study of Vesic indicates that the failure surfaces do not revert back to the shaft as shown in Fig. 15.6(b).

The total failure load $Q_u$ may be written as follows

$$Q_u = Q_a + W_p = Q_o + Q_f + W_p$$  \hspace{1cm} (15.4)

where

- $Q_a =$ load at failure applied to the pile
- $Q_o =$ base resistance
- $Q_f =$ shaft resistance
- $W_p =$ weight of the pile.

The general equation for the base resistance may be written as

$$Q_o = cN_c + q_o'N_q + \frac{1}{2}ydN_yA_b$$  \hspace{1cm} (15.5)

where

- $d =$ width or diameter of the shaft at base level
- $q_o' =$ effective overburden pressure at the base level of the pile
- $A_b =$ base area of pile
- $c =$ cohesion of soil
- $\gamma =$ effective unit weight of soil
- $N_c, N_q, N_y =$ bearing capacity factors which take into account the shape factor.

Cohesionless Soils

For cohesionless soils, $c = 0$ and the term $1/2ydN_y$ becomes insignificant in comparison with the term $q_o'N_q$ for deep foundations. Therefore Eq. (15.5) reduces to

$$Q_o = q_o'N_qA_b = q_bA_b$$  \hspace{1cm} (15.6)

Eq. (15.4) may now be written as

$$Q_o = Q_o + W_p = q_o'N_qA_b + W_p + Q_f$$  \hspace{1cm} (15.7)

The net ultimate load in excess of the overburden pressure load $q_o'N_q$ is

$$Q_o + W_p - q_oA_b = q_o'N_qA_b + W_p - q_oA_b + Q_f$$  \hspace{1cm} (15.8)

If we assume, for all practical purposes, $W_p$ and $q_oA_b$ are roughly equal for straight side or moderately tapered piles, Eq. (15.8) reduces to
Figure 15.6 The shapes of failure surfaces at the tips of piles as assumed by (a) Terzaghi, (b) Meyerhof, and (c) Vesic

\[ Q_u = q'_o N_A \tan \delta \]

or

\[ Q_u = q'_o N_A + A_s \tan \delta \]

where

- \( A_s \) = surface area of the embedded length of the pile
- \( q'_o \) = average effective overburden pressure over the embedded depth of the pile
- \( K_s \) = average lateral earth pressure coefficient
- \( \delta \) = angle of wall friction.

**Cohesive Soils**

For cohesive soils such as saturated clays (normally consolidated), we have for \( \phi = 0 \), \( N_q = 1 \) and \( N_f = 0 \). The ultimate base load from Eq. (15.5) is
\[
\bar{Q}_b = (c_bN_c + q'_o)A_b
\]  
(15.10)

The net ultimate base load is

\[
(Q_b - q'_o A_b) = Q_b = c_b N_c A_b
\]  
(15.11)

Therefore, the net ultimate load capacity of the pile, \( Q_u \), is

\[
Q_u = c_b N_c A_b + Q_f
\]
or
\[
Q_u = c_b N_c A_b + A_s \alpha \bar{f}_u
\]  
(15.12)

where 
\( \alpha = \) adhesion factor

\( c_u = \) average undrained shear strength of clay along the shaft

\( c_b = \) undrained shear strength of clay at the base level

\( N_c = \) bearing capacity factor

Equations (15.9) and (15.12) are used for analyzing the net ultimate load capacity of piles in cohesionless and cohesive soils respectively. In each case the following types of piles are considered.

1. Driven piles
2. Driven and cast-in-situ piles
3. Bored piles

### 15.10 ULTIMATE BEARING CAPACITY IN COHESIONLESS SOILS

#### Effect of Pile Installation on the Value of the Angle of Friction

When a pile is driven into loose sand its density is increased (Meyerhof, 1959), and the horizontal extent of the compacted zone has a width of about 6 to 8 times the pile diameter. However, in dense sand, pile driving decreases the relative density because of the dilatancy of the sand and the loosened sand along the shaft has a width of about 5 times the pile diameter (Kerisel, 1961). On the basis of field and model test results, Kishida (1967) proposed that the angle of internal friction decreases linearly from a maximum value of \( \phi_2 \) at the pile tip to a low value of \( \phi_1 \) at a distance of 3.5\( d \) from the tip where \( d \) is the diameter of the pile, \( \phi_1 \) is the angle of friction before the installation of the pile and \( \phi_2 \) after the installation as shown in Fig. 15.7. Based on the field data, the relationship between \( \phi_1 \) and \( \phi_2 \) in sands may be written as

\[
\phi_2 = \frac{\phi_1 + 40}{2}
\]  
(15.13)

Figure 15.7 The effect of driving a pile on \( \phi \)
An angle of $\phi_1 = \phi_2 = 40^\circ$ in Eq. (15.13) means no change of relative density due to pile driving. Values of $\phi_1$ are obtained from insitu penetration tests (with no correction due to overburden pressure, but corrected for field procedure) by using the relationships established between $\phi$ and SPT or CPT values. Kishida (1967) has suggested the following relationship between $\phi$ and the SPT value $N_{cor}$ as

$$\phi^* = \sqrt{20N_{cor} + 15^*}$$  \hspace{1cm} (15.14)

However, Tomlinson (1986) is of the opinion that it is unwise to use higher values for $\phi$ due to pile driving. His argument is that the sand may not get compacted, as for example, when piles are driven into loose sand, the resistance is so low and little compaction is given to the soil. He suggests that the value of $\phi$ used for the design should represent the in situ condition that existed before driving.

With regard to driven and cast-in-situ piles, there is no suggestion by any investigator as to what value of $\phi$ should be used for calculating the base resistance. However, it is safer to assume the insitu $\phi$ value for computing the base resistance.

With regard to bored and cast-in-situ piles, the soil gets loosened during boring. Tomlinson (1986) suggests that the $\phi$ value for calculating both the base and skin resistance should represent the loose state. However, Poulos et al., (1980) suggests that for bored piles, the value of $\phi$ be taken as

$$\phi = \phi_1 - 3$$  \hspace{1cm} (15.15)

where $\phi_1$ = angle of internal friction prior to installation of the pile.

15.11 CRITICAL DEPTH

The ultimate bearing capacity $Q_u$ in cohesionless soils as per Eq. (15.9) is

$$Q_u = q'_o N' q A_b + \frac{1}{2} K A_s \tan \delta A_s$$  \hspace{1cm} (15.16a)

or

$$Q_u = q_b A_b + f_s A_s$$  \hspace{1cm} (15.16b)

Eq. (15.16b) implies that both the point resistance $q_b$ and the skin resistance $f_s$ are functions of the effective overburden pressure $q_o$ in cohesionless soils and increase linearly with the depth of embedment, $L_e$, of the pile. However, extensive research work carried out by Vesic (1967) has revealed that the base and frictional resistances remain almost constant beyond a certain depth of embedment which is a function of $\phi$. This phenomenon was attributed to arching by Vesic. One conclusion from the investigation of Vesic is that in cohesionless soils, the bearing capacity factor, $N_q$, is not a constant depending on $\phi$ only, but also on the ratio $L/d$ (where $L = $ length of embedment of pile, $d = $ diameter or width of pile). In a similar way, the frictional resistance, $f_s$, increases with the $L/d$ ratio and remains constant beyond a particular depth. Let $L_c$ be the depth, which may be called the critical depth, beyond which both $q_b$ and $f_s$ remain constant. Experiments of Vesic have indicated that $L_c$ is a function of $\phi$. The $L_c/d$ ratio as a function of $\phi$ may be expressed as follows (Poulos and Davis, 1980)

For $28^\circ < \phi < 36.5^\circ$

$$\frac{L_c}{d} = 5 + 0.24 (\phi^* - 28^\circ)$$  \hspace{1cm} (15.17a)

For $36.5^\circ < \phi < 42^\circ$

$$\frac{L_c}{d} = 7 + 2.35(\phi^* - 36.5^\circ)$$  \hspace{1cm} (15.17b)

The above expressions have been developed based on the curve given by Poulos and Davis, (1980) giving the relationship between $L_c/d$ and $\phi^*$. 
The Eqs. (15.17) indicate

\[ L_c/d = 5 \quad \text{at } \phi = 28^\circ \]

\[ L_c/d = 7 \quad \text{at } \phi = 36.5^\circ \]

\[ L_c/d = 20 \quad \text{at } \phi = 42^\circ \]

The values to be used for obtaining \( L_c/d \) are as follows (Poulos and Davis, 1980)

for driven piles \( \phi = 0.75 \phi_i + 10^\circ \) \hspace{1cm} (15.18a)

for bored piles: \( \phi = \phi_i - 3^\circ \) \hspace{1cm} (15.18b)

where \( \phi_i \) = angle of internal friction prior to the installation of the pile.

15.12 TOMLINSON’S SOLUTION FOR \( Q_b \) IN SAND

**Driven Piles**

The theoretical \( N_q \) factor in Eq. (15.9) is a function of \( \phi \). There is great variation in the values of \( N_q \) derived by different investigators as shown in Fig. 15.8. Comparison of observed base resistances of piles by Nordlund (1963) and Vesic (1964) have shown (Tomlinson, 1986) that \( N_q \) values established by Berezantsev et al., (1961) which take into account the depth to width ratio of the pile,

![Figure 15.8](image-url)  
Figure 15.8 Bearing capacity factors for circular deep foundations (after Kézdi, 1975)
most nearly conform to practical criteria of pile failure. Berezantsev’s values of $N_q$ as adopted by Tomlinson (1986) are given in Fig. 15.9.

It may be seen from Fig. 15.9 that there is a rapid increase in $N_q$ for high values of $\phi$, giving thereby high values of base resistance. As a general rule (Tomlinson, 1986), the allowable working load on an isolated pile driven to virtual refusal, using normal driving equipment, in a dense sand or gravel consisting predominantly of quartz particles, is given by the allowable load on the pile considered as a structural member rather than by consideration of failure of the supporting soil, or if the permissible working stress on the material of the pile is not exceeded, then the pile will not fail.

As per Tomlinson, the maximum base resistance $q_b$ is normally limited to 11000 kN/m$^2$ (110 tf/ft$^2$) whatever might be the penetration depth of the pile.

**Bored and Cast-in-situ Piles in Cohesionless Soils**

Bored piles are formed in cohesionless soils by drilling with rigs. The sides of the holes might be supported by the use of casing pipes. When casing is used, the concrete is placed in the drilled hole and the casing is gradually withdrawn. In all the cases the sides and bottom if the hole will be loosened as a result of the boring operations, even though it may be initially be in a dense or medium dense state. Tomlinson suggests that the values of the parameters in Eq. (15.9) must be calculated by assuming that the $\phi$ value will represent the loose condition.

However, when piles are installed by rotary drilling under a bentonite slurry for stabilizing the sides, it may be assumed that the $\phi$ value used to calculate both the skin friction and base resistance will correspond to the undisturbed soil condition (Tomlinson, 1986).

The assumption of loose conditions for calculating skin friction and base resistance means that the ultimate carrying capacity of a bored pile in a cohesionless soil will be considerably lower than that of a pile driven in the same soil type. As per De Beer (1965), the base resistance $q_b$ of a bored and cast-in-situ pile is about one third of that of a driven pile.
We may write,

\[ q_b \text{ (bored pile)} = \frac{1}{3} q_h \text{ (driven pile)} \]

So far as friction load is concerned, the frictional parameter may be calculated by assuming a value of \( \phi \) equal to 28° which represents the loose condition of the soil.

The same Eq. (15.9) may be used to compute \( Q_u \) based on the modifications explained above.

### 15.13 MEYERHOF'S METHOD OF DETERMINING \( Q_b \) FOR PILES IN SAND

Meyerhof (1976) takes into account the critical depth ratio \( (L/d) \) for estimating the value of \( Q_b \). Fig. 15.10 shows the variation of \( L/d \) for both the bearing capacity factors \( N_c \) and \( N_q \) as a function of \( \phi \). According to Meyerhof, the bearing capacity factors increase with \( L/d \) and reach a maximum value at \( L/d \) equal to about 0.5 \( (L/d) \), where \( L_b \) is the actual thickness of the bearing stratum. For example, in a homogeneous soil (15.6c) \( L_b \) is equal to \( L \), the actual embedded length of pile; whereas in Fig. 15.6b, \( L_b \) is less than \( L \).

![Figure 15.10 Bearing capacity factors and critical depth ratios \( L/d \) for driven piles (after Meyerhof, 1976)](image-url)
As per Fig. 15.10, the value of \( L_{Jd} \) is about 25 for \( \phi = 45^\circ \) and it decreases with a decrease in the angle of friction \( \phi \). Normally, the magnitude of \( L_{Jd} \) for piles is greater than 0.5 \( (L_{Jd}) \) so that maximum values of \( N'_{c} \) and \( N'_{q} \) may apply for the calculation of \( q_b \), the unit bearing pressure of the pile. Meyerhof prescribes a limiting value for \( q_b \) based on his findings on static cone penetration resistance. The expression for the limiting value, \( q_{bl} \) is

\[
\begin{align*}
\text{for dense sand: } & q_{bl} = 50 N'_{q} \tan \phi \text{ kN/m}^2 \\
\text{for loose sand: } & q_{bl} = 25 N'_{q} \tan \phi \text{ kN/m}^2
\end{align*}
\]

(15.19a)

(15.19b)

where \( \phi \) is the angle of shearing resistance of the bearing stratum. The limiting \( q_{bl} \) values given by Eqs (15.9a and b) remain practically independent of the effective overburden pressure and groundwater conditions beyond the critical depth.

The equation for base resistance in sand may now be expressed as

\[
Q_b = q'_o N_{q} A_b \leq q_{bl} A_b
\]

(15.20)

where \( q'_o \) = effective overburden pressure at the tip of the pile \( L_{Jd} \) and \( N_{q} \) = bearing capacity factor (Fig. 15.10).

Eq. (15.20) is applicable only for driven piles in sand. For bored cast-in-situ piles the value of \( q_b \) is to be reduced by one third to one-half.

Clay Soil (\( \phi = 0 \))

The base resistance \( Q_b \) for piles in saturated clay soil may be expressed as

\[
Q_b = N'_{c} c_u A_b = 9 c_u A_b
\]

(15.21)

where \( N'_{c} = 9 \), and \( c_u \) = undrained shear strength of the soil at the base level of the pile.

**15.14 VESIC’S METHOD OF DETERMINING \( Q_b \)**

The unit base resistance of a pile in a \((c - \phi)\) soil may be expressed as (Vesic, 1977)

\[
q_b = c N'_{c} + q'_o N'^{*}_{q}
\]

(15.22)

where

- \( c \) = unit cohesion
- \( q'_o \) = effective vertical pressure at the base level of the pile
- \( N'^{*}_{c} \) and \( N'^{*}_{q} \) = bearing capacity factors related to each other by the equation

\[
N'^{*}_{c} = (N'^{*}_{q} - 1) \cot \phi
\]

(15.23)

As per Vesic, the base resistance is not governed by the vertical ground pressure \( q'_o \) but by the mean effective normal ground stress \( \sigma_m \) expressed as

\[
\sigma_m = \frac{1 + 2 K_o}{3} q'_o
\]

(15.24)

in which \( K_o \) = coefficient of earth pressure for the at rest condition = 1 - \( \sin \phi \).

Now the bearing capacity in Eq. (15.22) may be expressed as

\[
q_b = c N'^*_{c} + \sigma_m N'^*_{q}
\]

(15.25)
An equation for $N^*_{\sigma}$ can be obtained from Eqs. (15.22), (15.24) and (15.25) as

$$N^*_{\sigma} = \frac{3N^*_{\sigma}}{1 + 2K_o}$$  \hspace{1cm} (15.26)

Vesic has developed an expression for $N^*_{\sigma}$ based on the ultimate pressure needed to expand a spherical cavity in an infinite soil mass as

$$N^*_{\sigma} = \alpha_1 e^{\alpha_2 N^*_\phi (I_{rr})^{\alpha_3}}$$  \hspace{1cm} (15.27)

where

$$\alpha_1 = \frac{3}{3 - \sin\phi}, \alpha_2 = \left(\frac{\pi}{2} - \phi\right) \tan \phi, \alpha_3 = \frac{1.33 \sin \phi}{(1 + \sin \phi)}, \text{ and } N^*_\phi = \tan^2(45^\circ + \phi/2)$$

According to Vesic

$$I_{rr} = \frac{I_r}{1 + I_r \Delta}$$  \hspace{1cm} (15.28)

where

- $I_r = \text{rigidity index} = \frac{E_s}{2(1 + \mu)(c + q'_o \tan \phi)} = \frac{G}{(c + q'_o \tan \phi)}$  \hspace{1cm} (15.29)

where $I_{rr} = \text{reduced rigidity index for the soil}$

- $\Delta = \text{average volumetric strain in the plastic zone below the pile point}$
- $E_s = \text{modulus of elasticity of soil}$
- $G = \text{shear modulus of soil}$
- $\mu = \text{Poisson's ratio of soil}$

Figures 15.11(a) and 15.11(b) give plots of $N^*_{\sigma}$ versus $\phi$, and $N^*_c$ versus $\phi$ for various values of $I_{rr}$ respectively.

The values of rigidity index can be computed knowing the values of shear modulus $G$, and the shear strength $s (= c + q'_o \tan \phi)$.

When an undrained condition exists in the saturated clay soil or the soil is cohesionless and is in a dense state we have $\Delta = 0$ and in such a case $I_r = I_{rr}$.

For $\phi = 0$ (undrained condition), we have

$$N^*_{c} = 1.33(\ln I_{rr} + 1) + \frac{\pi}{2} + 1$$  \hspace{1cm} (15.30)

The value of $I_r$ depends upon the soil state, (a) for sand, loose or dense and (b) for clay low, medium or high plasticity. For preliminary estimates the following values of $I_r$ may be used.

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$I_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand ($D_r = 0.5-0.8$)</td>
<td>75–150</td>
</tr>
<tr>
<td>Silt</td>
<td>50–75</td>
</tr>
<tr>
<td>Clay</td>
<td>150–250</td>
</tr>
</tbody>
</table>
Figure 15.11  (a) Bearing capacity factor $N^*_a$ (b) Bearing capacity factor $N^*_c$  
(Vesic, 1977)
Table 15.1 Bearing capacity factors $N'_q$ and $N'_c$ by Janbu

<table>
<thead>
<tr>
<th>$\phi^\circ$</th>
<th>$N'_q$</th>
<th>$N'_c$</th>
<th>$N'_q$</th>
<th>$N'_c$</th>
<th>$N'_q$</th>
<th>$N'_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75°</td>
<td>1.00</td>
<td>5.74</td>
<td>1.00</td>
<td>5.74</td>
<td>1.00</td>
<td>5.74</td>
</tr>
<tr>
<td>90°</td>
<td>1.50</td>
<td>6.25</td>
<td>1.57</td>
<td>6.49</td>
<td>1.64</td>
<td>7.33</td>
</tr>
<tr>
<td>105°</td>
<td>2.25</td>
<td>7.11</td>
<td>2.47</td>
<td>8.34</td>
<td>2.71</td>
<td>9.70</td>
</tr>
<tr>
<td>10</td>
<td>5.29</td>
<td>11.78</td>
<td>6.40</td>
<td>14.83</td>
<td>7.74</td>
<td>18.53</td>
</tr>
<tr>
<td>20</td>
<td>13.60</td>
<td>21.82</td>
<td>18.40</td>
<td>30.14</td>
<td>24.90</td>
<td>41.39</td>
</tr>
<tr>
<td>30</td>
<td>23.08</td>
<td>31.53</td>
<td>33.30</td>
<td>46.12</td>
<td>48.04</td>
<td>67.18</td>
</tr>
<tr>
<td>40</td>
<td>41.37</td>
<td>48.11</td>
<td>64.20</td>
<td>75.31</td>
<td>99.61</td>
<td>117.52</td>
</tr>
<tr>
<td>45</td>
<td>79.90</td>
<td>78.90</td>
<td>134.87</td>
<td>133.87</td>
<td>227.68</td>
<td>226.68</td>
</tr>
</tbody>
</table>

15.15 JANBU’S METHOD OF DETERMINING $Q_b$

The bearing capacity equation of Janbu (1976) is the same as Eq. (15.22) and is expressed as

$$Q_b = (cN'_c + q'_o N'_q)A_b$$

(15.31)

The shape of the failure surface as assumed by Janbu is similar to that given in Fig. 15.6(b). Janbu’s equation for $N'_q$ is

$$N'_q = \left[\tan \phi + \sqrt{1 + \tan^2 \phi}\right] e^{2\psi \tan \phi}$$

(15.32)

where $\psi$ = angle as shown in Fig. 15.6(b). This angle varies from 60° in soft compressible soil to 105° in dense sand. The values for $N'_q$ used by Janbu are the same as those given by Vesic (Eq. 15.23). Table 15.1 gives the bearing capacity factors of Janbu.

