CHAPTER 16

DEEP FOUNDATION II: BEHAVIOR OF LATERALLY LOADED VERTICAL AND BATTER PILES

16.1 INTRODUCTION

When a soil of low bearing capacity extends to a considerable depth, piles are generally used to transmit vertical and lateral loads to the surrounding soil media. Piles that are used under tall chimneys, television towers, high rise buildings, high retaining walls, offshore structures, etc. are normally subjected to high lateral loads. These piles or pile groups should resist not only vertical movements but also lateral movements. The requirements for a satisfactory foundation are,

1. The vertical settlement or the horizontal movement should not exceed an acceptable maximum value,
2. There must not be failure by yield of the surrounding soil or the pile material.

Vertical piles are used in foundations to take normally vertical loads and small lateral loads. When the horizontal load per pile exceeds the value suitable for vertical piles, *batter piles* are used in combination with vertical piles. Batter piles are also called *inclined piles* or *raker piles*. The degree of batter, is the angle made by the pile with the vertical, may up to 30°. If the lateral load acts on the pile in the direction of batter, it is called an *in-batter* or *negative batter* pile. If the lateral load acts in the direction opposite to that of the batter, it is called an *out-batter* or *positive batter* pile. Fig. 16.1a shows the two types of batter piles.

Extensive theoretical and experimental investigation has been conducted on single vertical piles subjected to lateral loads by many investigators. Generalized solutions for laterally loaded vertical piles are given by Matlock and Reese (1960). The effect of vertical loads in addition to lateral loads has been evaluated by Davisson (1960) in terms of non-dimensional parameters. Broms (1964a, 1964b) and Poulos and Davis (1980) have given different approaches for solving laterally loaded pile problems. Brom's method is ingenious and is based primarily on the use of

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limiting values of soil resistance. The method of Poulos and Davis is based on the theory of elasticity.

The finite difference method of solving the differential equation for a laterally loaded pile is very much in use where computer facilities are available. Reese et al., (1974) and Matlock (1970) have developed the concept of (p-y) curves for solving laterally loaded pile problems. This method is quite popular in the USA and in some other countries.

However, the work on batter piles is limited as compared to vertical piles. Three series of tests on single 'in' and 'out' batter piles subjected to lateral loads have been reported by Matsuo (1939). They were run at three scales. The small and medium scale tests were conducted using timber piles embedded in sand in the laboratory under controlled density conditions. Loos and Breth (1949) reported a few model tests in dry sand on vertical and batter piles. Model tests to determine the effect of batter on pile load capacity have been reported by Tschebotarioff (1953), Yoshimi (1964), and Awad and Petrasovits (1968). The effect of batter on deflections has been investigated by Kubo (1965) and Awad and Petrasovits (1968) for model piles in sand.

Full-scale field tests on single vertical and batter piles, and also groups of piles, have been made from time to time by many investigators in the past. The field test values have been used mostly to check the theories formulated for the behavior of vertical piles only. Murthy and Subba Rao (1995) made use of field and laboratory data and developed a new approach for solving the laterally loaded pile problem.

Reliable experimental data on batter piles are rather scarce compared to that of vertical piles. Though Kubo (1965) used instrumented model piles to study the deflection behavior of batter piles, his investigation in this field was quite limited. The work of Awad and Petrasovits (1968) was based on non-instrumented piles and as such does not throw much light on the behavior of batter piles.

The author (Murthy, 1965) conducted a comprehensive series of model tests on instrumented piles embedded in dry sand. The batter used by the author varied from –45° to +45°. A part of the author's study on the behavior of batter piles, based on his own research work, has been included in this chapter.

16.2 WINKLER'S HYPOTHESIS

Most of the theoretical solutions for laterally loaded piles involve the concept of *modulus of subgrade reaction* or otherwise termed as *soil modulus* which is based on Winkler's assumption that a soil medium may be approximated by a series of closely spaced independent elastic springs. Fig. 16.1(b) shows a loaded beam resting on a elastic foundation. The reaction at any point on the base of the beam is actually a function of every point along the beam since soil material exhibits varying degrees of continuity. The beam shown in Fig. 16.1(b) can be replaced by a beam in Fig. 16.1(c). In this figure the beam rests on a bed of elastic springs wherein each spring is independent of the other. According to Winkler's hypothesis, the reaction at any point on the base of the beam in Fig. 16.1(c) depends only on the deflection at that point. Vesic (1961) has shown that the error inherent in Winkler's hypothesis is not significant.

The problem of a laterally loaded pile embedded in soil is closely related to the beam on an elastic foundation. A beam can be loaded at one or more points along its length, whereas in the case of piles the external loads and moments are applied at or above the ground surface only.

The nature of a laterally loaded pile-soil system is illustrated in Fig. 16.1(d) for a vertical pile. The same principle applies to batter piles. A series of nonlinear springs represents the force-deformation characteristics of the soil. The springs attached to the blocks of different sizes indicate reaction increasing with deflection and then reaching a yield point, or a limiting value that depends on depth; the taper on the springs indicates a nonlinear variation of load with deflection. The gap between the pile and the springs indicates the molding away of the soil by repeated loadings and the

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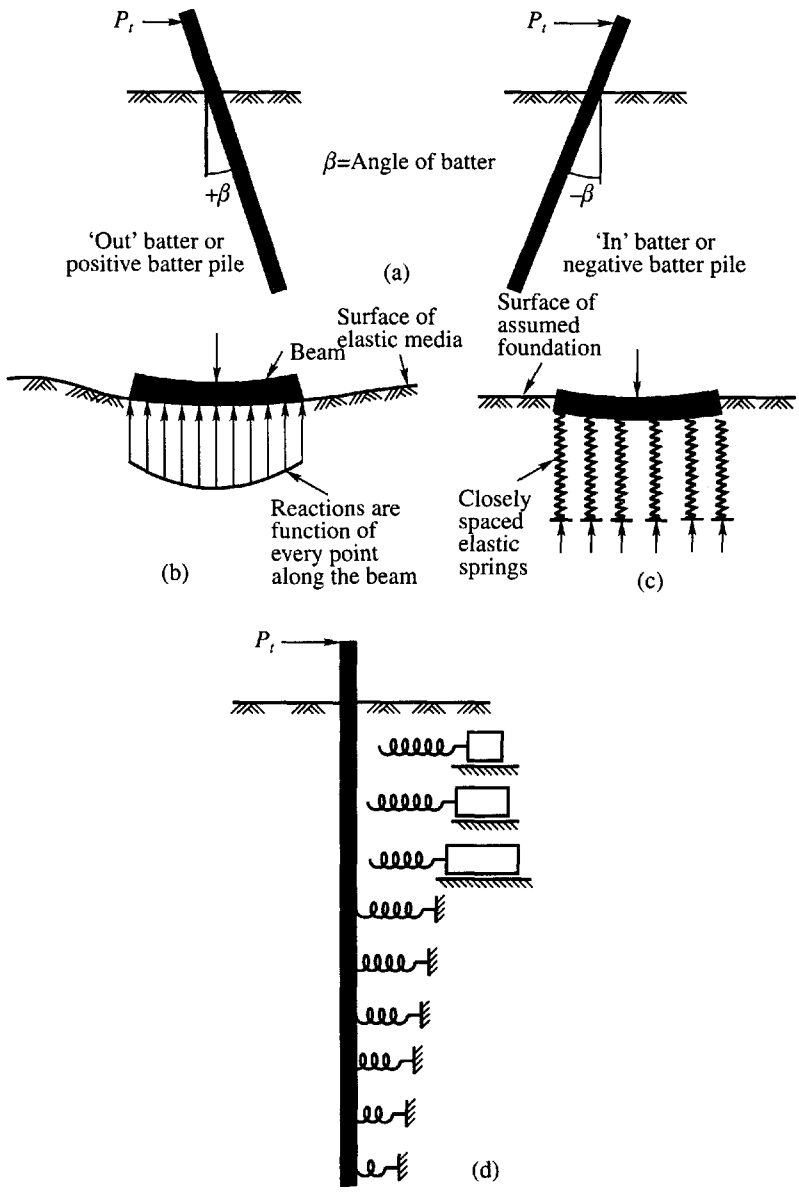

Figure 16.1 (a) Batter piles, (b, c) Winkler's hypothesis and (d) the concept of laterally loaded pile-soil system

increasing stiffness of the soil is shown by shortening of the springs as the depth below the surface increases.

16.3 THE DIFFERENTIAL EQUATION

Compatibility

As stated earlier, the problem of the laterally loaded pile is similar to the beam-on-elastic foundation problem. The interaction between the soil and the pile or the beam must be treated

quantitatively in the problem solution. The two conditions that must be satisfied for a rational analysis of the problem are,

1. Each element of the structure must be in equilibrium and
2. Compatibility must be maintained between the superstructure, the foundation and the supporting soil.