Since Janbu’s bearing capacity factor $N'_q$ depends on the angle $\psi$, there are two uncertainties involved in this procedure. They are

1. The difficulty in determining the values of $\psi$ for different situations at base level.
2. The settlement required at the base level of the pile for the full development of a plastic zone.

For full base load $Q_b$ to develop, at least a settlement of about 10 to 20 percent of the pile diameter is required which is considerable for larger diameter piles.

15.16 COYLE AND CASTELLO’S METHOD OF ESTIMATING $Q_b$ IN SAND

Coyle and Castello (1981) made use of the results of 24 full scale pile load tests driven in sand for evaluating the bearing capacity factors. The form of equation used by them is the same as Eq. (15.6) which may be expressed as

$$Q_b = q'_o N'_q A_b$$

(15.33)

where

$q'_o$ = effective overburden pressure at the base level of the pile

$N'_q$ = bearing capacity factor

Coyle and Castello collected data from the instrumented piles, and separated from the total load the base load and friction load. The total force at the top of the pile was applied by means of a jack. The soil at the site was generally fine sand with some percentage of silt. The lowest and the
highest relative densities were 40 to 100 percent respectively. The pile diameter was generally around 1.5 ft and pile penetration was about 50 ft. Closed end steel pipe was used for the tests in some places and precast square piles or steel \( H \) piles were used at other places.

The bearing capacity factor \( N'_q \) was evaluated with respect to depth ratio \( L/d \) in Fig. 15.12 for various values of \( \phi \).

### 15.17 THE ULTIMATE SKIN RESISTANCE OF A SINGLE PILE IN COHESIONLESS SOIL

#### Skin Resistance (Straight Shaft)

The ultimate skin resistance in a homogeneous soil as per Eq. (15.9) is expressed as

\[
Q_f = A_s \overline{q'_c} \overline{K}_s \tan \delta \tag{15.34a}
\]

In a layered system of soil \( \overline{q'_c}, \overline{K}_s \) and \( \delta \) vary with respect to depth. Equation (15.34a) may then be expressed as

\[
Q_f = \int_0^L P \overline{q'_c} \overline{K}_s \tan \delta \, dz \tag{15.34b}
\]

where \( \overline{q'_c}, \overline{K}_s \) and \( \delta \) refer to thickness \( dz \) of each layer and \( P \) is the perimeter of the pile.

As explained in Section 15.10 the effective overburden pressure does not increase linearly with depth and reaches a constant value beyond a particular depth \( L_c \), called the critical depth which is a function of \( \phi \). It is therefore natural to expect the skin resistance \( f_s \) also to remain constant beyond depth \( L_c \). The magnitude of \( L_c \) may be taken as equal to \( 20d \).
Table 15.2 Values of $K_s$ and $\delta$ (Broms, 1966)

<table>
<thead>
<tr>
<th>Pile material</th>
<th>$\delta$</th>
<th>$K_s$ Low $D_r$</th>
<th>$K_s$ High $D_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>20°</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Concrete</td>
<td>$3/4\phi$</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Wood</td>
<td>$2/3\phi$</td>
<td>1.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

![Graphs](image)

Figure 15.13 Values of $K_s \tan \delta$ in sand as per (a) Poulos and Davis 1980, (b) Meyerhof, 1976 and (c) taper factor $F_w$ (after Nordlund, 1963)

Eq. (15.17) can be used for determining the critical length $I_c$ for any given set of values of $\phi$ and $d$. $Q_j$ can be calculated from Eq. (15.34) if $K_s$ and $\delta$ are known.
The values of $K_s$ and $\delta$ vary not only with the relative density and pile material but also with the method of installation of the pile.

Broms (1966) has related the values of $K_s$ and $\delta$ to the effective angle of internal friction $\phi$ of cohesionless soils for various pile materials and relative densities ($D_r$) as shown in Table 15.2. The values are applicable to driven piles. As per the present state of knowledge, the maximum skin friction is limited to 110 kN/m$^2$ (Tomlinson, 1986).

Eq. (15.34) may also be written as

$$Q_f = \int_0^L P \bar{q}' \beta \, dz$$  \hspace{1cm} (15.35)$$

where, $\beta = \frac{K}{15} \tan \delta$.

Poulos and Davis, (1980) have given a curve giving the relationship between $\beta$ and $\phi^e$ which is applicable for driven piles and all types of material surfaces. According to them there is not sufficient evidence to show that $\beta$ would vary with the pile material. The relationship between $\beta$ and $\phi$ is given in Fig. 15.13(a). For bored piles, Poulos et al, recommend the relationship given by Meyerhof (1976) between $\phi$ and $\beta$ (Fig. 15.13(b)).

Skin Resistance on Tapered Piles

Nordlund (1963) has shown that even a small taper of 1° on the shaft gives a four fold increase in unit friction in medium dense sand under compression loading. Based on Nordlund's analysis, curves have been developed (Poulos and Davis, 1980) giving a relationship between taper angle $\phi^e$ and a taper correction factor $F_\phi$ which can be used in Eq. (15.35) as

$$Q_f = \int_0^L P \bar{q}' \beta \, dz$$  \hspace{1cm} (15.36)$$

Eq. (15.36) gives the ultimate skin load for tapered piles. The correction factor $F_\phi$ can be obtained from Fig. 15.13(c). The value of $\phi$ to be used for obtaining $F_\phi$ is as per Eq. (15.18a) for driven piles.

15.18 Skin Resistance $Q_f$ by Coyle and Castello Method (1981)

For evaluating frictional resistance, $Q_f$ for piles in sand, Coyle and Castello (1981) made use of the results obtained from 24 field tests on piles. The expression for $Q_f$ is the one given in Eq. (15.34a). They developed a chart (Fig. 15.14) giving relationships between $K_s$ and $\phi$ for various $L/d$ ratios. The angle of wall friction $\delta$ is assumed equal to 0.8$\phi$. The expression for $Q_f$ is

$$Q_f = A_s \bar{q}' \frac{K_s}{15} \tan \delta$$  \hspace{1cm} (15.34a)$$

where $\bar{q}' = \text{average effective overburden pressure}$ and $\delta = \text{angle of wall friction} = 0.8\phi$.

The value of $K_s$ can be obtained Fig. 15.14.

15.19 Static Bearing Capacity of Piles in Clay Soil

Equation for Ultimate Bearing Capacity

The static ultimate bearing capacity of piles in clay as per Eq. (15.12) is

$$Q_u = Q_h + Q_f = c_b N_c A_b + a \bar{c}_u A_s$$  \hspace{1cm} (15.37)$$
For layered clay soils where the cohesive strength varies along the shaft, Eq. (15.37) may be written as

\[ Q_u = c_b N_c A_b + \frac{L}{\alpha} \bar{c}_u A_s \]  

(15.38)

**Bearing Capacity Factor** \( N_c \)

The value of the bearing capacity factor \( N_c \) that is generally accepted is 9 which is the value proposed by Skempton (1951) for circular foundations for a \( L/B \) ratio greater than 4. The base capacity of a pile in clay soil may now be expressed as

\[ Q_b = 9c_b A_b \]  

(15.39)

The value of \( c_b \) may be obtained either from laboratory tests on undisturbed samples or from the relationships established between \( c_u \) and field penetration tests. Eq. (15.39) is applicable for all types of pile installations.

**Skin Resistance by \( \alpha \)-Method**

Tomlinson (1986) has given some empirical correlations for evaluating \( \alpha \) in Eq. (15.37) for different types of soil conditions and \( L/d \) ratios. His procedure requires a great deal of judgment of the soil conditions in the field and may lead to different interpretations by different geotechnical engineers. A simplified approach for such problems would be needed. Dennis and Olson (1983b)
made use of the information provided by Tomlinson and developed a single curve giving the relationship between $\alpha$ and the undrained shear strength $c_u$ of clay as shown in Fig. 15.15.

This curve can be used to estimate the values of $\alpha$ for piles with penetration lengths less than 30 m. As the length of the embedment increases beyond 30 m, the value of $\alpha$ decreases. Piles of such great length experience elastic shortening that results in small shear strain or slip at great depth as compared to that at shallow depth. Investigation indicates that for embedment greater than about 50 m the value of $\alpha$ from Fig 15.15 should be multiplied by a factor 0.56. For embedments between 30 and 50 m, the reduction factor may be considered to vary linearly from 1.0 to 0.56 (Dennis and Olson, 1983a, b)

**Skin Resistance by $\lambda$-Method**

Vijayvergiya and Focht (1972) have suggested a different approach for computing skin load $Q_f$ for steel-pipe piles on the basis of examination of load test results on such piles. The equation is of the form

$$Q_f = \lambda(q_o' + 2c_u)A_s$$

Where

- $\lambda$ = frictional capacity coefficient,
- $q_o'$ = mean effective vertical stress between the ground surface and pile tip.

The other terms are already defined. $\lambda$ is plotted against pile penetration as shown in Fig. 15.16.

Eq. (15.40) has been found very useful for the design of heavily loaded pipe piles for offshore structures.

**$\beta$-Method or the Effective Stress Method of Computing Skin Resistance**

In this method, the unit skin friction $f_s$ is defined as

$$f_s = K_s \tan \delta q_o' = \beta q_o'$$

Where

- $\beta$ = the skin factor = $K_s \tan \delta$,
- $K_s$ = lateral earth pressure coefficient,

**Figure 15.15** Adhesion factor $\alpha$ for piles with penetration lengths less than 50 m in clay.
(Data from Dennis and Olson 1983 a, b; Stas and Kulhawy, 1984)
Figure 15.16 Frictional capacity coefficient $\lambda$ vs pile penetration (Vijayvergiya and Focht, 1972)

$\delta$ = angle of wall friction,
$q'_o$ = average effective overburden pressure.

Burland (1973) discusses the values to be used for $\beta$ and demonstrates that a lower limit for this factor for normally consolidated clay can be written as

$$\beta_o = K_o \tan \phi'$$

As per Jaky (1944)

$$K_o = (1 - \sin \phi')$$

therefore

$$\beta = (1 - \sin \phi') \tan \phi'$$

where $\phi'$ = effective angle of internal friction.

Since the concept of this method is based on effective stresses, the cohesion intercept on a Mohr circle is equal to zero. For driven piles in stiff overconsolidated clay, $K_o$ is roughly 1.5 times greater than $K_o$. For overconsolidated clays $K_o$ may be found from the expression

$$K_o = (1 - \sin \phi') \sqrt{R_{oc}}$$

where $R_{oc}$ = overconsolidation ratio of clay.

For clays, $\phi'$ may be taken in the range of 20 to 30 degrees. In such a case the value of $\beta$ in Eq. (15.42d) varies between 0.24 and 0.29.
Meyerhof's Method (1976)

Meyerhof has suggested a semi-empirical relationship for estimating skin friction in clays.

For driven piles:

\[ f_s = 1.5c_u \tan \phi' \]  
(15.43)

For bored piles:

\[ f_s = c_u \tan \phi' \]  
(15.44)

By utilizing a value of 20° for \( \phi' \) for the stiff to very stiff clays, the expressions reduce to:

For driven piles:

\[ f_s = 0.55c_u \]  
(15.45)

For bored piles:

\[ f_s = 0.36c_u \]  
(15.46)

In practice the maximum value of unit friction for bored piles is restricted to 100 kPa.

15.20 BEARING CAPACITY OF PILES IN GRANULAR SOILS BASED ON SPT VALUE

Meyerhof (1956) suggests the following equations for single piles in granular soils based on SPT values.

For displacement piles:

\[ Q_u = Q_b + Q_f = 40N_{cor}(L/d)A_b + 2\overline{N}_{cor}A_s \]  
(15.47a)

for \( H \)-piles:

\[ Q_u = 40N_{cor}(L/d)A_b + \overline{N}_{cor}A_s \]  
(15.47b)

where \( q_b = 40N_{cor}(L/d) \leq 400N_{cor} \)

For bored piles:

\[ Q_u = 133N_{cor}A_b + 0.67\overline{N}_{cor}A_s \]  
(15.48)

where \( Q_u \) = ultimate total load in kN

\( N_{cor} \) = average corrected SPT value below pile tip

\( \overline{N}_{cor} \) = corrected average SPT value along the pile shaft

\( A_b \) = base area of pile in m² (for \( H \)-piles including the soil between the flanges)

\( A_s \) = shaft surface area in m²

In English units \( Q_u \) for a displacement pile is

\[ Q_u \text{(kip)} = Q_b + Q_f = 0.80N_{cor}(L/d)A_b + 0.04\overline{N}_{cor}A_s \]  
(15.49a)

where \( A_b \) = base area in ft² and \( A_s \) = surface area in ft²

and \( 0.80N_{cor}(\frac{L}{d})A_b \leq 8N_{cor}A_b \text{(kip)} \)  
(15.49b)
A minimum factor of safety of 4 is recommended. The allowable load \( Q_a \) is

\[
Q_a = \frac{Q_a}{4}
\]  
(15.50)

**Example 15.1**

A concrete pile of 45 cm diameter was driven into sand of loose to medium density to a depth of 15 m. The following properties are known:

(a) Average unit weight of soil along the length of the pile, \( \bar{\gamma} = 17.5 \text{ kN/m}^3 \), average \( \phi = 30^\circ \),

(b) average \( K_s = 1.0 \) and \( \delta = 0.75\phi \).

Calculate (a) the ultimate bearing capacity of the pile, and (b) the allowable load with \( F_s = 2.5 \). Assume the water table is at great depth. Use Berezantsev’s method.

**Solution**

From Eq. (15.9)

\[
Q_u = Q_b + Q_f = q'_0 A_b N_q + q'_0 A_s K_s \tan \delta
\]

where \( q'_0 = \bar{\gamma} L = 17.5 \times 15 = 262.5 \text{ kN/m}^2 \)

\[
q'_0 = \frac{1}{2} \bar{\gamma} L = \frac{262.5}{2} = 131.25 \text{ kN/m}^2
\]

\[
A_b = \frac{3.14}{4} \times 0.45^2 = 0.159 \text{ m}^2
\]
Deep Foundation I: Pile Foundation

$$A_s = 3.14 \times 0.45 \times 15 = 21.195 \ m^2$$

$$\delta = 0.75\phi = 0.75 \times 30 = 22.5^\circ$$

$$\tan \delta = 0.4142$$

From Fig. 15.9, $N_q$ for $\frac{L}{d} = \frac{15}{0.45} = 33.3$ and $\phi = 30^\circ$ is equal to 16.5.

Substituting the known values, we have

$$Q_u = Q_b + Q_f = 262.5 \times 0.159 \times 16.5 + 131.25 \times 21.195 \times 1.0 \times 0.4142$$

$$= 689 + 1152 = 1841 \ kN$$

$$Q_u = 1841 \ \frac{2.5}{2} = 736 \ kN$$

Example 15.2

Assume in Ex. 15.1 that the water table is at the ground surface and $\gamma_{sat} = 18.5 \ kN/m^3$. All the other data remain the same. Calculate $Q_u$ and $Q_a$.

Solution

Water table at the ground surface $\gamma_{sat} = 18.5 \ kN/m^3$

$$\gamma_b = \gamma_{sat} - \gamma_w = 18.5 - 9.81 = 8.69 \ kN/m^3$$

$$q'_b = 8.69 \times 15 = 130.35 \ kN/m^2$$

$$\overline{q'}_b = \frac{1}{2} \times 130.35 = 65.18 \ kN/m^2$$

Substituting the known values

$$Q_u = 130.35 \times 0.159 \times 16.5 + 65.18 \times 21.195 \times 1.0 \times 0.4142$$

$$= 342 + 572 = 914 \ kN$$

$$Q_u = \frac{914}{2.5} = 366 \ kN$$

Note: It may be noted here that the presence of a water table at the ground surface in cohesionless soil reduces the ultimate load capacity of pile by about 50 percent.

Example 15.3

A concrete pile of 45 cm diameter is driven to a depth of 16 m through a layered system of sandy soil ($c = 0$). The following data are available.

Top layer 1: Thickness = 8 m, $\gamma_d = 16.5 \ kN/m^3$, $e = 0.60$ and $\phi = 30^\circ$.

Layer 2: Thickness = 6 m, $\gamma_d = 15.5 \ kN/m^3$, $e = 0.65$ and $\phi = 35^\circ$.

Layer 3: Extends to a great depth, $\gamma_d = 16.00 \ kN/m^3$, $e = 0.65$ and $\phi = 38^\circ$. 
Assume that the value of $\delta$ in all the layers of sand is equal to 0.75$\phi$. The value of $K_s$ for each layer as equal to half of the passive earth pressure coefficient. The water table is at ground level.

Calculate the values of $Q_u$ and $Q_a$ with $F_s = 2.5$ by the conventional method for $Q_f$ and Berezantsev’s method for $Q_b$.

**Solution**

The soil is submerged throughout the soil profile. The specific gravity $G_s$ is required for calculating $y_{sat}$.

(a) Using the equation $y_d = \frac{y_w G_s}{1+e}$, calculate $G_s$ for each layer since $y_w$, $y_d$, and $e$ are known.

(b) Using the equation $y_{sat} = \frac{y_w (G_s + e)}{1+e}$, calculate $y_{sat}$ for each layer and then $y_o = y_{sat} - y_w$ for each layer.

(c) For a layered system of soil, the ultimate load can be determined by making use of Eq. (15.9). Now

$Q_o = Q_o + Q_f = q'_o N_q A_b + \sum_p \frac{L}{o} q'_o K_s \tan \delta \Delta L$

(d) $q'_o$ at the tip of the pile is

$q'_o = \gamma_{d_1} \Delta L_1 + \gamma_{d_2} \Delta L_2 + \gamma_{d_3} \Delta L_3$
(e) $q'_0$ at the middle of each layer is

$$q'_0 = \frac{1}{2} \Delta L \gamma_{b1}$$

$$q'_{02} = \Delta L \gamma_{b1} + \frac{1}{2} \Delta L \gamma_{b2}$$

$$q'_{03} = \Delta L \gamma_{b1} + \Delta L \gamma_{b2} + \frac{1}{2} \Delta L \gamma_{b3}$$

(f) $N_a = 95$ for $\phi = 38^\circ$ and $L = \frac{15}{0.45} = 33.33$ from Fig. 15.9.

(g) $A_k = 0.159 \, m^2$, $P = 1.413 \, m$.

(h) \( \bar{K}_s = \frac{1}{2} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = \frac{1}{2} K_p \cdot \bar{K}_s \) for each layer can be calculated.

(i) $\delta = 0.75 \phi$. The values of $\tan \delta$ can be calculated for each layer.

The computed values for all the layers are given below in a tabular form.