If the assumption is made that the structure can be maintained by selecting appropriate boundary conditions at the top of the pile, the remaining problem is to obtain a solution that insures equilibrium and compatibility of each element of the pile, taking into account the soil response along the pile. Such a solution can be made by solving the differential equation that describes the pile behavior.

The Differential Equation of the Elastic Curve

The standard differential equations for slope, moment, shear and soil reaction for a beam on an elastic foundation are equally applicable to laterally loaded piles.

The deflection of a point on the elastic curve of a pile is given by y. The x-axis is along the pile axis and deflection is measured normal to the pile-axis.

The relationships between y, slope, moment, shear and soil reaction at any point on the deflected pile may be written as follows.

deflection of the pile = y

slope of the deflected pile $S = \dfrac{dy}{dx}$ (16.1)

moment of pile $M = EI\dfrac{d^2y}{dx^2}$ (16.2)

shear $V = EI\dfrac{d^3y}{dx^3}$ (16.3)

soil reaction, $p = EI\dfrac{d^4y}{dx^4}$ (16.4)

where EI is the flexural rigidity of the pile material.

The soil reaction p at any point at a distance x along the axis of the pile may be expressed as

$$p = -E_s y \tag{16.5}$$

where y is the deflection at point x, and E_s is the soil *modulus*. Eqs (16.4) and (16.5) when combined gives

$$EI\frac{d^4y}{dx^4} + E_s y = 0 \tag{16.6}$$

which is called the *differential equation* for the *elastic curve* with zero axial load.

The key to the solution of laterally loaded pile problems lies in the determination of the value of the modulus of subgrade reaction (soil modulus) with respect to depth along the pile. Fig. 16.2(a) shows a vertical pile subjected to a lateral load at ground level. The deflected position of the pile and the corresponding soil reaction curve are also shown in the same figure. The soil modulus E_s at any point x below the surface along the pile as per Eq. (16.5) is

$$E_s = -\frac{p}{y} \tag{16.7}$$

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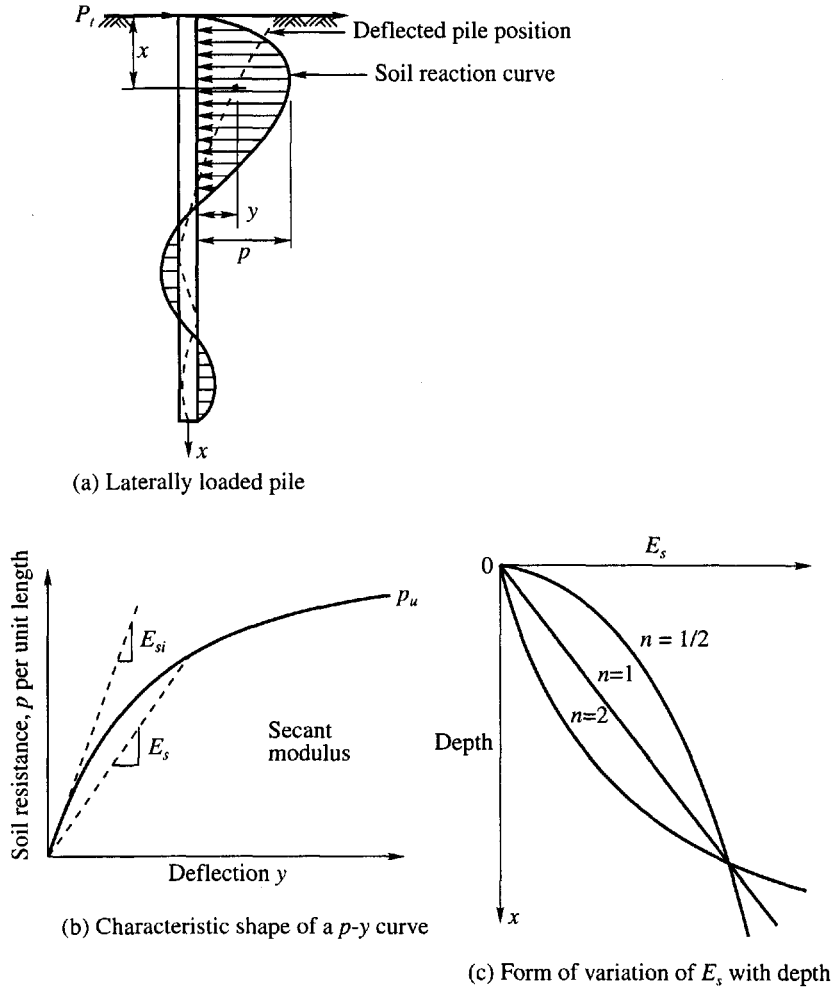

Figure 16.2 The concept of (p-y) curves: (a) a laterally loaded pile, (b) characteristic shape of a p–y curve, and (c) the form of variation of E_s with depth

As the load P_t at the top of the pile increases the deflection y and the corresponding soil reaction p increase. A relationship between p and y at any depth x may be established as shown in Fig. 16.2(b). It can be seen that the curve is strongly non-linear, changing from an initial tangent modulus E_{si} to an ultimate resistance p_u. E_s is not a constant and changes with deflection.