<table>
<thead>
<tr>
<th>Layer no.</th>
<th>$G_s$</th>
<th>$\gamma_0$</th>
<th>$q'_0$</th>
<th>$\bar{K}_s$</th>
<th>$\tan \delta$</th>
<th>$\Delta L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>2.69</td>
<td>10.36</td>
<td>41.44</td>
<td>1.5</td>
<td>0.414</td>
<td>8</td>
</tr>
<tr>
<td>2.</td>
<td>2.61</td>
<td>9.57</td>
<td>111.59</td>
<td>1.845</td>
<td>0.493</td>
<td>6</td>
</tr>
<tr>
<td>3.</td>
<td>2.69</td>
<td>10.05</td>
<td>150.35</td>
<td>2.10</td>
<td>0.543</td>
<td>2</td>
</tr>
</tbody>
</table>

From middle of layer 3 to tip of pile = 10.05

At the tip of pile $q'_0 = 160.40 \, kN/m^2$

$$Q_u = (160.4 \times 95 \times 0.159 + 1.413 (41.44 \times 1.5 \times 0.414 \times 8$$

$$+ 111.59 \times 1.845 \times 0.493 \times 6 + 150.35 \times 2.10 \times 0.543 \times 2)$$

$$= 2423 + 1636 \approx 4059 \, kN$$

$$Q_a = \frac{Q_u}{2.5} = \frac{4059}{2.5} = 1624 \, kN$$

**Example 15.4**

If the pile in Ex. 15.2 is a bored and cast-in-situ, compute $Q_u$ and $Q_a$. All the other data remain the same. Water table is close to the ground surface.

**Solution**

Per Tomlinson (1986), the ultimate bearing capacity of a bored and cast-in-situ-pile in cohesionless soil is reduced considerably due to disturbance of the soil. Per Section 15.12, calculate the base resistance for a driven pile and take one-third of this as the ultimate base resistance for a bored and cast-in-situ pile.

For computing $\delta$, take $\phi = 28^\circ$ and $\bar{K}_s = 1.0$ from Table 15.2 for a concrete pile.
Base resistance for driven pile
For $\phi = 30^\circ$, $N_q = 16.5$ from Fig. 15.9.

\[ A_b = 0.159 \text{ m}^2 \]
\[ q'_0 = 130.35 \text{ kN/m}^2 \text{ (From Ex. 15.2)} \]
\[ Q_b = 130.35 \times 0.159 \times 16.5 = 342 \text{ kN} \]

For bored pile
\[ Q_b = \frac{1}{3} \times 342 = 114 \text{ kN} \]

Skin load
\[ Q_f = A_s \bar{q}' K_s \tan \delta \]
For $\phi = 28^\circ$, $\delta = 0.75 \times 28 = 21^\circ$, $\tan \delta = 0.384$
\[ A_s = 21.195 \text{ m}^2 \text{ (from Ex. 15.2)} \]
\[ \bar{q}'_0 = 65.18 \text{ kN/m}^2 \text{ (Ex. 15.2)} \]
Substituting the known values,
\[ Q_f = 21.195 \times 65.18 \times 1.0 \times 0.384 = 530 \text{ kN} \]
Therefore, \[ Q_u = 114 + 530 = 644 \text{ kN} \]
\[ Q_u = \frac{644}{2.5} \approx 258 \text{ kN} \]

Example 15.5
Solve the problem given in Example 15.1 by Meyerhof’s method. All the other data remain the same.

Solution
Per Table 9.3, the sand in-situ may be considered in a loose state for $\phi = 30^\circ$. The corrected SPT value $N_{cor} = 10$.

Point bearing capacity
From Eq. (15.20)
\[ q_b = q'_0 N_q \leq q_{bl} \]
From Eq. (15.19 b)
\[ q_{bl} = 25 N_q \tan \phi \text{ kN/m}^2 \]
Now From Fig. 15.10 $N_q = 60$ for $\phi = 30^\circ$
\[ q'_0 = \gamma L = 17.5 \times 15 = 262.5 \text{ kN/m}^2 \]
\[ q_b = 262.5 \times 60 = 15,750 \text{ kN/m}^2 \]
\[ q_{bl} = 25 \times 60 \times \tan 30^\circ = 866 \text{ kN/m}^2 \]
Hence the limiting value for \( q_b = 866 \text{ kN/m}^2 \)

Now, \( Q_b = A_b q_b = A_b q_b = \frac{314}{4} \times (0.45)^2 \times 866 = 138 \text{ kN} \)

**Frictional resistance**

Per Section 15.17, the unit skin resistance \( f_s \) is assumed to increase from 0 at ground level to a limiting value of \( f_{sl} \) at \( L_c = 20d \) where \( L_c \) = critical depth and \( d \) = diameter. Therefore \( L_c = 20 \times 0.45 = 9 \text{ m} \)

Now \( f_{sl} = q' K_s \tan \delta = \gamma L_c K_s \tan \delta \)

Given: \( \gamma = 17.5 \text{ kN/m}^3 \), \( L_c = 9 \text{ m} \), \( K_s = 1.0 \) and \( \delta = 22.5^\circ \text{ m} \).

Substituting and simplifying we have

\[ f_{sl} = 17.5 \times 9 \times 1.0 \times \tan 22.5 = 65 \text{ kN/m}^2 \]

The skin load \( Q_f = Q_{fi} + Q_{f2} = \frac{1}{2} f_{sl} P L_c + P f_{sl} (L - L_c) \)

Substituting \( Q_f = \frac{1}{2} \times 65 \times 3.14 \times 0.45 \times 9 + 3.14 \times 0.45 \times 65 (15 - 9) \)

\[ = 413 + 551 = 964 \text{ kN} \]

The failure load \( Q_u \) is

\[ Q_u = Q_b + Q_f = 138 + 964 = 1,102 \text{ kN} \]

with \( F_s = 2.5 \),

\[ Q_u = \frac{1102}{2.5} = 440 \text{ kN} \]

---

**Example 15.6**

Determine the base load of the problem in Example 15.5 by Vesic’s method. Assume \( l_r = l_r = 50 \).

Determine \( Q_a \) for \( F_s = 2.5 \) using the value of \( Q_f \) in Ex 15.5.

**Solution**

From Eq. (15.25) for \( c = 0 \) we have

\[ q_b = \sigma_m N_c^r \]

From Eq. (15.24)

\[ \sigma_m = \frac{1 + 2 K_0}{3} q'_0 = \frac{1 + 2(1 - \sin \phi)}{3} q'_0 \]

\[ q'_0 = 15 \times 17.5 = 262.5 \text{ kN/m}^2 \]

\[ \sigma_m = \left[ \frac{1 + 2(1 - \sin 30^\circ)}{3} \right] \times 262.5 = 175 \text{ kN/m}^2 \]
From Fig. 15.11a, \( N_q^* = 36 \) for \( \phi = 30^\circ \) and \( L_r = 50 \)
Substituting
\[
q_b = 175 \times 36 = 6300 \text{ kN/m}^2
\]
\[
Q_b = A_b q_b = \frac{3.14}{4} \times (0.45)^2 \times 6300 = 1001 \text{ kN}
\]
\[
Q_u = Q_b + Q_f = 1001 + 964 = 1965 \text{ kN}
\]
\[
Q_u = \frac{1965}{2.5} = 786 \text{ kN}
\]

Example 15.7
Determine the base load of the problem in Example 15.1 by Janbu’s method. Use \( \psi = 90^\circ \).
Determine \( Q_u \) for \( F_s = 2.5 \) using the \( Q_f \) estimated in Example 15.5.

Solution
From Eq (15.31), for \( c = 0 \) we have
\[
q_b = q_0^* N_q^*
\]
For \( \phi = 30^\circ \) and \( \psi = 90^\circ \), we have \( N_q^* = 18.4 \) from Table 15.1. \( q_0^* = 262.5 \) kN/m² as in Ex. 15.5.
Therefore \( q_b = 262.5 \times 18.4 = 4830 \) kN/m²
\[
Q_b = A_b q_b = 0.159 \times 4830 = 768 \text{ kN}
\]
\[
Q_u = Q_b + Q_f = 768 + 964 = 1732 \text{ kN}
\]
\[
Q_u = \frac{1732}{2.5} = 693 \text{ kN}
\]

Example 15.8
Estimate \( Q_b, Q_f, Q_u \) and \( Q_a \) by the Coyle and Castello method using the data given in Example 15.1.

Solution
Base load \( Q_b \) from Eq. (15.33)
\[
q_b = q_0^* N_q^*
\]
From Fig. 15.12, \( N_q^* = 29 \) for \( \phi = 30^\circ \) and \( L/d = 33.3 \)
\[
q_0^* = 262.5 \) kN/m² as in Ex. 15.5.
Therefore \( q_b = 262.5 \times 29 = 7612 \) kN/m²
\[
Q_b = A_b q_b = 0.159 \times 7612 = 1210 \text{ kN}
\]
From Eq. (15.34a)
\[
Q_f = A_f \overline{q}_f K \tan \delta
\]
where \( A_s = 3.14 \times 0.45 \times 15 = 21.2 \text{ m}^2 \)

\[
\overline{q'} = \frac{1}{2} \times 262.5 = 131.25 \text{ kN/m}^2
\]

\( \phi = 0.8 \times 30^\circ = 24^\circ \)

From Fig. 15.14, \( K_s = 0.35 \) for \( \phi = 30^\circ \) and \( L/d = 33.3 \)

Therefore \( Q_f = 21.2 \times 131.25 \times 0.35 \tan 24^\circ = 434 \text{ kN} \)

\( Q_a = Q_b + Q_f = 1210 + 434 = 1644 \text{ kN} \)

\( Q_u = Q_a = \frac{1644}{2.5} = 658 \text{ kN} \)

**Example 15.9**

Determine \( Q_b, Q_f, Q_u \) and \( Q_a \) by using the SPT value for \( \phi = 30^\circ \) from Fig. 12.8.

**Solution**

From Fig. 12.8, \( N_{cor} = 10 \) for \( \phi = 30^\circ \). Use Eq. (15.47a) for \( Q_u \)

\[
Q_u = Q_b + Q_f = 40N_{cor} \frac{L}{d} A_b + 2N_{cor} A_s
\]

where \( Q_b \leq Q_{bl} = 400N_{cor} A_b \)

Given: \( L = 15 \text{ m}, d = 0.45 \text{ m}, A_b = 0.159 \text{ m}^2, A_s = 21.2 \text{ m}^2 \)

\[ Q_b = 40 \times 10 \times \frac{15}{0.45} \times 0.159 = 2120 \text{ kN} \]

\[ Q_{bl} = 400 \times 10 \times 0.159 = 636 \text{ kN} \]

Since \( Q_b > Q_{bl} \) use \( Q_{bl} \)

\[ Q_f = 2 \times 10 \times 21.2 = 424 \text{ kN} \]

Now \( Q_u = 636 + 424 = 1060 \text{ kN} \)

\[ Q_a = \frac{1060}{2.5} = 424 \text{ kN} \]

**Example 15.10**

Compare the values of \( Q_b, Q_f, Q_a \) obtained by the different methods in Examples 15.1, and 15.5 through 15.9 and make appropriate comments.
Comparison

The values obtained by different methods are tabulated below.

<table>
<thead>
<tr>
<th>Method No</th>
<th>Example No</th>
<th>Investigator</th>
<th>$Q_b$</th>
<th>$Q_f$</th>
<th>$Q_u$</th>
<th>$Q_u (F_s = 2.5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>15.1</td>
<td>Berezantsev</td>
<td>689</td>
<td>1152</td>
<td>1841</td>
<td>736</td>
</tr>
<tr>
<td>2.</td>
<td>15.5</td>
<td>Meyerhof</td>
<td>138</td>
<td>964</td>
<td>1102</td>
<td>440</td>
</tr>
<tr>
<td>3.</td>
<td>15.6</td>
<td>Vesic</td>
<td>1001</td>
<td>964</td>
<td>1965</td>
<td>786</td>
</tr>
<tr>
<td>4.</td>
<td>15.7</td>
<td>Janbu</td>
<td>768</td>
<td>964</td>
<td>1732</td>
<td>693</td>
</tr>
<tr>
<td>5.</td>
<td>15.8</td>
<td>Coyle &amp; Castello</td>
<td>1210</td>
<td>434</td>
<td>1644</td>
<td>658</td>
</tr>
<tr>
<td>6.</td>
<td>15.9</td>
<td>Meyerhof (Based on SPT)</td>
<td>636</td>
<td>424</td>
<td>1060</td>
<td>424</td>
</tr>
</tbody>
</table>

Comments

It may be seen from the table above that there are wide variations in the values of $Q_b$ and $Q_f$ between the different methods.

Method 1 Tomlinson (1986) recommends Berezantsev’s method for computing $Q_b$ as this method conforms to the practical criteria of pile failure. Tomlinson does not recommend the critical depth concept.

Method 2 Meyerhof’s method takes into account the critical depth concept. Eq. (15.19) is based on this concept. The equation [Eq. (15.20)] $q_b = q'_o N_q$ does not consider the critical depth concept where $q'_o =$ effective overburden pressure at the pile tip level of the pile. The value of $Q_b$ per this equation is

$$Q_b = q_b A_b = 15,750 \times 0.159 = 2504 \text{ kN}$$

which is very high and this is close to the value of $Q_b$ (= 2120 kN) by the SPT method. However Eq. (15.19 b) gives a limiting value for $Q_b = 138$ kN (here the sand is considered loose for $\phi = 30$).

$Q_f$ is computed by assuming $Q_f$ increases linearly with depth from 0 at $L = 0$ to $Q_{Jf}$ at depth $L_c = 20d$ and then remains constant to the end of the pile.

Though some investigators have accepted the critical depth concept for computing $Q_b$ and $Q_f$, it is difficult to generalize this concept as applicable to all types of conditions prevailing in the field.

Method 3 Vesic’s method is based on many assumptions for determining the values of $I_p$, $I_{pr}$, $\sigma_m$, $N^*$ etc. There are many assumptions in this method. Are these assumptions valid for field conditions? Designers have to answer this question.

Method 4 The uncertainties involved in Janbu’s method are given in Section 15.15 and as such difficult to assess the validity of this method.

Method 5 Coyle and Castello’s method is based on full scale field tests on a number of driven piles. Their bearing capacity factors vary with depth. Of the first five methods listed above, the value of $Q_a$ obtained by these is much higher than the other four methods whereas the value of $Q_f$ is very much lower. But on the whole the value of $Q_u$ is lower than the other methods.

Method 6 This method was developed by Meyerhof based on SPT values. The $Q_b$ value (= 2120 kN) by this method is very much higher than the preceding methods,
whereas the value of $Q_p$ is the lowest of all the methods. In the table given above the limiting value of $Q_{bl} = 636$ kN is considered. In all the six methods $F_s = 2.5$ has been taken to evaluate $Q_a$ whereas in method 6, $F_s = 4$ is recommended by Meyerhof.

**Which Method to Use**

There are wide variations in the values of $Q_b$, $Q_f$, $Q_u$, and $Q_a$ between the different methods. The relative proportions of loads carried by skin friction and base resistance also vary between the methods. The order of preference of the methods may be listed as follows:

<table>
<thead>
<tr>
<th>Preference No.</th>
<th>Method No.</th>
<th>Name of the investigator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Berezantsev for $Q_u$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>Meyerhof</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>Coyle and Castello</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>Meyerhof (SPT)</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>Vesic</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>Janbu</td>
</tr>
</tbody>
</table>

**Example 15.11**

A concrete pile 18 in. in diameter and 50 ft long is driven into a homogeneous mass of clay soil of medium consistency. The water table is at the ground surface. The unit cohesion of the soil under undrained condition is 1050 lb/ft$^2$ and the adhesion factor $\alpha = 0.75$. Compute $Q_u$ and $Q_a$ with $F_s = 2.5$.

**Solution**

Given: $L = 50$ ft, $d = 1.5$ ft, $c_u = 1050$ lb/ft$^2$, $\alpha = 0.75$.

From Eq. (15.37), we have

$$Q = Q_b + Q_f = c_b N_c A_b + A_s \alpha \bar{c}_u$$

where, $c_b = \bar{c}_u = 1050$ lb/ft$^2$; $N_c = 9$; $A_b = 1.766$ ft$^2$; $A_s = 235.5$ ft$^2$

Substituting the known values, we have

$$Q_u = \frac{1050 \times 9 \times 1.766}{1000} + \frac{235.5 \times 0.75 \times 1050}{1000}$$

$$= 16.69 + 185.46 = 202.15$$

$$Q_a = \frac{202.15}{2.5} = 81$$

**Example 15.12**

A concrete pile of 45 cm diameter is driven through a system of layered cohesive soils. The length of the pile is 16 m. The following data are available. The water table is close to the ground surface.

Top layer 1: Soft clay, thickness = 8 m, unit cohesion $\bar{c}_u = 30$ kN/m$^2$ and adhesion factor $\alpha = 0.90$.

Layer 2: Medium stiff, thickness = 6 m, unit cohesion $\bar{c}_u = 50$ kN/m$^2$ and $\alpha = 0.75$. 

---

*Image and content adherence to the text is maintained.*
Layer 3: Stiff stratum extends to a great depth, unit cohesion $\bar{c}_u = 105 \text{kN/m}^2$ and $\alpha = 0.50$.

Compute $Q_u$ and $Q_a$ with $F_s = 2.5$.

**Solution**

Here, the pile is driven through clay soils of different consistencies.

The equations for $Q_u$ expressed as (Eq. 15.38) yield

$$Q_u = 9c_bA_b + P \int_0^L \alpha \bar{c}_u \Delta L$$

Here, $c_b = \bar{c}_u$ of layer 3, $P = 1.413 \text{ m}$, $A_b = 0.159 \text{ m}^2$.

Substituting the known values, we have

$$Q_u = 9 \times 105 \times 0.159 + 1.413 (0.90 \times 30 \times 8$$

$$+ 0.75 \times 50 \times 6 + 0.50 \times 105 \times 2)$$

$$= 150.25 + 771.5 = 921.75 \text{ kN}$$

$$Q_u = \frac{921.75}{2.5} = 369 \text{ kN}$$

---

**Diagram:**

- Layer 1
  - Soft clay
  - $\bar{c}_u = 30 \text{kN/m}^2$
  - $\alpha = 0.90$
  - $\Delta L_1 = 8 \text{ m}$
  - 45 cm

- Layer 2
  - Medium stiff clay
  - $\bar{c}_u = 50 \text{kN/m}^2$
  - $\alpha = 0.75$
  - $\Delta L_2 = 6 \text{ m}$

- Layer 3
  - Stiff clay
  - $\bar{c}_u = 105 \text{kN/m}^2$
  - $\alpha = 0.50$
  - $\Delta L_3 = 2 \text{ m}$

**Figure Ex. 15.12**
Example 15.13

A precast concrete pile of size $18 \times 18$ in is driven into stiff clay. The unconfined compressive strength of the clay is $4.2 \text{kips/ft}^2$. Determine the length of pile required to carry a safe working load of $90 \text{kips}$ with $F_s = 2.5$.

Solution

The equation for $Q_u$ is

$$Q_u = N_c c_u A_b + \alpha \bar{c}_u A,$$

we have

$$Q_u = 2.5 \times 90 = 225 \text{kips},$$

$$N_c = 9, \ c_u = 2.1 \text{kips/ft}^2$$

$$\alpha = 0.48 \text{ from Fig. 15.15}, \ \bar{c}_u = c_u = 2.1 \text{kips/ft}^2, \ A_b = 2.25 \text{ ft}^2$$

Assume the length of pile $= L \text{ ft}$

Now, $A_s = 4 \times 1.5L = 6L$

Substituting the known values, we have

$$225 = 9 \times 2.1 \times 2.25 + 0.48 \times 2.1 \times 6L$$

or

$$225 = 42.525 + 6.05L$$

Simplifying, we have

$$L = \frac{225 - 42.525}{6.05} = 30.2 \text{ ft}$$

Figure Ex. 15.13
Example 15.14
For the problem given in Example 15.11 determine the skin friction load by the \( \lambda \)-method. All the other data remain the same. Assume the average unit weight of the soil is 110 lb/ft\(^3\). Use \( Q_b \) given in Ex. 15.11 and determine \( Q_a \) for \( F_s = 2.5 \).

Solution
Per Eq. (15.40)
\[
Q_f = \lambda (\bar{q}_n + 2c_u)A_x
\]
\[
\bar{q}_n = \frac{1}{2} \times 50 \times 110 = 2,750 \text{ lb/ft}^2
\]
Depth = 50 ft = 15.24 m
From Fig. 15.16, \( \lambda = 0.2 \) for depth \( L = 15.24 \) m
Now \( Q_f = \frac{0.2(2750 + 2 \times 1050) \times 235.5}{1000} = 228.44 \) kips
Now \( Q_a = Q_b + Q_f = 16.69 + 228.44 = 245 \) kips
\[
Q_a = \frac{245}{2.5} = 98 \text{ kips}
\]

Example 15.15
A reinforced concrete pile of size 30 \( \times \) 30 cm and 10 m long is driven into coarse sand extending to a great depth. The average total unit weight of the soil is 18 kN/m\(^3\) and the average \( N_{cor} \) value is 15. Determine the allowable load on the pile by the static formula. Use \( F_s = 2.5 \). The water table is close to the ground surface.

Solution
In this example only the \( N \)-value is given. The corresponding \( \phi \) value can be found from Fig. 12.8 which is equal to 32\(^\circ\).

Now from Fig. 15.9, for \( \phi = 32^\circ \), and \( \frac{L}{d} = \frac{10}{0.3} = 33.33 \), the value of \( N_q = 25 \).
\[
A_b = 0.3 \times 0.3 = 0.09 \text{ m}^2
\]
\[
A_s = 10 \times 4 \times 0.3 = 12 \text{ m}^2
\]
\[
\delta = 0.75 \times 32 = 24^\circ \text{, tan} \delta = 0.445
\]
The relative density is loose to medium dense. From Table 15.2, we may take
\[
\bar{K}_s = 1 + \frac{1}{3}(2 - 1) = 1.33
\]
Now, \( Q_a = q_b^\prime N_q A_b + \bar{q}_n^\prime \bar{K}_s \tan \delta A_s \)
\[
\gamma_b = \gamma_{sat} - \gamma_w = 18.0 - 9.81 = 8.19 \text{ kN/m}^3
\]
Medium dense sand
\( \gamma_{sat} = 18 \text{ kN/m}^3 \)
\( \phi = 32^\circ \)

**Figure Ex. 15.15**

\[
q_0' = \gamma_b L = 8.19 \times 10 = 81.9 \text{ kN/m}^2
\]

\[
\bar{q}_0' = \frac{q_0'}{2} = \frac{81.9}{2} = 40.95 \text{ kN/m}^2
\]

Substituting the known values, we have

\[
Q_u = 81.9 \times 25 \times 0.09 + 40.95 \times 1.33 \times 0.445 \times 12
\]

\[
= 184 + 291 = 475 \text{ kN}
\]

\[
Q_a = \frac{475}{25} = 190 \text{ kN}
\]

**Example 15.16**

Determine the allowable load on the pile given in Ex. 15.15 by making use of the SPT approach by Meyerhof.

**Solution**

Per Ex. 15.15, \( N_{cor} = 15 \)

Expression for \( Q_u \) is (Eq. 15.47a)

\[
Q_u = Q_b + Q_f = 40N_{cor}(L/d)A_b + 2\bar{N}_{cor}A_s
\]

Here, we have to assume \( N_{cor} = \bar{N}_{cor} = 15 \)
\[ A_b = 0.3 \times 0.3 = 0.09 \, \text{m}^2, \quad A_s = 4 \times 0.3 \times 10 = 12 \, \text{m}^2 \]

Substituting, we have

\[ Q_b = 40 \times 15 \times \frac{10}{0.3} \times 0.09 = 1800 \, \text{kN} \]

\[ Q_{bl} = 400N_{cor} \times A_b = 400 \times 15 \times 0.09 = 540 \, \text{kN} < Q_b \]

Hence, \( Q_{bl} \) governs.

\[ Q_f = 2N_{cor}A_s = 2 \times 15 \times 12 = 360 \, \text{kN} \]

A minimum \( F_y = 4 \) is recommended, thus,

\[ Q_u = \frac{Q_b + Q_f}{4} = \frac{540 + 360}{4} = 225 \, \text{kN} \]

**Example 15.17**

Precast concrete piles 16 in. in diameter are required to be driven for a building foundation. The design load on a single pile is 100 kips. Determine the length of the pile if the soil is loose to medium dense sand with an average \( N_{cor} \) value of 15 along the pile and 21 at the tip of the pile. The water table may be taken at the ground level. The average saturated unit weight of soil is equal to 120 lb/ft\(^3\). Use the static formula and \( F_y = 2.5 \).

**Solution**

It is required to determine the length of a pile to carry an ultimate load of \( Q_u = 2.5 \times 100 = 250 \, \text{kips} \).

The equation for \( Q_u \) is

\[ Q_u = q_0'N_{cor}A_b + \bar{q}_0'K \tan \delta A_s \]

The average value of \( \phi \) along the pile and the value at the tip may be determined from Fig. 12.8.

For \( N_{cor} = 15, \phi = 32^\circ; \) for \( N_{cor} = 21, \phi = 33.5^\circ. \)

Since the soil is submerged

\[ \gamma_b = 120 - 62.4 = 57.6 \, \text{lb/ft}^3 \]

Now

\[ q_0' = \gamma_bL = 57.6L \, \text{lb/ft}^2 \]

\[ \bar{q}_0' = \frac{57.6L}{2} = 28.8L \, \text{lb/ft}^2 \]

\[ A_b = \frac{3.14}{4} \times (1.33)^2 = 1.39 \, \text{ft}^2 \]

\[ A_t = 3.14 \times 1.33 \times L = 4.176L \, \text{ft}^2 \]
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Figure Ex. 15.17

From Fig. 15.9, $N_q = 40$ for $L/d = 20$ (assumed)

From Table 15.2, $K_f = 1.33$ for the lower side of medium dense sand

$\delta = \frac{3}{4} \times 33.5 = 25.1^\circ$, $\tan \delta = 0.469$

Now by substituting the known values, we have

$$Q_u = \frac{(57.6L) \times 40 \times 1.39}{1000} + \frac{(28.8L) \times 0.469 \times 1.33 \times (4.176L)}{1000}$$

$$= 3.203L + 0.075L^2$$

or $L^2 + 42.71L - 3333 = 0$

Solving this equation gives a value of $L = 40.2$ ft or say 41 ft.

Example 15.18
Refer to the problem in Example 15.17. Determine directly the ultimate and the allowable loads using $N_{cor}$. All the other data remain the same.

Solution
Use Eq. (15.49a)

$$Q_u = Q_b + Q_f = 0.8N_{cor}(L/d)A_b + 0.04N_{cor}A_f \text{ (kips)}$$

Given: $N_{cor} = 21$, $N_{cor} = 15$, $d = 16$ in, $L = 41$ ft
Substituting

\[ Q_b = 0.80 \times 21 \frac{41}{1.33} \times 1.39 = 720 \leq 8N_{cw}A_p \text{ kips} \]

\[ Q_{bl} = 8 \times 21 \times 1.39 = 234 \text{ kips} \]

The limiting value of \( Q_b = 234 \text{ kips} \)

\[ Q_f = 0.04N_{cw}A_s = 0.04 \times 15 \times 171.65 = 103 \text{ kips} \]

\[ Q_a = Q_b + Q_f = 234 + 103 = 337 \text{ kips} \]

\[ Q_a = \frac{337}{4} = 84 \text{ kips} \]

## 15.21 BEARING CAPACITY OF PILES BASED ON STATIC CONE PENETRATION TESTS (CPT)

### Methods of Determining Pile Capacity

The cone penetration test may be considered as a small scale pile load test. As such the results of this test yield the necessary parameters for the design of piles subjected to vertical load. The types of static cone penetrometers and the methods of conducting the tests have been discussed in detail in Chapter 9. Various methods for using CPT results to predict vertical pile capacity have been proposed. The following methods will be discussed:

1. Vander Veen’s method.
2. Schmertmann’s method.

### Vander Veen’s Method for Piles in Cohesionless Soils

In the Vander Veen et al., (1957) method, the ultimate end-bearing resistance of a pile is taken, equal to the point resistance of the cone. To allow for the variation of cone resistance which normally occurs, the method considers average cone resistance over a depth equal to three times the diameter of the pile above the pile point level and one pile diameter below point level as shown in Fig. 15.17(a). Experience has shown that if a safety factor of 2.5 is applied to the ultimate end resistance as determined from cone resistance, the pile is unlikely to settle more than 15 mm under the working load (Tomlinson, 1986). The equations for ultimate bearing capacity and allowable load may be written as,

\[ q_b = q_p \text{ (cone)} \]  \hspace{1cm} (15.51a)

\[ Q_b = A_bq_p \]  \hspace{1cm} (15.51b)

\[ Q_a = \frac{A_bq_p}{F_s} \]  \hspace{1cm} (15.51c)

where, \( q_p \) = average cone resistance over a depth \( 4d \) as shown in Fig. 15.17(a) and \( F_s \) = factor of safety.
The skin friction on the pile shaft in cohesionless soils is obtained from the relationships established by Meyerhof (1956) as follows.

For displacement piles, the ultimate skin friction, $f_s$, is given by

$$f_s = \frac{q_c}{2} \text{ (kPa)} \quad (15.52a)$$

and for H-section piles, the ultimate limiting skin friction is given by

$$f_s = \frac{q_c}{4} \text{ (kPa)} \quad (15.52b)$$

where $q_c = \text{average cone resistance in kg/cm}^2 \text{ over the length of the pile shaft under consideration.}$

Meyerhof states that for straight sided displacement piles, the ultimate unit skin friction, $f_s$, has a maximum value of 107 kPa and for H-sections, a maximum of 54 kPa (calculated on all faces of flanges and web). The ultimate skin load is

$$Q_f = A_s f_s \quad (15.53a)$$

The ultimate load capacity of a pile is

$$Q_u = Q_b + Q_f \quad (15.53b)$$

The allowable load is

$$Q_a = \frac{Q_b + Q_f}{2.5} \quad (15.53c)$$

If the working load, $Q_a$, obtained for a particular position of pile in Fig. 15.17(a), is less than that required for the structural designer’s loading conditions, then the pile must be taken to a greater depth to increase the skin friction $f_s$ or the base resistance $Q_b$.

**Schmertmann’s Method for Cohesionless and Cohesive Soils**

Schmertmann (1978) recommends one procedure for all types of soil for computing the point bearing capacity of piles. However, for computing side friction, Schmertmann gives two different approaches, one for sand and one for clay soils.

**Point Bearing Capacity $Q_b$ in All Types of Soil**

The method suggested by Schmertmann (1978) is similar to the procedures developed by De Ruiter and Beringen (1979) for sand. The principle of this method is based on the one suggested by Vander Veen (1957) and explained earlier. The procedure used in this case involves determining a representative cone point penetration value, $q_p$, within a depth between 0.7 to 4d below the tip level of the pile and 8d above the tip level as shown in Fig. 15.17(b) and (c). The value of $q_p$ may be expressed as

$$q_p = \frac{(q_{c1} + q_{c2})/2 + q_{c3}}{2} \quad (15.54)$$

where $q_{c1} = \text{average cone resistance below the tip of the pile over a depth which may vary between 0.7d and 4d, where d = diameter of pile,}$

$q_{c2} = \text{minimum cone resistance recorded below the pile tip over the same depth 0.7d to 4d,}$
\[ q_{c3} = \text{average of the envelope of minimum cone resistance recorded above the pile tip to a height of } 8d. \]

Now, the unit point resistance of the pile, \( q_b \), is

\[ q_b (\text{pile}) = q_p (\text{cone}) \quad (15.55a) \]

The ultimate base resistance, \( Q_b \), of a pile is

\[ Q_b = A_g q_p \quad (15.55b) \]

The allowable base load, \( Q_a \) is

\[ Q_a = \frac{A_g q_p}{F_s} \quad (15.55c) \]

**Method of Computing the Average Cone Point Resistance \( q_p \)**

The method of computing \( q_{c1}, q_{c2} \) and \( q_{c3} \) with respect to a typical \( q_c \)-plot shown in Fig. 15.17(b) and (c) is explained below.

**Case 1:** When the cone point resistance \( q_c \) below the tip of a pile is lower than that at the tip (Fig. 15.17(b)) within depth \( 4d \).

\[ q_{c1} = \frac{d_2(q_o + q_b) + d_3(q_o + q_c) / 2 + d_4(q_d + q_c) / 2}{4d} \quad (15.56a) \]

where \( q_o, q_b \) etc., refer to the points \( o, b \) etc, on the \( q_c \)-profile, \( q_{c2} = q_c = \text{minimum value below tip within a depth of } 4d \) at point \( c \) on the \( q_c \)-profile.

The envelope of minimum cone resistance above the pile tip is as shown by the arrow mark along (15.17b) aefghk.

\[ q_{c3} = \frac{d_2(q_o + q_b) + d_3(q_o + q_f) / 2 + d_4(q_f + q_b) / 2}{8d} \quad (15.56b) \]

where \( q_o = q_e, \quad q_f = q_g, \quad q_b = q_k \).

**Case 2:** When the cone resistance \( q_c \) below the pile tip is greater than that at the tip within a depth \( 4d \). (Fig. 15.17(c)).

In this case \( q_p \) is found within a total depth of \( 0.7d \) as shown in Fig. 15.17(c).

\[ q_{c1} = \frac{q_o + q_b}{2} \]

\( q_{c2} = q_o = \text{minimum value at the pile tip itself, } q_{c3} = \text{average of the minimum values along the envelope } ocde \) as before.

In determining the average \( q_c \) above, the minimum values \( q_{c2} \) selected under Case 1 or 2 are to be disregarded.

**Effect of Overconsolidation Ratio in Sand**

Reduction factors have been developed that should be applied to the theoretical end bearing of a pile as determined from the CPT if the bearing layer consists of overconsolidated sand. The problem in many cases will be to make a reasonable estimate of the overconsolidation ratio in sand. In sands with a high \( q_c \), some conservation in this respect is desirable, in particular for shallow foundations. The influence of overconsolidation on pile end bearing is one of the reasons for applying a limiting value to pile end bearing, irrespective of the cone resistances recorded in the
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Figure 15.17  Pile capacity by use of CPT values (a) Vander Veen’s method, and (b,c) Schmertmann’s method

bearing layer. A limit pile end bearing of 15 MN/m² is generally accepted (De Ruiter and Beringen, 1979), although in dense sands cone resistance may be greater than 50 MN/m². It is unlikely that in dense normally consolidated sand ultimate end bearing values higher than 15 MN/m² can occur but this has not been adequately confirmed by load tests.

Design CPT Values for Sand and Clay

The application of CPT in evaluating the design values for skin friction and bearing as recommended by De Ruiter and Beringen (1979) is summarized in Table. 15.3.
Table 15.3 Application of CPT in Pile Design (After De Ruiter and Beringen, 1979)

<table>
<thead>
<tr>
<th>Item</th>
<th>Sand</th>
<th>Clay</th>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Friction</td>
<td>Minimum of $f_s$</td>
<td>$f_s = \alpha' c_u$, where $q_c = \text{cone resistance}$</td>
<td></td>
</tr>
<tr>
<td>$f_i$</td>
<td>$f_i = 0.12 \text{ MPa}$</td>
<td>$q = \text{cone resistance below pile tip}$</td>
<td></td>
</tr>
<tr>
<td>$f_s$</td>
<td>$f_s = \text{CPT sleeve friction}$</td>
<td>$\alpha' = 1 \text{ in N.C. clay}$</td>
<td>$c_u = q_f/N_c$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>$f_s = q_s/300 \text{ (compression)}$</td>
<td>$f_s = q_s/400 \text{ (tension)}$</td>
<td></td>
</tr>
<tr>
<td>Unit end bearing $q_p$</td>
<td>Minimum of $q_p$ from Fig. 15.17(b) and c</td>
<td>$q_p = N_c u$</td>
<td>$q_p = \text{ultimate resistance of pile}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Ultimate Skin Load $Q_f$ in Cohesionless Soils**

For the computation of skin load, $Q_p$, Schmertmann (1978) presents the following equation

$$Q_f = \frac{K}{z=8d} \int_{z=0}^{z=L} f_c A_z + \int_{z=8d}^{z=L} f_c A_z$$ (15.56c)

where $K = f_s/f_c = \text{correction factor for } f_c$

- $f_c = \text{unit pile friction}$
- $f_c = \text{unit sleeve friction measured by the friction jacket}$
- $z = \text{depth to } f_c \text{ value considered from ground surface}$
- $d = \text{pile diameter or width}$
- $A_z = \text{pile-soil contact area per } f_c \text{ depth interval}$
- $L = \text{embedded depth of pile}$

When $f_c$ does not vary significantly with depth, Eq. (15.56c) can be written in a simplified form as

$$Q_f = K \frac{1}{2} \left[ f_c A_z \right]_{z=8d}^{z=L} + \left[ f_c A_z \right]_{8d-L}$$ (15.56d)

where $f_c$ is the average value within the depths specified. The correction factor $K$ is given in Fig. 15.18(a).

**Ultimate Skin Load $Q_f$ for Piles in Clay Soil**

For piles in clay Schmertmann gives the expression

$$Q_f = \alpha' f_c A_z$$ (15.57)

where $\alpha' = \text{ratio of pile to penetrometer sleeve friction}$,

- $f_c = \text{average sleeve friction}$,
- $A_z = \text{pile to soil contact area}$.

Fig. 15.18(b) gives values of $\alpha'$. 
Example 15.19

A concrete pile of 40 cm diameter is driven into a homogeneous mass of cohesionless soil. The pile carries a safe load of 650 kN. A static cone penetration test conducted at the site indicates an average value of $q_c = 40$ kg/cm$^2$ along the pile and 120 kg/cm$^2$ below the pile tip. Compute the length of the pile with $F_s = 2.5$. (Fig. Ex. 15.19)
Solution

From Eq. (15.51a)

\[ q_b \text{ (pile)} = q_p \text{ (cone)} \]

Given, \( q_p = 120 \text{ kg/cm}^2 \), therefore,

\[ q_b = 120 \text{ kg/cm}^2 = 120 \times 100 = 12000 \text{ kN/m}^2 \]

Per Section 15.12, \( q_b \) is restricted to 11,000 kN/m\(^2\).

Therefore,

\[ Q_b = A_b q_b = \frac{3.14}{4} \times 0.4^2 \times 11000 = 1382 \text{ kN} \]

Assume the length of the pile = \( L \) m

The average, \( q_c = 40 \text{ kg/cm}^2 \)

Per Eq. (15.52a),

\[ f_s = \frac{q_c}{2} \text{ kN/m}^2 = \frac{40}{2} = 20 \text{ kN/m}^2 \]

Now,

\[ Q_f = f_s A_s = 20 \times 3.14 \times 0.4 \times L = 25.12L \text{ kN} \]

Given \( Q_a = 650 \text{ kN} \). With \( F_s = 2.5 \), \( Q_u = 650 \times 2.5 = 1625 \text{ kN} \)

Now,

\[ 1625 = Q_b + Q_f = (1382 + 25.12L) \text{ kN} \]

or

\[ L = \frac{1625 - 1382}{25.12} = 9.67 \text{ m} \text{ or say } 10 \text{ m} \]

The pile has to be driven to a depth of 10 m to carry a safe load of 650 kN with \( F_s = 2.5 \).

---

\( Q_u \)

\( L \)

\( \bar{q}_c = 40 \text{ kg/cm}^2 \)

\( q_b = 120 \text{ kg/cm}^2 \)

\( \bar{q}_c \)

Figure Ex. 15.19
Example 15.20

A concrete pile of size $0.4 \times 0.4$ m is driven to a depth of 12 m into medium dense sand. The water table is close to the ground surface. Static cone penetration tests were carried out at this site by using an electric cone penetrometer. The values of $q_c$ and $f_c$ as obtained from the test have been plotted against depth and shown in Fig. Ex. 15.20. Determine the safe load on this pile with $F_s = 2.5$ by Schmertmann’s method (Section 15.21).