There are many factors that influence the value of E_s such as the pile width d, the flexural stiffness EI, the magnitude of loading P_t and the soil properties.

The variation of E_s with depth for any particular load level may be expressed as

$$E_s = n_h x^n \tag{16.8a}$$

in which n_h is termed the *coefficient of soil modulus variation.* The value of the power n depends upon the type of soil and the batter of the pile. Typical curves for the form of variation of E_s with depth for values of n equal to 1/2, 1, and 2 are given 16.2(c). The most useful form of variation of E_s is the linear relationship expressed as

$$E_s = n_h x \tag{16.8b}$$

which is normally used by investigators for vertical piles.

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Table 16.1 Typical values of n_h for cohesive soils (Taken from Poulos and Davis, 1980)

Soil type	n_h lb/in^3	Reference
Soft NC clay	0.6 to 12.7	Reese and Matlock, 1956
	1.0 to 2.0	Davisson and Prakash, 1963
NC organic clay	0.4 to 1.0	Peck and Davisson, 1962
	0.4 to 3.0	Davisson, 1970
Peat	0.2	Davisson, 1970
	0.1 to 0.4	Wilson and Hills, 1967
Loess	29 to 40	Bowles, 1968

Table 16.1 gives some typical values for cohesive soils for n_h and Fig. 16.3 gives the relationship between n_h and the relative density of sand (Reese, 1975).

16.4 NON-DIMENSIONAL SOLUTIONS FOR VERTICAL PILES SUBJECTED TO LATERAL LOADS

Matlock and Reese (1960) have given equations for the determination of y, S, M, V, and p at any point x along the pile based on dimensional analysis. The equations are

deflection, $$y = \left[\frac{P_t T^3}{EI}\right] A_y + \left[\frac{M_t T^2}{EI}\right] B_y \tag{16.9}$$

slope, $$S = \left[\frac{P_t T^2}{EI}\right] A_s + \left[\frac{M_t T}{EI}\right] B_s \tag{16.10}$$

moment, $$M = \left[P_t T\right] A_m + \left[M_t\right] B_m \tag{16.11}$$

shear, $$V = \left[P_t\right] A_v + \left[\frac{M_t}{T}\right] B_v \tag{16.12}$$

soil reaction, $$p = \frac{P_t}{T} A_p + \frac{M_t}{T^2} B_p \tag{16.13}$$

where T is the relative stiffness factor expressed as

$$T = \left[\frac{EI}{n_h}\right]^{\frac{1}{5}} \tag{16.14a}$$

for $E_s = n_h x$

For a general case $$T = \left[\frac{EI}{n_h}\right]^{\frac{1}{n+4}} \tag{16.14b}$$

In Eqs (16.9) through (16.13), A and B are the sets of non-dimensional coefficients whose values are given in Table 16.2. The principle of superposition for the deflection of a laterally loaded

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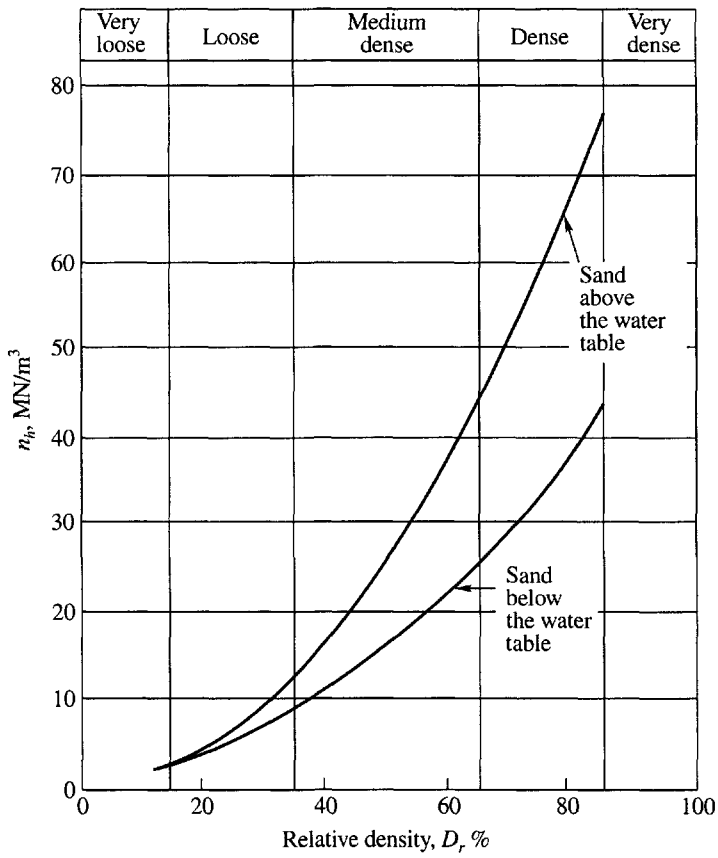