Solution

First determine the representative cone penetration value $q_p$ by using Eq. (15.54)

$$q_p = \frac{(q_{c1} + q_{c2}) + q_{c3}}{2}$$

Now from Fig. Ex. 15.20 and Eq (15.56a)

$$q_{c1} = \frac{d_1(q_a + q_b) + d_2(q_b + q_d) + d_3(q_d + q_c)}{4d}$$

$$= \frac{0.7(76 + 85) / 2 + 0.3(85 + 71) / 2 + 0.6(71 + 80) / 2}{4 \times 0.4}$$

$$= 78 \text{ kg/cm}^2$$

$q_{c2} = q_d$ = the minimum value below the tip of pile within $4d$ depth = 71 kg/cm$^2$.
From Eq. (15.56b)

\[ q_c = \frac{d_a q_m + d_z (q_m + q_n) / 2 + d_y q_n + d_z (q_y + q_n) / 2}{8d} \]

\[ = \frac{0.4 \times 71 + 0.3(71+65) / 2 + 2.1 \times 65 + 0.4(65+60) / 2}{8 \times 0.4} \]

\[ = 66 \text{ kg/cm}^2 = 660 \text{ t/m}^2 \text{ (metric)} \]

From Eq. (15.54)

\[ q_p = \frac{(78+71)/2+66}{2} = 70 \text{ kg/cm}^2 = 700 \text{ t/m}^2 \text{ (metric)} \]

Ultimate Base Load

\[ Q_b = q_b A_b = q_p A_b = 700 \times 0.4^2 = 112 \text{ t (metric)} \]

Frictional Load \( Q_f \)

From Eq. (15.56d)

\[ Q_f = K \left( \frac{1}{2} (\tilde{f}_c A_s)_{0-8d} + (\tilde{f}_c A_s)_{8d-L} \right) \]

where \( K = \) correction factor from Fig. (15.18a) for electrical penetrometer.

For \( \frac{L}{d} = \frac{12}{0.4} = 30 \), \( K = 0.75 \) for concrete pile. It is now necessary to determine the average sleeve friction \( \tilde{f}_c \) between depths \( z = 0 \) and \( z = 8d \), and \( z = 8d \) and \( z = L \) from the top of pile from \( f_c \) profile given in Fig. Ex. 15.20.

\[ Q_f = 0.75 \left[ \frac{1}{2} \times 0.34 \times 10 \times 4 \times 0.4 \times 3.2 + 0.71 \times 10 \times 4 \times 0.4 \times 8.8 \right] \]

\[ = 0.75 \times [8.7 + 99.97] = 81.5 \text{ t (metric)} \]

\[ Q_u = Q_b + Q_f = 112 + 81.5 = 193.5 \text{ t} \]

\[ Q_u = \frac{Q_b + Q_f}{2.5} = \frac{193.5}{2.5} = 77.4 \text{ t (metric)} = 759 \text{ kN} \]

Example 15.21

A concrete pile of section 0.4 x 0.4 m is driven into normally consolidated clay to a depth of 10 m. The water table is at ground level. A static cone penetration test (CPT) was conducted at the site with an electric cone penetrometer. Fig. Ex. 15.21 gives a profile of \( q_c \) and \( f_c \) values with respect to depth. Determine safe loads on the pile by the following methods:

(a) \( \alpha \)-method (b) \( \lambda \)-method, given: \( \gamma_b = 8.5 \text{ kN/m}^3 \) and (c) Schmertmann’s method. Use a factor of safety of 2.5.

Solution

(a) \( \alpha \)-method

The \( \alpha \)-method requires the undrained shear strength of the soil. Since this is not given, it has to be determined by using the relation between \( q_c \) and \( c_n \) given in Eq. (9.14).
$c_u = \frac{q_c}{N_k}$ by neglecting the overburden effect,

where $N_k = \text{cone factor} = 20$.

It is necessary to determine the average $c_u$ along the pile shaft and $c_b$ at the base level of the pile. For this purpose find the corresponding $f_c$ (sleeve friction) values from Fig. Ex. 15.21.

Average $\overline{q_c}$ along the shaft, $\overline{q_c} = \frac{1+16}{2} = 8.5 \text{ kg/cm}^2$.

Average of $q_c$ within a depth $3d$ above the base and $d$ below the base of the pile (Refer to Fig. 15.17a)

$q_p = \frac{15+18.5}{2} = 17 \text{ kg/cm}^2$

$c_u = \frac{8.5}{20} = 0.43 \text{ kg/cm}^2 = 43 \text{ kN/m}^2$
$c_b = \frac{17}{20} = 0.85 \text{ kg/cm}^2 \approx 85 \text{ kN/m}^2$.

**Ultimate Base Load, $Q_b$**

From Eq. (15.39)

$Q_b = 9c_bA_b = 9 \times 85 \times 0.40^2 = 122 \text{ kN}.$

**Ultimate Friction Load, $Q_f$**

From Eq. (15.37)

$Q_f = \alpha \bar{c}_u A_s$

From Fig. 15.15 for $\bar{c}_u = 43 \text{ kN/m}^2$, $\alpha = 70$

$Q_f = 0.70 \times 43 \times 10 \times 4 \times 0.4 = 481.6 \text{ kN}$ or say 482 kN

$Q_u = 122 + 482 = 604 \text{ kN}$

$Q_a = \frac{604}{2.5} = 241.6 \text{ kN}$ or say 242 kN

**b) $\lambda$-method**

**Base Load $Q_b$**

In this method the base load is the same as in (a) above. That is

$Q_b = 122 \text{ kN}$

**Friction Load**

From Fig. 15.17 $\lambda = 0.25$ for $L = 10 \text{ m}$ (= 32.4 ft). From Eq. 15.40

$f_s = \lambda (\bar{q}_o + 2\bar{c}_u)$

$\bar{q}_o = \frac{1}{2} \times 10 \times 8.5 = 42.5 \text{ kN/m}^2$

$f_s = 0.25(42.5 + 2 \times 43) = 32 \text{ kN/m}^2$

$Q_f = f_s A_s = 32 \times 10 \times 4 \times 0.4 = 512 \text{ kN}$

$Q_u = 122 + 512 = 634 \text{ kN}$

$Q_a = \frac{634}{2.5} = 254 \text{ kN}.$

**c) Schmertmann's Method**

**Base load $Q_b$**

Use Eq. (15.54) for determining the representative value for $q_p$. Here, the minimum value for $q_e$ is at point $O$ on the $q_e$-profile in Fig. Ex. 15.21 which is the base level of the pile. Now $q_{c_1}$ is the average $q_c$ at the base and 0.7$d$ below the base of the pile, that is,

$q_{c_1} = \frac{q_o + q_e}{2} = \frac{16 + 18.5}{2} = 17.25 \text{ kg/cm}^2$

$q_{c_2} = q_o = 16 \text{ kg/cm}^2$

$q_{c_3} = \text{The average of } q_e \text{ within a depth } 8d \text{ above the base level}$
\[ q_p = \frac{q_o + q_k}{2} = \frac{16 + 11}{2} = 13.5 \text{ kg/cm}^2 \]

From Eq. (15.45), \[ q_p = \frac{(17.25+16) / 2 + 13.5}{2} = 15 \text{ kg/cm}^2. \]

From Eq. (15.55a)
\[ q_s(pile) = q_p (cone) = 15 \text{ kg/cm}^2 = 1500 \text{ kN/m}^2 \]
\[ Q_b = q_s A_b = 1500 \times (0.4)^2 = 240 \text{ kN} \]

**Friction Load \( Q_f \)**

Use Eq. (15.57)
\[ Q_f = \alpha' f_c A_s \]

where \( \alpha' \) = ratio of pile to penetrometer sleeve friction.

From Fig. Ex. 15.21 \( f_c = \frac{0+115}{2} = 0.58 \text{ kg/cm}^2 = 58 \text{ kN/m}^2. \)

From Fig. 16.18b for \( f_c = 58 \text{ kN/m}^2, \alpha' = 0.70 \)
\[ Q_f = 0.70 \times 58 \times 10 \times 4 \times 0.4 = 650 \text{ kN} \]
\[ Q_u = 240 + 650 = 890 \text{ kN} \]
\[ Q_a = \frac{890}{2.5} = 356 \text{ kN} \]

Note: The values given in the examples are only illustrative and not factual.

### 15.22 BEARING CAPACITY OF A SINGLE PILE BY LOAD TEST

A pile load test is the most acceptable method to determine the load carrying capacity of a pile. The load test may be carried out either on a driven pile or a *cast-in-situ* pile. Load tests may be made either on a single pile or a group of piles. Load tests on a pile group are very costly and may be undertaken only in very important projects.

Pile load tests on a single pile or a group of piles are conducted for the determination of

1. Vertical load bearing capacity,
2. Uplift load capacity,
3. Lateral load capacity.

Generally load tests are made to determine the bearing capacity and to establish the load-settlement relationship under a compressive load. The other two types of tests may be carried out only when piles are required to resist large uplift or lateral forces.

Usually pile foundations are designed with an estimated capacity which is determined from a thorough study of the site conditions. At the beginning of construction, load tests are made for the purpose of verifying the adequacy of the design capacity. If the test results show an inadequate factor of safety or excessive settlement, the design must be revised before construction is under way.

Load tests may be carried out either on

1. A working pile or
2. A test pile.
A working pile is a pile driven or cast-in-situ along with the other piles to carry the loads from the superstructure. The maximum test load on such piles should not exceed one and a half times the design load.

A test pile is a pile which does not carry the loads coming from the structure. The maximum load that can be put on such piles may be about 2\(\frac{1}{2}\) times the design load or the load imposed must be such as to give a total settlement not less than one-tenth the pile diameter.

**Method of Carrying Out Vertical Pile Load Test**

A vertical pile load test assembly is shown in Fig. 15.19(a). It consists of

1. An arrangement to take the reaction of the load applied on the pile head,
2. A hydraulic jack of sufficient capacity to apply load on the pile head, and
3. A set of three dial gauges to measure settlement of the pile head.

**Load Application**

A load test may be of two types:

1. Continuous load test.
2. Cyclic load test.

In the case of a continuous load test, continuous increments of load are applied to the pile head. Settlement of the pile head is recorded at each load level.

In the case of the cyclic load test, the load is raised to a particular level, then reduced to zero, again raised to a higher level and reduced to zero. Settlements are recorded at each increment or decrement of load. Cyclic load tests help to separate frictional load from point load.

The total elastic recovery or settlement \(S_e\) is due to

1. The total plastic recovery of the pile material,
2. Elastic recovery of the soil at the tip of the pile, \(S_{el}\)

The total settlement \(S\) due to any load can be separated into elastic and plastic settlements by carrying out cyclic load tests as shown in Fig. 15.19(b).

A pile loaded to \(Q_1\) gives a total settlement \(S_1\). When this load is reduced to zero, there is an elastic recovery which is equal to \(S_{el}\). This elastic recovery is due to the elastic compression of the pile material and the soil. The net settlement or plastic compression is \(S_p\). The pile is loaded again from zero to the next higher load \(Q_2\) and reduced to zero thereafter. The corresponding settlements may be found as before. The method of loading and unloading may be repeated as before.

**Allowable Load from Single Pile Load Test Data**

There are many methods by which allowable loads on a single pile may be determined by making use of load test data. If the ultimate load can be determined from load-settlement curves, allowable loads are found by dividing the ultimate load by a suitable factor of safety which varies from 2 to 3. A factor of safety of 2.5 is normally recommended. A few of the methods that are useful for the determination of ultimate or allowable loads on a single pile are given below:

1. The ultimate load, \(Q_u\) can be determined as the abscissa of the point where the curved part of the load-settlement curve changes to a falling straight line, Fig. 15.20(a).
2. \(Q_u\) is the abscissa of the point of intersection of the initial and final tangents of the load-settlement curve, Fig. 15.20(b).
3. The allowable load \(Q_a\) is 50 percent of the ultimate load at which the total settlement amounts to one-tenth of the diameter of the pile for uniform diameter piles.
4. The allowable load $Q_a$ is sometimes taken as equal to two-thirds of the load which causes a total settlement of 12 mm.

5. The allowable load $Q_a$ is sometimes taken as equal to two-thirds of the load which causes a net (plastic) settlement of 6 mm.

If pile groups are loaded to failure, the ultimate load of the group, $Q_{gu}$, may be found by any one of the first two methods mentioned above for single piles. However, if the groups are subjected to only
one and a half-times the design load of the group, the allowable load on the group cannot be found on
the basis of 12 or 6 mm settlement criteria applicable to single piles. In the case of a group with piles
spaced at less than 6 to 8 times the pile diameter, the stress interaction of the adjacent piles affects the
settlement considerably. The settlement criteria applicable to pile groups should be the same as that
applicable to shallow foundations at design loads.

15.23 PILE BEARING CAPACITY FROM DYNAMIC PILE DRIVING
FORMULAS
The resistance offered by a soil to penetration of a pile during driving gives an indication of its
bearing capacity. Qualitatively speaking, a pile which meets greater resistance during driving is
capable of carrying a greater load. A number of dynamic formulae have been developed which
equate pile capacity in terms of driving energy.

The basis of all these formulae is the simple energy relationship which may be stated by the
following equation. (Fig. 15.21).

\[ Wh = Q_u s \]

or

\[ Q_u = \frac{Wh}{s} \]  \hspace{1cm} (15.58)

where
- \( W \) = weight of the driving hammer
- \( h \) = height of fall of hammer
- \( Wh \) = energy of hammer blow
- \( Q_u \) = ultimate resistance to penetration
- \( s \) = pile penetration under one hammer blow
- \( Q_u s \) = resisting energy of the pile

**Hiley Formula**

Equation (15.58) holds only if the system is 100 percent efficient. Since the driving of a pile
involves many losses, the energy of the system may be written as
Energy input = Energy used + Energy losses

or

Energy used = Energy input – Energy losses.

The expressions for the various energy terms used are

1. Energy used = $Q_u s$,
2. Energy input = $\eta_h W h$, where $\eta_h$ is the efficiency of the hammer.
3. The energy losses are due to the following:
   (i) The energy loss $E_1$ due to the elastic compressions of the pile cap, pile material and the soil surrounding the pile. The expression for $E_1$ may be written as

   \[ E_1 = \frac{1}{2} Q_u (c_1 + c_2 + c_3) = Q_u C \]

   where
   - $c_1 =$ elastic compression of the pile cap
   - $c_2 =$ elastic compression of the pile
   - $c_3 =$ elastic compression of the soil.

   (ii) The energy loss $E_2$ due to the interaction of the pile hammer system (impact of two bodies). The expression for $E_2$ may be written as

   \[ E_2 = W h W_p \frac{1 - C_r^2}{W + W_p} \]

   where
   - $W_p =$ weight of pile
   - $C_r =$ coefficient of restitution.
Substituting the various expressions in the energy equation and simplifying, we have

\[ Q_u = \frac{\eta_h Wh}{s + C} \times \frac{1 + C^2 R}{1 + R} \]

where \( R = \frac{W_p}{W} \)

Equation (15.59) is called the Hiley formula. The allowable load \( Q_u \) may be obtained by dividing \( Q_u \) by a suitable factor of safety.

If the pile tip rests on rock or relatively impenetrable material, Eq. (15.59) is not valid. Chellis (1961) suggests for this condition that the use of \( W_p^2 \) instead of \( W_p \) may be more correct. The various coefficients used in the Eq. (15.59) are as given below:

1. **Elastic compression \( c_1 \) of cap and pile head**

<table>
<thead>
<tr>
<th>Pile Material</th>
<th>Range of Driving Stress kg/cm(^2)</th>
<th>Range of ( c_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precast concrete pile with packing inside cap</td>
<td>30-150</td>
<td>0.12-0.50</td>
</tr>
<tr>
<td>Timber pile without cap</td>
<td>30-150</td>
<td>0.05-0.20</td>
</tr>
<tr>
<td>Steel H-pile</td>
<td>30-150</td>
<td>0.04-0.16</td>
</tr>
</tbody>
</table>

2. **Elastic compression \( c_2 \) of pile.**

This may be computed using the equation

\[ c_2 = \frac{Q_u L}{AE} \]

where \( L = \) embedded length of the pile, \( A = \) average cross-sectional area of the pile, \( E = \) Young's modulus.

3. **Elastic compression \( c_3 \) of soil.**

The average value of \( c_3 \) may be taken as 0.1 (the value ranges from 0.0 for hard soil to 0.2 for resilient soils).

4. **Pile-hammer efficiency**

<table>
<thead>
<tr>
<th>Hammer Type</th>
<th>( \eta_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drop</td>
<td>1.00</td>
</tr>
<tr>
<td>Single acting</td>
<td>0.75-0.85</td>
</tr>
<tr>
<td>Double acting</td>
<td>0.85</td>
</tr>
<tr>
<td>Diesel</td>
<td>1.00</td>
</tr>
</tbody>
</table>

5. **Coefficient of restitution \( C_r \)**

<table>
<thead>
<tr>
<th>Material</th>
<th>( C_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood pile</td>
<td>0.25</td>
</tr>
<tr>
<td>Compact wood cushion on steel pile</td>
<td>0.32</td>
</tr>
<tr>
<td>Cast iron hammer on concrete pile without cap</td>
<td>0.40</td>
</tr>
<tr>
<td>Cast iron hammer on steel pipe without cushion</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Engineering News Record (ENR) Formula

The general form of the Engineering News Record Formula for the allowable load $Q_a$ may be obtained from Eq. (15.59) by putting $\eta = 1$ and $C_r = 1$ and a factor of safety equal to 6. The formula proposed by A.M. Wellington, editor of the Engineering News, in 1886, is

$$Q_a = \frac{Wh}{6(s + C)}$$  \hspace{1cm} (15.60)

where $Q_a$ = allowable load in kg,
$W$ = weight of hammer in kg,
$h$ = height of fall of hammer in cm,
$s$ = final penetration in cm per blow (which is termed as set). The set is taken as the average penetration per blow for the last 5 blows of a drop hammer or 20 blows of a steam hammer,
$C$ = empirical constant,
$= 2.5$ cm for a drop hammer,
$= 0.25$ cm for single and double acting hammers.

The equations for the various types of hammers may be written as:

1. Drop hammer

$$Q_a = \frac{Wh}{6(s + 2.5)}$$  \hspace{1cm} (15.61)

2. Single-acting hammer

$$Q_a = \frac{Wh}{6(s + 0.25)}$$  \hspace{1cm} (15.62)

3. Double-acting hammer

$$Q_a = \frac{(W + ap)}{6(s + 0.25)}$$  \hspace{1cm} (15.63)

$a$ = effective area of the piston in sq. cm,
$p$ = mean effective steam pressure in kg/cm$^2$.

Comments on the Use of Dynamic Formulae

1. Detailed investigations carried out by Vesic (1967) on deep foundations in granular soils indicate that the Engineering News Record Formula applicable to drop hammers, Eq. (15.61), gives pile loads as low as 44% of the actual loads. In order to obtain better agreement between the one computed and observed loads, Vesic suggests the following values for the coefficient $C$ in Eq. (15.60).

For steel pipe piles, $C = 1$ cm.
For precast concrete piles $C = 1.5$ cm.

2. The tests carried out by Vesic in granular soils indicate that Hiley's formula does not give consistent results. The values computed from Eq. (15.59) are sometimes higher and sometimes lower than the observed values.
3. Dynamic formulae in general have limited value in pile foundation work mainly because the
dynamic resistance of soil does not represent the static resistance, and because often the results
obtained from the use of dynamic equations are of questionable dependability. However,
engineers prefer to use the Engineering News Record Formula because of its simplicity.

4. Dynamic formulae could be used with more confidence in freely draining materials such as
coarse sand. If the pile is driven to saturated loose sand and silt, there is every
possibility of development of liquefaction which reduces the bearing capacity of the pile.

5. Dynamic formulae are not recommended for computing allowable loads of piles driven
into cohesive soils. In cohesive soils, the resistance to driving increases through the sudden
increase in stress in pore water and decreases because of the decreased value of the internal
friction between soil and pile because of pore water. These two oppositely directed forces
do not lend themselves to analytical treatment and as such the dynamic penetration
resistance to pile driving has no relationship to static bearing capacity.

There is another effect of pile driving in cohesive soils. During driving the soil becomes
remolded and the shear strength of the soil is reduced considerably. Though there will be a
regaining of shear strength after a lapse of some days after the driving operation, this will not be
reflected in the resistance value obtained from the dynamic formulae.

Example 15.22

A 40 x 40 cm reinforced concrete pile 20 m long is driven through loose sand and then into dense
gravel to a final set of 3 mm/blow, using a 30 kN single-acting hammer with a stroke of 1.5 m.
Determine the ultimate driving resistance of the pile if it is fitted with a helmet, plastic dolly and 50
mm packing on the top of the pile. The weight of the helmet and dolly is 4 kN. The other details are:
weight of pile = 74 kN; weight of hammer = 30 kN; pile hammer efficiency \( \eta_h = 0.80 \) and
coefficient of restitution \( C_r = 0.40 \).

Use the Hiley formula. The sum of the elastic compression \( \overline{C} \) is

\[ \overline{C} = c_1 + c_2 + c_3 = 19.6 \text{ mm.} \]

Solution

Hiley Formula

Use Eq. (15.59)

\[ Q_u = \frac{\eta_h Wh}{s + C} \times \frac{1 + C^2 R}{1 + R} \]

where \( \eta_h = 0.80, \ W = 30 \text{ kN}, \ h = 1.5 \text{ m}, \ R = \frac{W_p}{W} = \frac{(74 + 4)}{30} = 2.6, \ C_r = 0.40, \ s = 0.30 \text{ cm}. \)

Substituting we have,

\[ Q_u = \frac{0.8 \times 30 \times 150}{0.3 + 19.6 / 2} \times \frac{1 + 0.4^2 \times 2.6}{1 + 2.6} = 2813 \times 0.393 = 1105 \text{ kN} \]

15.24 BEARING CAPACITY OF PILES FOUND ON A ROCKY BED

Piles are at times required to be driven through weak layers of soil until the tips meet a hard strata
for bearing. If the bearing strata happens to be rock, the piles are to be driven to refusal in order to
obtain the maximum carrying capacity from the piles. If the rock is strong at its surface, the pile will
refuse further driving at a negligible penetration. In such cases the carrying capacity of the piles is
governed by the strength of the pile shaft regarded as a column as shown in Fig. 15.6(a). If the soil
mass through which the piles are driven happens to be stiff clays or sands, the piles can be regarded
as being supported on all sides from buckling as a strut. In such cases, the carrying capacity of a pile
is calculated from the safe load on the material of the pile at the point of minimum cross-section. In
practice, it is necessary to limit the safe load on piles regarded as short columns because of the
likely deviations from the vertical and the possibility of damage to the pile during driving.

If piles are driven to weak rocks, working loads as determined by the available stress on the
material of the pile shaft may not be possible. In such cases the frictional resistance developed over
the penetration into the rock and the end bearing resistance are required to be calculated. Tomlinson
(1986) suggests an equation for computing the end bearing resistance of piles resting on rocky
strata as

\[ q_u = 2N_0 q_{ur} \] (15.64)

where \( N_0 = \tan^2 (45 + \phi/2) \),
\( q_{ur} = \) unconfined compressive strength of the rock.

Boring of a hole in rocky strata for constructing bored piles may weaken the bearing strata of
some types of rock. In such cases low values of skin friction should be used and normally may not
be more than 20 kN/m² (Tomlinson, 1986) when the boring is through friable chalk or mud stone.
In the case of moderately weak to strong rocks where it is possible to obtain core samples for
unconfined compression tests, the end bearing resistance can be calculated by making use of
Eq. (15.64).

15.25 UPLIFT RESISTANCE OF PILES

Piles are also used to resist uplift loads. Piles used for this purpose are called tension piles, uplift
piles or anchor piles. Uplift forces are developed due to hydrostatic pressure or overturning
moments as shown in Fig. 15.22.

Figure 15.22 shows a straight edged pile subjected to uplift force. The equation for the uplift
force \( P_{ul} \) may be written as

\[ P_{ul} = W_p + A_s f_r \] (15.65)

where, \( P_{ul} = \) uplift capacity of pile,
\( W_p = \) weight of pile,
\( f_r = \) unit resisting force
\( A_s = \) effective area of the embedded length of pile.

Uplift Resistance of Pile in Clay

For piles embedded in clay, Eq. (15.65) may written as

\[ P_{ul} = W_p + A_s c_u \] (15.66)

where, \( c_u = \) average undrained shear strength of clay along the pile shaft,
\( \alpha = \) adhesion factor \( (= c_d/c_u) \),
\( c_d = \) average adhesion.

Figure 15.23 gives the relationship between \( \alpha \) and \( c_u \) based on pull out test results as collected
by Sowa (1970). As per Sowa, the values of \( c_u \) agree reasonably well with the values for piles
subjected to compression loadings.
Uplift Resistance of Pile in Sand

Adequate confirmatory data are not available for evaluating the uplift resistance of piles embedded in cohesionless soils. Ireland (1957) reports that the average skin friction for piles under compression loading and uplift loading are equal, but data collected by Sowa (1970) and Downs and Chieurzzi (1966) indicate lower values for upward loading as compared to downward loading especially for cast-in-situ piles. Poulos and Davis (1980) suggest that the skin friction of upward loading may be taken as two-thirds of the calculated shaft resistance for downward loading.

A safety factor of 3 is normally assumed for calculating the safe uplift load for both piles in clay and sand.
Example 15.23

A reinforced concrete pile 30 ft long and 15 in. in diameter is embedded in a saturated clay of very stiff consistency. Laboratory tests on samples of undisturbed soil gave an average undrained cohesive strength $c_u = 2500 \text{ lb/ft}^2$. Determine the net pullout capacity and the allowable pullout load with $F_s = 3$.

Solution

Given: $L = 30 \text{ ft}$, $d = 15 \text{ in. diameter}$, $c_u = 2500 \text{ lb/ft}^2$, $F_s = 3$.

From Fig. 15.23 $c_d/c_u = 0.41$ for $c_u = 2500 \times 0.0479 = 120 \text{ kN/m}^2$ for concrete pile.

From Eq. (15.66)

$$P_{ul} (\text{net}) = \alpha c_u A_s$$

where $\alpha = c_d / c_u = 0.41$, $c_u = 2500 \text{ lb/ft}^2$

$$A_s = 3.14 \times \frac{15}{12} \times 30 = 117.75 \text{ ft}^2$$

Substituting

$$P_{ul} (\text{net}) = \frac{0.41 \times 2500 \times 117.75}{1000} = 120.69 \text{ kips}$$

$$P_{ul} (\text{allowed}) = \frac{120.69}{3} = 40 \text{ kips}$$

Example 15.24

Refer to Ex. 15.23. If the pile is embedded in medium dense sand, determine the net pullout capacity and the net allowable pullout load with $F_s = 3$.

Given: $L = 30 \text{ ft}$, $\phi = 38^\circ$, $K_s = 1.5$, and $\delta = 25^\circ$, $\gamma$ (average) = $110 \text{ lb/ft}^3$.

The water table is at great depth. Refer to Section 15.25.

Solution

Downward skin resistance $Q_f$

$$Q_f = \frac{1}{2} \alpha \gamma' \tan \delta A_s$$

where $\gamma' = \frac{1}{2} \times 30 \times 110 = 1650 \text{ lb/ft}^2$

$$A_s = 3.14 \times 1.25 \times 30 = 117.75 \text{ ft}^2$$

$$Q_f (\text{down}) = \frac{1650 \times 1.5 \tan 25^\circ \times 117.75}{1000} = 136 \text{ kips}$$

Based on the recommendations of Poulos and Davis (1980)

$$Q_f (\text{up}) = \frac{2}{3} Q_f (\text{down}) = \frac{2}{3} \times 136 = 91 \text{ kips}$$
PART B—PILE GROUP

15.26 NUMBER AND SPACING OF PILES IN A GROUP

Very rarely are structures founded on single piles. Normally, there will be a minimum of three piles under a column or a foundation element because of alignment problems and inadvertent eccentricities. The spacing of piles in a group depends upon many factors such as

\[ Q_{pu} \text{(up)} = \frac{91}{3} = 30 \text{ kips} \]

Figure 15.24 Pressure isobars of (a) single pile, (b) group of piles, closely spaced, and (c) group of piles with piles far apart.
1. overlapping of stresses of adjacent piles,
2. cost of foundation,
3. efficiency of the pile group.

The pressure isobars of a single pile with load $Q$ acting on the top are shown in Fig. 15.24(a). When piles are placed in a group, there is a possibility the pressure isobars of adjacent piles will overlap each other as shown in Fig. 15.24(b). The soil is highly stressed in the zones of overlapping of pressures. With sufficient overlap, either the soil will fail or the pile group will settle excessively since the combined pressure bulb extends to a considerable depth below the base of the piles. It is possible to avoid overlap by installing the piles further apart as shown in Fig. 15.24(c). Large spacings are not recommended sometimes, since this would result in a larger pile cap which would increase the cost of the foundation.

The spacing of piles depends upon the method of installing the piles and the type of soil. The piles can be driven piles or \textit{cast-in-situ} piles. When the piles are driven there will be greater overlapping of stresses due to the displacement of soil. If the displacement of soil compacts the soil in between the piles as in the case of loose sandy soils, the piles may be placed at closer intervals.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Typical_arrangements_of_piles_in_groups}\caption{Typical arrangements of piles in groups}
\end{figure}
But if the piles are driven into saturated clay or silty soils, the displaced soil will not compact the soil between the piles. As a result the soil between the piles may move upwards and in this process lift the pile cap. Greater spacing between piles is required in soils of this type to avoid lifting of piles. When piles are cast-in-situ, the soils adjacent to the piles are not stressed to that extent and as such smaller spacings are permitted.

Generally, the spacing for point bearing piles, such as piles founded on rock, can be much less than for friction piles since the high-point-bearing stresses and the superposition effect of overlap of the point stresses will most likely not overstress the underlying material nor cause excessive settlements.

The minimum allowable spacing of piles is usually stipulated in building codes. The spacings for straight uniform diameter piles may vary from 2 to 6 times the diameter of the shaft. For friction piles, the minimum spacing recommended is 3d where d is the diameter of the pile. For end bearing piles passing through relatively compressible strata, the spacing of piles shall not be less than 2.5d. For end bearing piles passing through compressible strata and resting in stiff clay, the spacing may be increased to 3.5d. For compaction piles, the spacing may be 2d. Typical arrangements of piles in groups are shown in Fig. 15.25.

15.27 PILE GROUP EFFICIENCY

The spacing of piles is usually predetermined by practical and economical considerations. The design of a pile foundation subjected to vertical loads consists of

1. The determination of the ultimate load bearing capacity of the group $Q_{qu}$.
2. Determination of the settlement of the group, $S_g$, under an allowable load $Q_{go}$.

The ultimate load of the group is generally different from the sum of the ultimate loads of individual piles $Q_u$.

The factor

$$E_g = \frac{Q_{gu}}{\Sigma Q_u}$$  \hspace{1cm} (15.67)

is called group efficiency which depends on parameters such as type of soil in which the piles are embedded, method of installation of piles i.e. either driven or cast-in-situ piles, and spacing of piles.

There is no acceptable "efficiency formula" for group bearing capacity. There are a few formulae such as the Converse-Labarre formula that are sometimes used by engineers. These formulae are empirical and give efficiency factors less than unity. But when piles are installed in sand, efficiency factors greater than unity can be obtained as shown by Vesic (1967) by his experimental investigation on groups of piles in sand. There is not sufficient experimental evidence to determine group efficiency for piles embedded in clay soils.

**Efficiency of Pile Groups in Sand**

Vesic (1967) carried out tests on 4 and 9 pile groups driven into sand under controlled conditions. Piles with spacings 2, 3, 4, and 6 times the diameter were used in the tests. The tests were conducted in homogeneous, medium dense sand. His findings are given in Fig. 15.26. The figure gives the following:

1. The efficiencies of 4 and 9 pile groups when the pile caps do not rest on the surface.
2. The efficiencies of 4 and 9 pile groups when the pile caps rest on the surface.
3. The skin efficiency of 4 and 9 pile groups.
4. The average point efficiency of all the pile groups.

It may be mentioned here that a pile group with the pile cap resting on the surface takes more load than one with free standing piles above the surface. In the former case, a part of the load is taken by the soil directly under the cap and the rest is taken by the piles. The pile cap behaves the same way as a shallow foundation of the same size. Though the percentage of load taken by the group is quite considerable, building codes have not so far considered the contribution made by the cap.

It may be seen from the Fig. 15.26 that the overall efficiency of a four pile group with a cap resting on the surface increases to a maximum of about 1.7 at pile spacings of 3 to 4 pile diameters, becoming somewhat lower with a further increase in spacing. A sizable part of the increased bearing capacity comes from the caps. If the loads transmitted by the caps are reduced, the group efficiency drops to a maximum of about 1.3.

Very similar results are indicated from tests with 9 pile groups. Since the tests in this case were carried out only up to a spacing of 3 pile diameters, the full picture of the curve is not available. However, it may be seen that the contribution of the cap for the bearing capacity is relatively smaller.

Vesic measured the skin loads of all the piles. The skin efficiencies for both the 4 and 9-pile groups indicate an increasing trend. For the 4-pile group the efficiency increases from about 1.8 at 2 pile diameters to a maximum of about 3 at 5 pile diameters and beyond. In contrast to this, the average point load efficiency for the groups is about 1.01. Vesic showed for the first time that the
increasing bearing capacity of a pile group for piles driven in sand comes primarily from an increase in skin loads. The point loads seem to be virtually unaffected by group action.

**Pile Group Efficiency Equation**

There are many pile group equations. These equations are to be used very cautiously, and may in many cases be no better than a good guess. The Converse-Labarre Formula is one of the most widely used group-efficiency equations which is expressed as

\[ E_g = 1 - \frac{\theta(n-1)m + (m-1)n}{90 mn} \]  

(15.68)

where  
- \( m \) = number of columns of piles in a group,
- \( n \) = number of rows,
- \( \theta \) = \( \tan^{-1}(d/s) \) in degrees,
- \( d \) = diameter of pile,
- \( s \) = spacing of piles center to center.

### 15.28 VERTICAL BEARING CAPACITY OF PILE GROUPS EMBEDDED IN SANDS AND GRAVELS

**Driven piles.** If piles are driven into loose sands and gravel, the soil around the piles to a radius of at least three times the pile diameter is compacted. When piles are driven in a group at close spacing, the soil around and between them becomes highly compacted. When the group is loaded, the piles and the soil between them move together as a unit. Thus, the pile group acts as a pier foundation having a base area equal to the gross plan area contained by the piles. The efficiency of the pile group will be greater than unity as explained earlier. It is normally assumed that the efficiency falls to unity when the spacing is increased to five or six diameters. Since present knowledge is not sufficient to evaluate the efficiency for different spacing of piles, it is conservative to assume an efficiency factor of unity for all practical purposes. We may, therefore, write

\[ Q_gu = nQ_u \]  

(15.69)

where \( n \) = the number of piles in the group.

The procedure explained above is not applicable if the pile tips rest on compressible soil such as silts or clays. When the pile tips rest on compressible soils, the stresses transferred to the compressible soils from the pile group might result in over-stressing or extensive consolidation. The carrying capacity of pile groups under these conditions is governed by the shear strength and compressibility of the soil, rather than by the 'efficiency' of the group within the sand or gravel stratum.

**Bored Pile Groups In Sand And Gravel**

Bored piles are *cast-in-situ* concrete piles. The method of installation involves

1. Boring a hole of the required diameter and depth,
2. Pouring in concrete.

There will always be a general loosening of the soil during boring and then too when the boring has to be done below the water table. Though bentonite slurry (sometimes called as *drilling mud*) is used for stabilizing the sides and bottom of the bores, loosening of the soil cannot be avoided. Cleaning of the bottom of the bore hole prior to concreting is always a problem which will never be achieved quite satisfactorily. Since bored piles do not compact the soil between the piles,
the efficiency factor will never be greater than unity. However, for all practical purposes, the efficiency may be taken as unity.

**Pile Groups In Cohesive Soils**

The effect of driving piles into cohesive soils (clays and silts) is very different from that of cohesionless soils. It has already been explained that when piles are driven into clay soils, particularly when the soil is soft and sensitive, there will be considerable remolding of the soil. Besides there will be heaving of the soil between the piles since compaction during driving cannot be achieved in soils of such low permeability. There is every possibility of lifting of the pile during this process of heaving of the soil. Bored piles are, therefore, preferred to driven piles in cohesive soils. In case driven piles are to be used, the following steps should be favored:

1. Piles should be spaced at greater distances apart.
2. Piles should be driven from the center of the group towards the edges, and
3. The rate of driving of each pile should be adjusted as to minimize the development of pore water pressure.

Experimental results have indicated that when a pile group installed in cohesive soils is loaded, it may fail by any one of the following ways:

1. May fail as a block (called block failure).
2. Individual piles in the group may fail.
When piles are spaced at closer intervals, the soil contained between the piles move downward with the piles and at failure, piles and soil move together to give the typical 'block failure'. Normally this type of failure occurs when piles are placed within 2 to 3 pile diameters. For wider spacings, the piles fail individually. The efficiency ratio is less than unity at closer spacings and may reach unity at a spacing of about 8 diameters.

The equation for block failure may be written as (Fig. 15.27).

\[ Q_{gu} = cN_cA_g + P_gL \bar{e} \]  
(15.70)

where

- \( c \) = cohesive strength of clay beneath the pile group,
- \( \bar{c} \) = average cohesive strength of clay around the group,
- \( L \) = length of pile,
- \( P_g \) = perimeter of pile group,
- \( A_g \) = sectional area of group,
- \( N_c \) = bearing capacity factor which may be assumed as 9 for deep foundations.

The bearing capacity of a pile group on the basis of individual pile failure may be written as

\[ Q_{gu} = nQ_u \]  
(15.71)

where

- \( n \) = number of piles in the group,
- \( Q_u \) = bearing capacity of an individual pile.

The bearing capacity of a pile group is normally taken as the smaller of the two given by Eqs. (15.70) and (15.71).

**Example 15.25**

A group of 9 piles with 3 piles in a row was driven into a soft clay extending from ground level to a great depth. The diameter and the length of the piles were 30 cm and 10 m respectively. The unconfined compressive strength of the clay is 70 kPa. If the piles were placed 90 cm center to center, compute the allowable load on the pile group on the basis of a shear failure criterion for a factor of safety of 2.5.

**Solution**

The allowable load on the group is to be calculated for two conditions: (a) block failure and (b) individual pile failure. The least of the two gives the allowable load on the group.

(a) Block failure (Fig. 15.27). Use Eq. (15.70),

\[ Q_{gu} = cN_cA_g + P_gL \bar{e} \]  
where \( N_c = 9, c = \bar{c} = 70/2 = 35 \text{ kN/m}^2 \)

\[ A_g = 2.1 \times 2.1 = 4.4 \text{ m}^2, \quad P_g = 4 \times 2.1 = 8.4 \text{ m}, \quad L = 10 \text{ m} \]

\[ Q_{gu} = 35 \times 9 \times 4.4 + 8.4 \times 10 \times 35 = 4326 \text{ kN}, \quad Q_u = \frac{4326}{2.5} = 1730 \text{ kN} \]

(b) Individual pile failure

\[ Q_u = Q_b + Q_f = q_bA_b + \alpha \bar{c}A_g \]  
Assume \( \alpha = 1 \).

Now,

\[ q_b = cN_c = 35 \times 9 = 315 \text{ kN/m}^2, \quad A_b = 0.07 \text{ m}^2, \]
\[ A_s = 3.14 \times 0.3 \times 10 = 9.42 \text{ m}^2 \]

Substituting,

\[ Q_u = 315 \times 0.07 + 1 \times 35 \times 9.42 = 352 \text{ kN} \]

\[ Q_{gw} = nQ_u = 9 \times 352 = 3168 \text{ kN}, \quad Q_u = \frac{3168}{2.5} = 1267 \text{ kN} \]

The allowable load is 1267 kN.

### 15.29 SETTLEMENT OF PILES AND PILE GROUPS IN SANDS AND GRAVELS

normally it is not necessary to compute the settlement of a single pile as this settlement under a working load will be within the tolerable limits. However, settlement analysis of a pile group is very much essential. The total settlement analysis of a pile group does not bear any relationship with that of a single pile since in a group the settlement is very much affected due to the interaction stresses between piles and the stressed zone below the tips of piles.