Figure 16.3 Variation of n_h with relative density (Reese, 1975)

pile is shown in Fig. 16.4. The A and B coefficients are given as a function of the depth coefficient, Z, expressed as

$$Z = \frac{x}{T} \tag{16.14c}$$

The A and B coefficients tend to zero when the depth coefficient Z is equal to or greater than 5 or otherwise the length of the pile is more than $5T$. Such piles are called long or flexible piles. The length of a pile loses its significance beyond $5T$.

Normally we need deflection and slope at ground level. The corresponding equations for these may be expressed as

$$y_g = 2.43\frac{P_t T^3}{EI} + 1.62\frac{M_t T^2}{EI} \tag{16.15a}$$

$$S_g = 1.62\frac{P_t T^2}{EI} + 1.75\frac{M_t T}{EI} \tag{16.15b}$$

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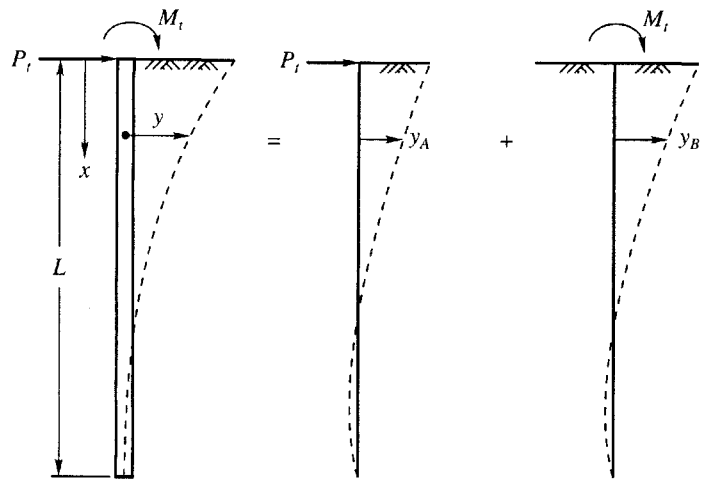

Figure 16.4 Principle of superposition for the deflection of laterally loaded piles

y_g for fixed head is

$$y_g = 0.93\frac{P_t T^3}{EI} \tag{16.16a}$$

Moment at ground level for fixed head is

$$M_t = -0.93[P_t T] \tag{16.16b}$$

16.5 *p-y* CURVES FOR THE SOLUTION OF LATERALLY LOADED PILES

Section 16.4 explains the methods of computing deflection, slope, moment, shear and soil reaction by making use of equations developed by non-dimensional methods. The prediction of the various curves depends primarily on the single parameter n_h. If it is possible to obtain the value of n_h independently for each stage of loading P_t, the *p-y* curves at different depths along the pile can be constructed as follows:

1. Determine the value of n_h for a particular stage of loading P_t.
2. Compute T from Eq. (16.14a) for the linear variation of E_s with depth.
3. Compute y at specific depths $x = x_1$, $x = x_2$, etc. along the pile by making use of Eq. (16.9), where A and B parameters can be obtained from Table 16.2 for various depth coefficients Z.
4. Compute p by making use of Eq. (16.13), since T is known, for each of the depths $x = x_1$, $x = x_2$, etc.
5. Since the values of p and y are known at each of the depths x_1, x_2 etc., one point on the *p-y* curve at each of these depths is also known.
6. Repeat steps 1 through 5 for different stages of loading and obtain the values of p and y for each stage of loading and plot to determine *p-y* curves at each depth.