Settlement analysis of single piles by Poulos and Davis (1980) indicates that immediate settlement contributes the major part of the final settlement (which includes the consolidation settlement for saturated clay soils) even for piles in clay. As far as piles in sand is concerned, the immediate settlement is almost equal to the final settlement.

However, it may be noted here that consolidation settlement becomes more important for pile groups in saturated clay soils.

Immediate settlement of a single pile may be computed by making use of semi-empirical methods. The method as suggested by Vesic (1977) has been discussed here.

In recent years, with the advent of computers, more sophisticated methods of analysis have been developed to predict the settlement and load distribution in a single pile. The following three methods are often used.

1. 'Load transfer' method which is also called as the 't-z' method.
2. Elastic method based on Mindlin's (1936) equations for the effects of subsurface loadings within a semi-infinite mass.
3. The finite element method.

This chapter discusses only the 't-z' method. The analysis of settlement by the elastic method is quite complicated and is beyond the scope of this book. Poulos and Davis, (1980) have discussed this procedure in detail. The finite element method of analysis of a single pile axially loaded has been discussed by many investigators such as Ellison et al., (1971), Desai (1974), Balaam et al., (1975), etc. The finite element approach is a generalization of the elastic approach. The power of this method lies in its capability to model complicated conditions and to represent non-linear stress/strain behavior of the soil over the whole zone of the soil modelled. Use of computer programs is essential and the method is more suited to research or investigation of particularly complex problems than to general design.

Present knowledge is not sufficient to evaluate the settlements of piles and pile groups. For most engineering structures, the loads to be applied to a pile group will be governed by consideration of consolidation settlement rather than by bearing capacity of the group divided by an arbitrary factor of safety of 2 or 3. It has been found from field observation that the settlement of a pile group is many times the settlement of a single pile at the corresponding working load. The settlement of a group is affected by the shape and size of the group, length of piles, method of installation of piles and possibly many other factors.
Semi-Empirical Formulas and Curves

Vesic (1977) proposed an equation to determine the settlement of a single pile. The equation has been developed on the basis of experimental results he obtained from tests on piles. Tests on piles of diameters ranging from 2 to 18 inches were carried out in sands of different relative densities. Tests were carried out on driven piles, jacked piles, and bored piles (jacked piles are those that are pushed into the ground by using a jack). The equation for total settlement of a single pile may be expressed as

\[ S = S_p + S_f \]  

(15.72)

where

- \( S \) = total settlement,
- \( S_p \) = settlement of the pile tip,
- \( S_f \) = settlement due to the deformation of the pile shaft.

The equation for \( S_p \) is

\[ S_p = \frac{C_w Q_p}{(1 + D_r^2)q_{pu}} \]  

(15.73)

The equation for \( S_f \) is

\[ S_f = (Q_p + \alpha Q_f) \frac{L}{AE} \]  

(15.74)

where

- \( Q_p \) = point load,
- \( d \) = diameter of the pile at the base,
- \( q_{pu} \) = ultimate point resistance per unit area,
- \( D_r \) = relative density of the sand,
- \( C_w \) = settlement coefficient, = 0.04 for driven piles
  = 0.05 for jacked piles
  = 0.18 for bored piles,
- \( Q_f \) = friction load,
- \( L \) = pile length,
- \( A \) = cross-sectional area of the pile,
- \( E \) = modulus of deformation of the pile shaft,
- \( \alpha \) = coefficient which depends on the distribution of skin friction along the shaft and can be taken equal to 0.6.

Settlement of piles cannot be predicted accurately by making use of equations such as the ones given here. One should use such equations with caution. It is better to rely on load tests for piles in sands.

Settlement of Pile Groups in Sand

The relation between the settlement of a group and a single pile at corresponding working loads may be expressed as

\[ F_g = \frac{S_g}{S} \]  

(15.75)

where

- \( F_g \) = group settlement factor,
- \( S_g \) = settlement of group,
- \( S \) = settlement of a single pile.
Vesic (1967) obtained the curve given in Fig. 15.28 by plotting \( F_g \) against \( B/d \) where \( d \) is the diameter of the pile and \( B \), the distance between the center to center of the outer piles in the group (only square pile groups are considered). It should be remembered here that the curve is based on the results obtained from tests on groups of piles embedded in medium dense sand. It is possible that groups in much looser or much denser deposits might give somewhat different behavior. The group settlement ratio is very likely be affected by the ratio of the pile point settlement \( S_p \) to total pile settlement.

Skempton et al., (1953) published curves relating \( F_g \) with the width of pile groups as shown in Fig. 15.29. These curves can be taken as applying to driven or bored piles. Since the abscissa for the curve in Fig. 15.29 is not expressed as a ratio, this curve cannot directly be compared with Vesic’s curve given in Fig. 15.28. According to Fig. 15.29 a pile group 3 m wide would settle 5 times that of a single test pile.

**t-z Method**

Consider a floating vertical pile of length \( L \) and diameter \( d \) subjected to a vertical load \( Q \) (Fig. 15.30). This load will be transferred to the surrounding and underlying soil layers as described in
Section 15.7. The pile load will be carried partly by skin friction (which will be mobilized through the increasing settlement on the mantle surface and the compression of the pile shaft) and partly through the pile tip, in the form of point resistance as can be seen from Fig. 15.30. Thus the load is taken as the sum of these components. The distribution of the point resistance is usually considered as uniform; however, the distribution of the mantle friction depends on many factors. Load transfer in pile-soil system is a very complex phenomenon involving a number of parameters which are difficult to evaluate in numerical terms. Yet, some numerical assessment of load transfer characteristics of a pile soil system is essential for the rational design of pile foundation.

The objective of a load transfer analysis is to obtain a load-settlement curve. The basic problem of load transfer is shown in Fig. 15.30. The following are to be determined:

1. The vertical movements $s$, of the pile cross-sections at any depth $z$ under loads acting on the top.
2. The corresponding pile load $Q$, at depth $z$ acting on the pile section.
3. The vertical movement of the base of the pile and the corresponding point stress.

The mobilization of skin shear stress $\tau$ at any depth $z$, from the ground surface depends on the vertical movement of the pile cross section at that level. The relationship between the two may be linear or non-linear. The shear stress $\tau$, reaches the maximum value, $\tau_p$, at that section when the vertical movement of the pile section is adequate. It is, therefore, essential to construct $(\tau - s)$ curves at various depths $z$ as required.

There will be settlement of the tip of pile after the full mobilization of skin friction. The movement of the tip, $s_h$, may be assumed to be linear with the point pressure $q_p$. When the movement of the tip is adequate, the point pressure reaches the maximum pressure $q_p$ (ultimate base pressure). In order to solve the load-transfer problem, it is essential to construct a $(q_p - s)$ curve.

$(\tau - s)$ and $(q_p - s)$ curves

Coyle and Reese (1966) proposed a set of average curves of load transfer based on laboratory test piles and instrumented field piles. These curves are limited to the case of steel-pipe friction piles in clay with an embedded depth not exceeding 100 ft. Coyle and Sulaiman (1967) have also given load transfer curves for piles in sand. These curves are meant for specific cases and therefore meant to solve specific problems and as such this approach cannot be considered as a general case.

Verbrugge (1981) proposed an elastic-plastic model for the $(\tau - s)$ and $(q_p - s)$ curves based on CPT results. The slopes of the elastic portion of the curves given are

$$ \frac{\tau}{s} = \frac{0.22E_s}{2R} $$

$$ \frac{q_p}{s} = \frac{3.125E_s}{2R} $$

The value of elastic modulus $E_s$ of cohesionless soils may be obtained by the following expressions (Verbrugge):

for bored piles $E_s = (36 + 2.2q_c)$ kg/cm$^2$

for driven piles $E_s = 3(36 + 2.2q_c)$ kg/cm$^2$

where $q_c$ = point resistance of static cone penetrometer in kg/cm$^2$.

The relationship recommended for $E_s$ is for $q_c > 4$ kg/cm$^2$. The maximum values of $\tau$, and $\tau_p$, for the plastic portion of $(\tau - s)$ curves are given in Table 15.4 and 15.5 for cohesionless and cohesive soils respectively. In Eqs (15.76) and (15.77) the value of $E_s$ can be obtained by any one of
Figure 15.30  t–z method of analysis of pile load-settlement relationship

the known methods. The maximum value of $q_p$ ($q_b$) may be obtained by any one of the known methods such as

1. From the relationship

   $q_b = q'_N$ for cohesionless soils
   $q_b = 9c_u$ for cohesive soils.

Table 15.4  Recommended maximum skin shear stress $\tau_{\text{max}}$ for piles in cohesionless soils (After Verbrugge, 1981)

<table>
<thead>
<tr>
<th>$\tau_{\text{max}}$ kN/m$^2$</th>
<th>Pile type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.011 $q_c$</td>
<td>Driven concrete piles</td>
</tr>
<tr>
<td>0.009 $q_c$</td>
<td>Driven steel piles</td>
</tr>
<tr>
<td>0.005 $q_c$</td>
<td>Bored concrete piles</td>
</tr>
</tbody>
</table>

Limiting values

$\tau_{\text{max}} = 80$ kN/m$^2$ for bored piles
$\tau_{\text{max}} = 120$ kN/m$^2$ for driven piles
Table 15.5  Recommended maximum skin shear stress $\tau_{\text{max}}$ for piles in cohesive soils (After Verbrugge, 1981) 

[Values recommended are for Dutch cone penetrometer]

<table>
<thead>
<tr>
<th>Type of pile</th>
<th>Material</th>
<th>Range of $q_c$, kN/m$^2$</th>
<th>$\tau_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driven</td>
<td>Concrete</td>
<td>$q_c \leq 375$</td>
<td>0.053 $q_c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$375 &lt; q_c &lt; 4500$</td>
<td>18 + 0.006 $q_c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$4500 &lt; q_c$</td>
<td>0.01 $q_c$</td>
</tr>
<tr>
<td></td>
<td>Steel</td>
<td>$q_c \leq 450$</td>
<td>0.033 $q_c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$450 &lt; q_c \leq 1500$</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1500 &lt; q_c$</td>
<td>0.01 $q_c$</td>
</tr>
<tr>
<td>Bored</td>
<td>Concrete</td>
<td>$q_c \leq 600$</td>
<td>0.037 $q_c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$600 &lt; q_c &lt; 4500$</td>
<td>18 + 0.006 $q_c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$4500 &lt; q_c$</td>
<td>0.01 $q_c$</td>
</tr>
<tr>
<td></td>
<td>Steel</td>
<td>$q_c \leq 500$</td>
<td>0.03 $q_c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$500 &lt; q_c &lt; 1500$</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1500 &lt; q_c$</td>
<td>0.01 $q_c$</td>
</tr>
</tbody>
</table>

2. From static cone penetration test results
3. From pressuremeter test results

Method of obtaining load-settlement curve (Fig. 15.30)

The approximate load-settlement curve is obtained point by point in the following manner:

1. Divide the pile into any convenient segments (possibly 10 for computer programming and less for hand calculations).
2. Assume a point pressure $q_p$ less than the maximum $q_b$.
3. Read the corresponding displacement $s_p$ from the $(q_p - s)$ curve.
4. Assume that the load in the pile segment closest to the point (segment $n$) is equal to the point load.
5. Compute the compression of the segment $n$ under that load by

$$\Delta s_n = \frac{Q_p L}{AE_p}$$

where, $Q_p = q_p A_b$,

$A$ = cross-sectional area of segment,
$E_p$ = modulus of elasticity of the pile material.
6. Calculate the settlement of the top of segment $n$ by

$$s_n = s_p + \Delta s_n$$

7. Use the $(\tau - s)$ curves to read the friction $\tau_n$ on segment $n$, at displacement $s_n$.
8. Calculate the load in pile segment $(n - 1)$ by:
where \( \Delta z_n \) = length of segment \( n \),

\[ d_n = \text{average diameter of pile in segment } n \text{ (applies to tapered piles)}. \]

9. Do 4 through 8 up to the top segment. The load and displacement at the top of the pile provide one point on the load-settlement curve.

10. Repeat 1 through 9 for the other assumed values of the point pressure, \( q_p \).

\( E_p \) may also be obtained from the relationship established between \( E_p \) and the field tests such as SPT, CPT and PMT. It may be noted here that the accuracy of the results obtained depends upon the accuracy with which the values of \( E_p \) simulate the field conditions.

**Example 15.26**

A concrete pile of section 30 x 30 cm is driven into medium dense sand with the water table at ground level. The depth of embedment of the pile is 18 m. Static cone penetration test conducted at the site gives an average value \( q_c = 50 \text{ kg/cm}^2 \). Determine the load transfer curves and then calculate the settlement. The modulus of elasticity of the pile material \( E_p \) is \( 21 \times 10^4 \text{ kg/cm}^2 \) (Fig. Ex. 15.26).

**Solution**

It is first necessary to draw the \((q_p - s)\) and \((\tau - s)\) curves (see section 15.29). The curves can be constructed by determining the ratios of \( q_p/s \) and \( \tau/s \) from Eqs (15.77) and (15.76) respectively.

\[
\frac{q_p}{s} = \frac{3.125E}{2R}
\]
\[ \tau = \frac{0.22E}{s} \]

where \( R \) = radius or width of pile

The value of \( E_s \) for a driven pile may be determined from Eq. (15.78).

\[ E_s = 3(36 + 2.2q_p) \text{ kg/cm}^2 \]

\[ = 3(36 + 2.2 \times 50) = 438 \text{ kg/cm}^2 \]

Now \( \frac{q_p}{s} = \frac{3.125 \times 438}{2 \times 15} = 45.62 \text{ kg/cm}^3 \)

\[ \frac{\tau}{s} = \frac{0.22 \times 438}{2 \times 0.15} = 3.21 \text{ kg/cm}^3 \]

To construct the \((q_p-s)\) and \((\tau-s)\) curves, we have to know the maximum values of \(q_p/s\) and \(\tau\).

Given \( q_p = 50 \text{ kg/cm}^2 \) – the maximum value.

From Table 15.4 \( \tau_{\text{max}} = 0.011 \)

\( q_c = 0.011 \times 50 = 0.55 \text{ kg/cm}^2 \)

Now the theoretical maximum settlement \( s \) for \( q_p(\text{max}) = 50 \text{ kg/cm}^2 \) is

\[ s_{(\text{max})} = \frac{50}{45.62} = 1.096 \text{ cm} = 10.96 \text{ mm} \]

The curve \((q_p-s)\) may be drawn as shown in Fig. Ex. 15.26. Similarly for \( \tau_{(\text{max})} = 0.55 \text{ kg/cm}^2 \)

\[ s_{(\text{max})} = \frac{0.55}{3.21} = 0.171 \text{ cm} = 1.71 \text{ mm} \]

Now the \((\tau-s)\) curve can be constructed as shown in Fig. Ex. 15.26.

**Calculation of pile settlement**

The various steps in the calculations are

1. Divide the pile length 18 m into three equal parts of 6 m each.
2. To start with assume a base pressure \( q_b = 5 \text{ kg/cm}^2 \).
3. From the \((q_p-s)\) curve \( s_1 = 0.12 \text{ cm} \) for \( q_p = 5 \text{ kg/cm}^2 \).
4. Assume that a load \( Q_1 = Q_p = 5 \times 900 = 4500 \text{ kg} \) acts axially on segment 1.
5. Now the compression \( \Delta s_1 \) of segment 1 is

\[ \Delta s_1 = \frac{Q_1 \Delta L}{AE_p} = \frac{4500 \times 600}{30 \times 30 \times 21 \times 10^6} = 0.014 \text{ cm} \]

6. Settlement of the top of segment 1 is

\[ s_2 = s_1 + \Delta s_1 = 0.12 + 0.014 = 0.134 \text{ cm} \]

7. Now from \((\tau-s)\) curve Fig. Ex. 15.26, \( \tau = 0.43 \text{ kg/cm}^2 \).
8. The pile load in segment 2 is

\[ Q_2 = 4 \times 30 \times 600 \times 0.43 + 4500 = 30,960 + 4500 = 35,460 \text{ kg} \]

9. Now the compression of segment 2 is

\[ \Delta s_2 = \frac{Q_2 \Delta L}{AE_p} = \frac{35,460 \times 600}{900 \times 21 \times 10^6} = 0.113 \text{ cm} \]

10. Settlement of the top of segment 2 is
$s_3 = s_2 + \Delta s_2 = 0.134 + 0.113 = 0.247 \text{ cm.}$

11. Now from $(\tau - s)$ curve, Fig. Ex. 15.26 $\tau = 0.55 \text{ kg/cm}^2$ for $s_3 = 0.247 \text{ cm}$. This is the maximum shear stress.

12. Now the pile load in segment 3 is

$$Q_3 = 4 \times 30 \times 600 \times 0.55 + 35,460 = 39,600 + 35,460 = 75,060 \text{ kg}$$

13. The compression of segment 3 is

$$\Delta s_3 = \frac{Q_3 \Delta L}{AE_p} = \frac{75,060 \times 600}{900 \times 21 \times 10^4} = 0.238 \text{ cm}$$

14. Settlement of top of segment 3 is

$$s_4 = s_3 + \Delta s_3 = 0.247 + 0.238 = 0.485 \text{ cm}.$$ 

15. Now from $(\tau - s)$ curve, $\tau_{\text{max}} = 0.55 \text{ kg/cm}^2$ for $s \geq 0.17 \text{ cm}$.

16. The pile load at the top of segment 3 is

$$Q_T = 4 \times 30 \times 600 \times 0.55 + 75,060 = 39,600 + 75,060 = 114,660 \text{ kg}$$

$$= 115 \text{ tones (metric)}$$

Total pile load $Q_T = 115 \text{ tones}$. 

This yields one point on the load settlement curve for the pile. Other points can be obtained in the same way by assuming different values for the base pressure $q_p$ in Step 2 above. For accurate results, the pile should be divided into smaller segments.

### 15.30 SETTLEMENT OF PILE GROUPS IN COHESIVE SOILS

The total settlements of pile groups may be calculated by making use of consolidation settlement equations. The problem involves evaluating the increase in stress $\Delta \rho$ beneath a pile group when the group is subjected to a vertical load $Q$. The computation of stresses depends on the type of soil through which the pile passes. The methods of computing the stresses are explained below:

---

**Figure 15.31 Settlement of pile groups in clay soils**
1. The soil in the first group given in Fig. 15.31 (a) is homogeneous clay. The load $Q_z$ is assumed to act on a fictitious footing at a depth $2/3L$ from the surface and distributed over the sectional area of the group. The load on the pile group acting at this level is assumed to spread out at a 2 Vert : 1 Horiz slope. The stress $\Delta p$ at any depth $z$ below the fictitious footing may be found as explained in Chapter 6.

2. In the second group given in (b) of the figure, the pile passes through a very weak layer of depth $L_1$ and the lower portion of length $L_2$ is embedded in a strong layer. In this case, the load $Q_z$ is assumed to act at a depth equal to $2/3 L_2$ below the surface of the strong layer and spreads at a 2 : 1 slope as before.

3. In the third case shown in (c) of the figure, the piles are point bearing piles. The load in this case is assumed to act at the level of the firm stratum and spreads out at a 2 : 1 slope.

15.31 ALLOWABLE LOADS ON GROUPS OF PILES

The basic criterion governing the design of a pile foundation should be the same as that of a shallow foundation, that is, the settlement of the foundation must not exceed some permissible value. The permissible values of settlements assumed for shallow foundations in Chapter 13 are also applicable to pile foundations. The allowable load on a group of piles should be the least of the values computed on the basis of the following two criteria.

1. Shear failure,
2. Settlement.

Procedures have been given in earlier chapters as to how to compute the allowable loads on the basis of a shear failure criterion. The settlement of pile groups should not exceed the permissible limits under these loads.