The individual *p-y* curves obtained by the above procedure at depths x_1, x_2, etc. can be plotted on a common pair of axes to give a family of curves for the selected depths below the surface. The *p-y* curve shown in Fig. 16.2b is strongly non-linear and this curve can be predicted only if the

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Table 16.2 The A and B coefficients as obtained by Reese and Matlock (1956) for long vertical piles on the assumption $E_s = n_h x$

Z	A_y	A_s	A_m	A_v	A_p
0.0	2.435	−1.623	0.000	1.000	0.000
0.1	2.273	−1.618	0.100	0.989	−0.227
0.2	2.112	−1.603	0.198	0.966	−0.422
0.3	1.952	−1.578	0.291	0.906	−0.586
0.4	1.796	−1.545	0.379	0.840	−0.718
0.5	1.644	−1.503	0.459	0.764	−0.822
0.6	1.496	−1.454	0.532	0.677	−0.897
0.7	1.353	−1.397	0.595	0.585	−0.947
0.8	1.216	−1.335	0.649	0.489	−0.973
0.9	1.086	−1.268	0.693	0.392	−0.977
1.0	0.962	−1.197	0.727	0.295	−0.962
1.2	0.738	−1.047	0.767	0.109	−0.885
1.4	0.544	−0.893	0.772	−0.056	−0.761
1.6	0.381	−0.741	0.746	−0.193	−0.609
1.8	0.247	−0.596	0.696	−0.298	−0.445
2.0	0.142	−0.464	0.628	−0.371	−0.283
3.0	−0.075	−0.040	0.225	−0.349	0.226
4.0	−0.050	0.052	0.000	−0.016	0.201
5.0	−0.009	0.025	−0.033	0.013	0.046
Z	B_y	B_s	B_m	B_v	B_p
0.0	1.623	−1.750	1.000	0.000	0.000
0.1	1.453	−1.650	1.000	−0.007	−0.145
0.2	1.293	−1.550	0.999	−0.028	−0.259
0.3	1.143	−1.450	0.994	−0.058	−0.343
0.4	1.003	−1.351	0.987	−0.095	−0.401
0.5	0.873	−1.253	0.976	−0.137	−0.436
0.6	0.752	−1.156	0.960	−0.181	−0.451
0.7	0.642	−1.061	0.939	−0.226	−0.449
0.8	0.540	−0.968	0.914	−0.270	−0.432
0.9	0.448	−0.878	0.885	−0.312	−0.403
1.0	0.364	−0.792	0.852	−0.350	−0.364
1.2	0.223	−0.629	0.775	−0.414	−0.268
1.4	0.112	−0.482	0.668	−0.456	−0.157
1.6	0.029	−0.354	0.594	−0.477	−0.047
1.8	−0.030	−0.245	0.498	−0.476	0.054
2.0	−0.070	−0.155	0.404	−0.456	0.140
3.0	−0.089	0.057	0.059	−0.0213	0.268
4.0	−0.028	0.049	0.042	0.017	0.112
5.0	0.000	0.011	0.026	0.029	−0.002

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values of n_h are known for each stage of loading. Further, the curve can be extended until the soil reaction, p, reaches an ultimate value, p_u, at any specific depth x below the ground surface.

If n_h values are not known to start with at different stages of loading, the above method cannot be followed. Supposing p-y curves can be constructed by some other independent method, then p-y curves are the starting points to obtain the curves of deflection, slope, moment and shear. This means we are proceeding in the reverse direction in the above method. The methods of constructing p-y curves and predicting the non-linear behavior of laterally loaded piles are beyond the scope of this book. This method has been dealt with in detail by Reese (1985).

Example 16.1

A steel pipe pile of 61 cm outside diameter with a wall thickness of 2.5 cm is driven into loose sand (D_r = 30%) under submerged conditions to a depth of 20 m. The submerged unit weight of the soil is 8.75 kN/m^3 and the angle of internal friction is 33°. The EI value of the pile is 4.35×10^{11} kg-cm^2 (4.35×10^2 MN-m^2). Compute the ground line deflection of the pile under a lateral load of 268 kN at ground level under a free head condition using the non-dimensional parameters of Matlock and Reese. The n_h value from Fig. 16.3 for D_r = 30% is 6 MN/m^3 for a submerged condition.

Solution

From Eq. (16.15a)

$$y_g = 2.43\frac{P_t T^3}{EI} \text{ for } M_t = 0$$

From Eq. (16.14a),

$$T = \frac{EI}{n_h}^{\frac{1}{5}}$$

where, P_t = 0.268 MN

$EI = 4.35 \times 10^2$ MN-m^2

n_h = 6 MN/m^3

$$T = \frac{4.35 \times 10^2}{6}^{\frac{1}{5}} = 2.35 \text{ m}$$

$$\text{Now } y_g = \frac{2.43 \times 0.268 \times (2.35)^3}{4.35 \times 10^2} = 0.0194 \text{ m} = 1.94 \text{ cm}$$

Example 16.2

If the pile in Ex. 16.1 is subjected to a lateral load at a height 2 m above ground level, what will be the ground line deflection?

Solution

From Eq. (16.15a)

$$y_g = 2.43\frac{P_t T^3}{EI} + 1.62\frac{M_t T^2}{EI}$$

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As in Ex. 16.1 $T = 2.35$ m, $M_t = 0.268 \times 2 = 0.536$ MN-m

Substituting, $$y_g = \frac{2.43 \times 0.268 \times (2.35)^3}{4.35 \times 10^2} + \frac{1.62 \times 0.536 \times (2.35)^2}{4.35 \times 10^2}$$

$$= 0.0194 + 0.0110 = 0.0304 \text{ m} = 3.04 \text{ cm}.$$

Example 16.3

If the pile in Ex. 16.1 is fixed against rotation, calculate the deflection at the ground line.

Solution

Use Eq. (16.16a)

$$y_g = \frac{0.93 P_t T^3}{EI}$$

The values of P_t, T and El are as given in Ex. 16.1. Substituting these values

$$y_g = \frac{0.93 \times 0.268 \times (2.35)^3}{4.35 \times 10^2} = 0.0075 \text{ m} = 0.75 \text{ cm}$$

16.6 BROMS' SOLUTIONS FOR LATERALLY LOADED PILES

Broms' (1964a, 1964b) solutions for laterally loaded piles deal with the following:

1. Lateral deflections of piles at ground level at working loads
2. Ultimate lateral resistance of piles under lateral loads

Broms' provided solutions for both short and long piles installed in cohesive and cohesionless soils respectively. He considered piles fixed or free to rotate at the head. Lateral deflections at working loads have been calculated using the concept of subgrade reaction. It is assumed that the deflection increases linearly with the applied loads when the loads applied are less than one-half to one-third of the ultimate lateral resistance of the pile.

Lateral Deflections at Working Loads

Lateral deflections at working loads can be obtained from Fig. 16.5 for cohesive soil and Fig. 16.6 for cohesionless soils respectively. For piles in saturated cohesive soils, the plot in Fig. 16.5 gives the relationships between the dimensionless quantity βL and $(y_o kdL)/P_t$ for free-head and restrained piles, where

$$\beta = \left(\frac{kd}{4EI}\right)^{1/4} \tag{16.17}$$

EI = stiffness of pile section
k = coefficient of horizontal subgrade reaction
d = width or diameter of pile
L = length of pile

A pile is considered long or short on the following conditions

Free-head Pile

Long pile when $\beta L > 2.50$
Short pile when $\beta L < 2.50$

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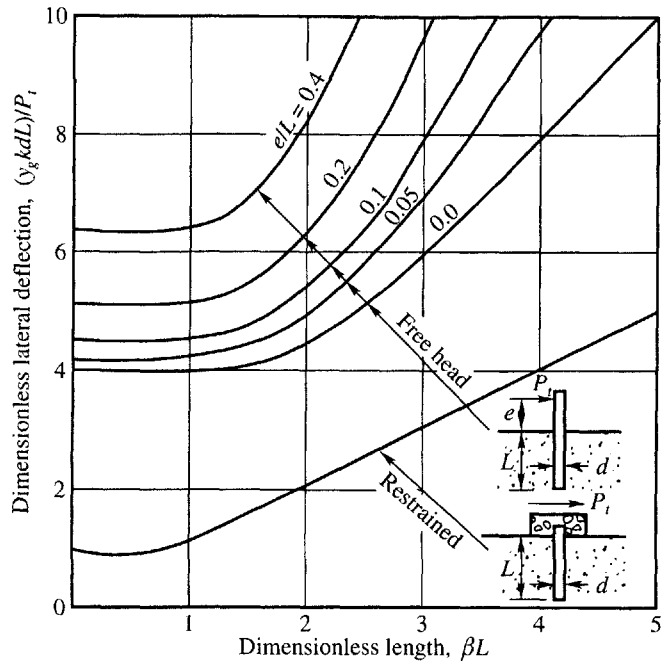