Example 15.27

It is required to construct a pile foundation comprised of 20 piles arranged in 5 columns at distances of 90 cm center to center. The diameter and lengths of the piles are 30 cm and 9 m respectively. The bottom of the pile cap is located at a depth of 2.0 m from the ground surface. The details of the soil properties etc. are as given below with reference to ground level as the datum. The water table was found at a depth of 4 m from ground level.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>To</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

Compute the consolidation settlement of the pile foundation if the total load imposed on the foundation is 2500 kN.
Solution

Assume that the total load 2500 kN acts at a depth \((2/3)L = (2/3) \times 9 = 6\) m from the bottom of the pile cap on a fictitious footing as shown in Fig. 15.31(a). This fictitious footing is now at a depth of 8 m below ground level. The size of the footing is 3.9 \times 3.0\ m. Now three layers are assumed to contribute to the settlement of the foundation. They are: Layer 1—from 8 m to 12 m (= 4 m thick) below ground level; Layer 2—from 12 m to 14 m = 2 m thick; Layer 3—from 14 m to 17 m = 3 m thick. The increase in pressure due to the load on the fictitious footing at the centers of each layer is computed on the assumption that the load is spread at an angle of 2 vertical to 1 horizontal [Fig. 15.31(a)] starting from the edges of the fictitious footing. The settlement is computed by making use of the equation

\[
S_i = H_i \frac{C_c}{1 + e_o} \log \frac{p_o + \Delta p}{p_o}
\]

where \(p_o\) = the effective overburden pressure at the middle of each layer,
\(\Delta p\) = the increase in pressure at the middle of each layer

Computation of \(p_o\)

For Layer 1,

\[
p_o = 2 \times 16 + 2 \times 19.2 + (10 - 4)(19.2 - 9.81) = 126.74 \text{ kN/m}^2
\]

For Layer 2,

\[
p_o = 126.74 + 2(19.2 - 9.81) + 1 \times (18.24 - 9.81) = 153.95 \text{ kN/m}^2
\]

For Layer 3,

\[
p_o = 153.95 + 1(18.24 - 9.81) + 1.5 \times (20.0 - 9.81) = 177.67 \text{ kN/m}^2
\]

Computation of \(\Delta p\)

For Layer 1

\[
\Delta p = \frac{2500}{29.5} = 84.75 \text{ kN/m}^2
\]

For Layer 2

\[
\Delta p = \frac{2500}{71.2} = 35.1 \text{ kN/m}^2
\]

For Layer 3

\[
\Delta p = \frac{2500}{119.7} = 20.9 \text{ kN/m}^2
\]

Settlement computation

\[
\text{Layer 1}\quad S_1 = \frac{4 \times 0.23}{1 + 0.80} \log \frac{126.74 + 84.75}{126.74} = 0.113 \text{ m}
\]

\[
\text{Layer 2}\quad S_2 = \frac{2 \times 0.34}{1 + 1.08} \log \frac{153.95 + 35.1}{153.95} = 0.029 \text{ m}
\]
15.32 NEGATIVE FRICTION

Figure 15.32(a) shows a single pile and (b) a group of piles passing through a recently constructed cohesive soil fill. The soil below the fill had completely consolidated under its overburden pressure.

When the fill starts consolidating under its own overburden pressure, it develops a drag on the surface of the pile. This drag on the surface of the pile is called 'negative friction'. Negative friction

\[ S_3 = \frac{3 \times 0.2}{1 + 0.7} \log \frac{177.67 + 20.9}{177.67} = 0.017 \text{ m} \]

Total = 0.159 m = 16 cm.

---

**Figure 15.32** Negative friction on piles
may develop if the fill material is loose cohesionless soil. Negative friction can also occur when fill is placed over peat or a soft clay stratum as shown in Fig. 15.32c. The superimposed loading on such compressible stratum causes heavy settlement of the fill with consequent drag on piles.

Negative friction may develop by lowering the ground water which increases the effective stress causing consolidation of the soil with resultant settlement and friction forces being developed on the pile.

Negative friction must be allowed when considering the factor of safety on the ultimate carrying capacity of a pile. The factor of safety, $F_s$, where negative friction is likely to occur may be written as

$$F_s = \frac{\text{Ultimate carrying capacity of a single pile or group of piles}}{\text{Working load} + \text{Negative skin friction load}}$$

**Computation of Negative Friction on a Single Pile**

The magnitude of negative friction $F_n$ for a single pile in a fill may be taken as (Fig. 15.32(a)).

(a) For cohesive soils

$$F_n = PL_n s.$$  \hspace{1cm} (15.80)

(b) For cohesionless soils

$$F_n = \frac{1}{2} PL_n^2 \gamma L \tan \delta$$  \hspace{1cm} (15.81)

where $L_n = \text{length of piles in the compressible material}$,
$s = \text{shear strength of cohesive soils in the fill}$,
$P = \text{perimeter of pile}$,
$K = \text{earth pressure coefficient normally lies between the active and the passive earth pressure coefficients}$,
$\delta = \text{angle of wall friction which may vary from } \phi/2 \text{ to } \phi$.

**Negative Friction on Pile Groups**

When a group of piles passes through a compressible fill, the negative friction, $F_{ng}$, on the group may be found by any of the following methods [Fig. 15.32b].

(a) $F_{ng} = nF_n$ \hspace{1cm} (15.82)

(b) $F_{ng} = sL_n P_g + \gamma L_n A_g$ \hspace{1cm} (15.83)

where $n = \text{number of piles in the group}$,
$\gamma = \text{unit weight of soil within the pile group to a depth } L_n$,
$P_g = \text{perimeter of pile group}$,
$A_g = \text{sectional area of pile group within the perimeter } P_g$,
$s = \text{shear strength of soil along the perimeter of the group}$.

Equation (15.82) gives the negative friction forces of the group as equal to the sum of the friction forces of all the single piles.

Eq. (15.83) assumes the possibility of block shear failure along the perimeter of the group which includes the volume of the soil $\gamma L_n A_g$ enclosed in the group. The maximum value obtained from Eqs (15.82) or (15.83) should be used in the design.

When the fill is underlain by a compressible stratum as shown in Fig. 15.32(c), the total negative friction may be found as follows:
\[ F_{ng} = n(F_{n1} + F_{n2}) \]  
\[ F_{ng} = s_1 L_1 P_g + s_2 L_2 P_g + \gamma_1 L_1 A_g + \gamma_2 L_2 A_g \]
\[ = P_g (s_1 L_1 + s_2 L_2) + A_g (\gamma_1 L_1 + \gamma_2 L_2) \]

where \( L_1 \) = depth of fill,
\( L_2 \) = depth of compressible natural soil,
\( s_1, s_2 \) = shear strengths of the fill and compressible soils respectively,
\( \gamma_1, \gamma_2 \) = unit weights of fill and compressible soils respectively,
\( F_{n1} \) = negative friction of a single pile in the fill,
\( F_{n2} \) = negative friction of a single pile in the compressible soil.

The maximum value of the negative friction obtained from Eqs. (15.84) or (15.85) should be used for the design of pile groups.

**Example 15.28**

A square pile group similar to the one shown in Fig. 15.27 passes through a recently constructed fill. The depth of fill \( L_n = 3 \) m. The diameter of the pile is 30 cm and the piles are spaced 90 cm center to center. If the soil is cohesive with \( q_u = 60 \) kN/m\(^2\), and \( \gamma = 15 \) kN/m\(^3\), compute the negative frictional load on the pile group.

**Solution**

The negative frictional load on the group is the maximum of [(Eqs (15.82) and (15.83)]

(a) \[ F_{ng} = nF_n \]
and (b) \[ F_{ng} = sL_n P_g + \gamma L_g A_g \]

where \( P_g = 4 \times 3 = 12 \) m, \( A_g = 3 \times 3 = 9 \) m\(^2\), \( c_u = 60/2 = 30 \) kN/m\(^2\)

(a) \[ F_{ng} = 9 \times 3.14 \times 0.3 \times 3 = 763 \text{ kN} \]
(b) \[ F_{ng} = 30 \times 3 \times 12 + 15 \times 3 \times 9 = 1485 \text{ kN} \]

The negative frictional load on the group = 1485 kN.

**15.33 UPLIFT CAPACITY OF A PILE GROUP**

The uplift capacity of a pile group, when the vertical piles are arranged in a closely spaced groups may not be equal to the sum of the uplift resistances of the individual piles. This is because, at ultimate load conditions, the block of soil enclosed by the pile group gets lifted. The manner in which the load is transferred from the pile to the soil is quite complex. A simplified way of calculating the uplift capacity of a pile group embedded in cohesionless soil is shown in Fig. 15.33(a). A spread of load of 1 Horiz : 4 Vert from the pile group base to the ground surface may be taken as the volume of the soil to be lifted by the pile group (Tomlinson, 1977). For simplicity in calculation, the weight of the pile embedded in the ground is assumed to be equal to that of the volume of soil it displaces. If the pile group is partly or fully submerged, the submerged weight of soil below the water table has to be taken.

In the case of cohesive soil, the uplift resistance of the block of soil in undrained shear enclosed by the pile group given in Fig. 15.33(b) has to be considered. The equation for the total uplift capacity \( P_{gu} \) of the group may be expressed by

\[ P_{gu} = 2L(\bar{L} + B) \bar{c}_u + W \]  
**15.33 UPLIFT CAPACITY OF A PILE GROUP**

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In the case of cohesive soil, the uplift resistance of the block of soil in undrained shear enclosed by the pile group given in Fig. 15.33(b) has to be considered. The equation for the total uplift capacity \( P_{gu} \) of the group may be expressed by

\[ P_{gu} = 2L(\bar{L} + B) \bar{c}_u + W \]
where \( L \) = depth of the pile block
\( \bar{L} \) and \( \bar{B} \) = overall length and width of the pile group
\( c_u \) = average undrained shear strength of soil around the sides of the group
\( W \) = combined weight of the block of soil enclosed by the pile group plus the weight of the piles and the pile cap.

A factor of safety of 2 may be used in both cases of piles in sand and clay.

The uplift efficiency \( E_{gu} \) of a group of piles may be expressed as

\[
E_{gu} = \frac{P_{gu}}{nP_{us}}
\]  

(15.89)

where \( P_{us} \) = uplift capacity of a single pile
\( n \) = number of piles in the group

The efficiency \( E_{gu} \) varies with the method of installation of the piles, length and spacing and the type of soil. The available data indicate that \( E_{gu} \) increases with the spacing of piles. Meyerhof and Adams (1968) presented some data on uplift efficiency of groups of two and four model circular...
footings in clay. The results indicate that the uplift efficiency increases with the spacing of the footings or bases and as the depth of embedment decreases, but decreases as the number of footings or bases in the group increases. How far the footings would represent the piles is a debatable point. For uplift loading on pile groups in sand, there appears to be little data from full scale field tests.

15.34 PROBLEMS

15.1 A 45 cm diameter pipe pile of length 12 m with closed end is driven into a cohesionless soil having $\phi = 35^\circ$. The void ratio of the soil is 0.48 and $G_s = 2.65$. The water table is at the ground surface. Estimate (a) the ultimate base load $Q_b$, (b) the frictional load $Q_f$, and (c) the allowable load $Q_a$ with $F_s = 2.5$.

Use the Berezantsev method for estimating $Q_b$. For estimating $Q_f$ use $K_s = 0.75$ and $\theta = 20^\circ$.

15.2 Refer to Problem 15.1. Compute $Q_b$ by Meyerhof’s method. Determine $Q_f$ using the critical depth concept, and $Q_a$ with $F_s = 2.5$. All the other data given in Prob. 15.1 remain the same.

15.3 Estimate $Q_b$ by Vesic’s method for the pile given in Prob. 15.1. Assume $I_r = I_r = 60$. Determine $Q_a$ for $F_s = 2.5$ and use $Q_f$ obtained in Prob. 15.1.

15.4 For Problem 15.1, estimate the ultimate base resistance $Q_b$ by Janbu’s method. Determine $Q_a$ with $F_s = 2.5$. Use $Q_f$ obtained in Prob. 15.1. Use $\psi = 90^\circ$.

15.5 For Problem 15.1, estimate $Q_b$, $Q_f$, and $Q_a$ by Coyle and Castello method. All the data given remain the same.

15.6 For problem 15.1, determine $Q_b$, $Q_f$, and $Q_a$ by Meyerhof’s method using the relationship between $N_{cor}$ and $\phi$ given in Fig 12.8.

---

Figure Prob. 15.11
15.7 A concrete pile 40 cm in diameter is driven into homogeneous sand extending to a great depth. Estimate the ultimate load bearing capacity and the allowable load with $F_s = 3.0$ by Coyle and Castello’s method. Given: $L = 15$ m, $\phi = 36^\circ$, $\gamma = 18.5$ kN/m$^3$.

15.8 Refer to Prob. 15.7. Estimate the allowable load by Meyerhof’s method using the relationship between $\phi$ and $N_{cor}$ given in Fig. 12.8.

15.9 A concrete pile of 15 in. diameter, 40 ft long is driven into a homogeneous stratum of clay with the water table at ground level. The clay is of medium stiff consistency with the undrained shear strength $c_u = 600$ lb/ft$^2$. Compute $Q_a$ by Skempton’s method and $Q_f$ by the $\alpha$-method. Determine $Q_a$ for $F_s = 2.5$.

15.10 Refer to Prob 15.9. Compute $Q_f$ by the $\lambda$-method. Determine $Q_a$ by using $Q_a$ computed in Prob. 15.9. Assume $\gamma_{sat} = 120$ lb/ft$^3$.

15.11 A pile of 40 cm diameter and 18.5 m long passes through two layers of clay and is embedded in a third layer. Fig. Prob. 15.11 gives the details of the soil system. Compute $Q_f$ by the $\alpha$-method and $Q_a$ by Skempton’s method. Determine $Q_a$ for $F_s = 2.5$.

15.12 A concrete pile of size 16 x 16 in. is driven into a homogeneous clay soil of medium consistency. The water table is at ground level. The undrained shear strength of the soil is 500 lb/ft$^2$. Determine the length of pile required to carry a safe load of 50 kips with $F_s = 3$. Use the $\alpha$-method.

15.13 Refer to Prob. 15.12. Compute the required length of pile by the $\lambda$-method. All the other data remain the same. Assume $\gamma_{sat} = 120$ lb/ft$^3$.

15.14 A concrete pile 50 cm in diameter is driven into a homogeneous mass of cohesionless soil. The pile is required to carry a safe load of 700 kN. A static cone penetration test conducted at the site gave an average value of $q_c = 35$ kg/cm$^2$ along the pile and 60 kg/cm$^2$ below the base of the pile. Compute the length of the pile with $F_s = 3$.

15.15 Refer to Problem 15.14. If the length of the pile driven is restricted to 12 m, estimate the ultimate load $Q_u$ and safe load $Q_a$ with $F_s = 3$. All the other data remain the same.

15.16 A reinforced concrete pile 20 in. in diameter penetrates 40 ft into a stratum of clay and rests on a medium dense sand stratum. Estimate the ultimate load. Given: for sand- $\phi = 35^\circ$, $\gamma_{sat} = 120$ lb/ft$^3$ for clay $\gamma_{sat} = 119$ lb/ft$^3$, $c_u = 800$ lb/ft$^2$. Use (a) the $\alpha$-method for computing the frictional load, (b) Meyerhof’s method for estimating $Q_a$. The water table is at ground level.

15.17 A ten-story building is to be constructed at a site where the water table is close to the ground surface. The foundation of the building will be supported on 30 cm diameter pipe piles. The bottom of the pile cap will be at a depth of 1.0 m below ground level. The soil investigation at the site and laboratory tests have provided the saturated unit weights, the shear strength values under undrained conditions (average), the corrected SPT values, and the soil profile of the soil to a depth of about 40 m. The soil profile and the other details are given below.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil</th>
<th>$\gamma_{sat}$ kN/m$^3$</th>
<th>$N_{cor}$</th>
<th>$\phi$ (average)</th>
<th>$c$ (average) kN/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>To</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>Sand</td>
<td>19</td>
<td>18</td>
<td>33$^\circ$</td>
</tr>
<tr>
<td>6.0</td>
<td>22</td>
<td>Med. stiff clay</td>
<td>18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>22</td>
<td>30</td>
<td>sand</td>
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<td>25</td>
<td>35$^\circ$</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
<td>stiff clay</td>
<td>18.5</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>
Determine the ultimate bearing capacity of a single pile for lengths of (a) 15 m, and (b) 25 m below the bottom of the cap.
Use $\alpha = 0.50$ and $K_s = 1.2$. Assume $\phi = 0.8$. 

15.18 For a pile designed for an allowable load of 400 kN driven by a steam hammer (single acting) with a rated energy of 2070 kN-cm, what is the approximate terminal set of the pile using the ENR formula?

15.19 A reinforced concrete pile of 40 cm diameter and 25 m long is driven through medium dense sand to a final set of 2.5 mm, using a 40 kN single - acting hammer with a stroke of 150 cm. Determine the ultimate driving resistance of the pile if it is fitted with a helmet, plastic dolly and 50 mm packing on the top of the pile. The weight of the helmet, with dolly is 4.5 kN. The other particulars are: weight of pile = 85 kN, weight of hammer 35 kN; pile hammer efficiency $\eta_k = 0.85$; the coefficient restitution $C_r = 0.45$. Use Hiley’s formula. The sum of elastic compression $C = c_1 + c_2 + c_3 = 20.1$ mm.

15.20 A reinforced concrete pile 45 ft long and 20 in. in diameter is driven into a stratum of homogeneous saturated clay having $c_u = 800$ lb/ft$^2$. Determine (a) the ultimate load capacity and the allowable load with $F_s = 3$; (b) the pullout capacity and the allowable pullout load with $F_s = 3$. Use the $\alpha$-method for estimating the compression load.

15.21 Refer to Prob. 15.20. If the pile is driven to medium dense sand, estimate (a) the ultimate compression load and the allowable load with $F_s = 3$, and (b) the pullout capacity and the allowable pullout load with $F_s = 3$. Use the Coyle and Castello method for computing $Q_b$ and $Q_p$. The other data available are: $\phi = 36^\circ$, and $\gamma = 115$ lb/ft$^3$. Assume the water table is at a great depth.

15.22 A group of nine friction piles arranged in a square pattern is to be proportioned in a deposit of medium stiff clay. Assuming that the piles are 30 cm diameter and 10 m long, find the optimum spacing for the piles. Assume $\alpha = 0.8$ and $c_u = 50$ kN/m$^3$.

15.23 A group of 9 piles with 3 in a row was driven into sand at a site. The diameter and length of the piles are 30 cm and 12 m respectively. The properties of the soil are: $\phi = 30^\circ$, $e = 0.7$, and $G_s = 2.64$. If the spacing of the piles is 90 cm, compute the allowable load on the pile group on the basis of shear failure for $F_s = 2.0$ with respect to skin resistance, and $F_s = 2.5$ with respect to base resistance. For $\phi = 30^\circ$, assume $N_q = 22.5$ and $N_y = 19.7$. The water table is at ground level.

15.24 Nine RCC piles of diameter 30 cm each are driven in a square pattern at 90 cm center to center to a depth of 12 m into a stratum of loose to medium dense sand. The bottom of the pile cap embedding all the piles rests at a depth of 1.5 m below the ground surface. At a depth of 15 m lies a clay stratum of thickness 3 m and below which lies sandy strata. The liquid limit of the clay is 45%. The saturated unit weights of sand and clay are 18.5 kN/m$^3$ and 19.5 kN/m$^3$ respectively. The initial void ratio of the clay is 0.65. Calculate the consolidation settlement of the pile group under the allowable load. The allowable load $Q_a = 120$ kN.

15.25 A square pile group consisting of 16 piles of 40 cm diameter passes through two layers of compressible soils as shown in Fig. 15.32(c). The thicknesses of the layers are: $L_1 = 2.5$ m and $L_2 = 3$ m. The piles are spaced at 100 cm center to center. The properties of the fill material are: top fill $c_u = 25$ kN/m$^2$; the bottom fill (peat), $c_u = 30$ kN/m$^2$. Assume $\gamma = 14$ kN/m$^3$ for both the fill materials. Compute the negative frictional load on the pile group.