Figure 16.5 Charts for calculating lateral deflection at the ground surface of horizontally loaded pile in cohesive soil (after Broms 1964a)

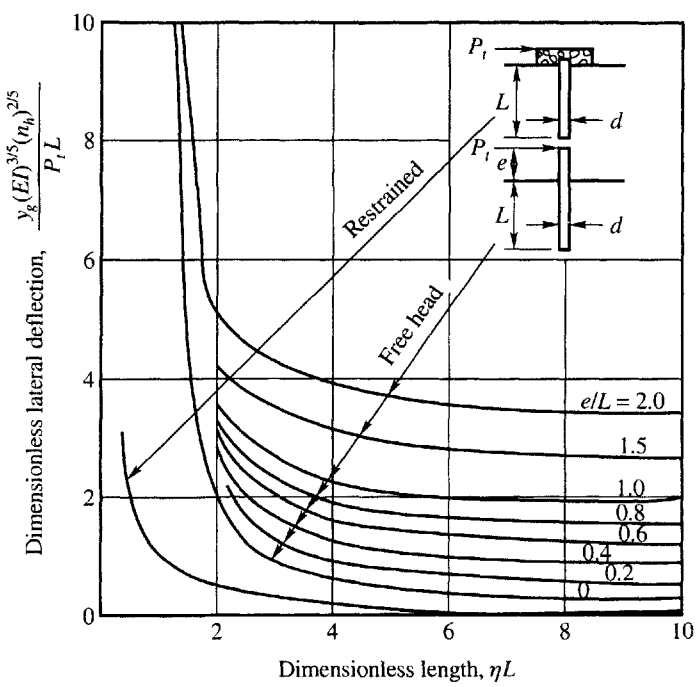

Figure 16.6 Charts for calculating lateral deflection at the ground surface of horizontally loaded piles in cohesionless soil (after Broms 1964b)

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Fixed-head Pile

Long pile when $\beta L > 1.5$

Short pile when $\beta L < 1.5$

Tomlinson (1977) suggests that it is sufficiently accurate to take the value of k in Eq. (16.17) as equal to k_1 given in Table 14.1(b).

Lateral deflections at working loads of piles embedded in cohesionless soils may be obtained from Fig. 16.6 Non-dimensionless factor $[y_g(EI)^{3/5}(n_h)^{2/5}]/P_tL$ is plotted as a function of ηL for various values of e/L

where y_g = deflection at ground level

$$\eta = \left(\frac{n_h}{EI}\right)^{1/5} \tag{16.18}$$

n_h = coefficient of soil modulus variation

P_t = lateral load applied at or above ground level

L = length of pile

e = eccentricity of load.

Ultimate Lateral Resistance of Piles in Saturated Cohesive Soils

The ultimate soil resistance of piles in cohesive soils increases with depth from $2c_u$ (c_u = undrained shear strength) to 8 to 12 c_u at a depth of three pile diameters (3d) below the surface. Broms (1964a) suggests a constant value of $9c_u$ below a depth of $1.5d$ as the ultimate soil resistance. Figure 16.7 gives solutions for short piles and Fig. 16.8 for long piles. The solution for long piles

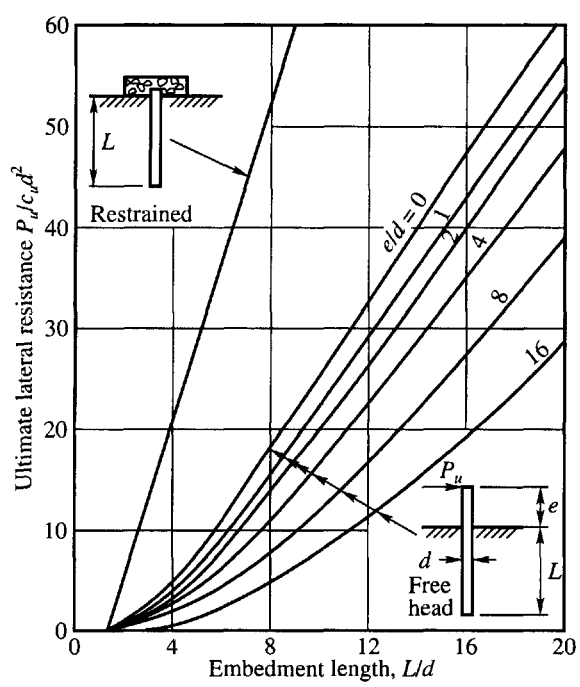

Figure 16.7 Ultimate lateral resistance of a short pile in cohesive soil related to embedded length (after Broms (1964a))

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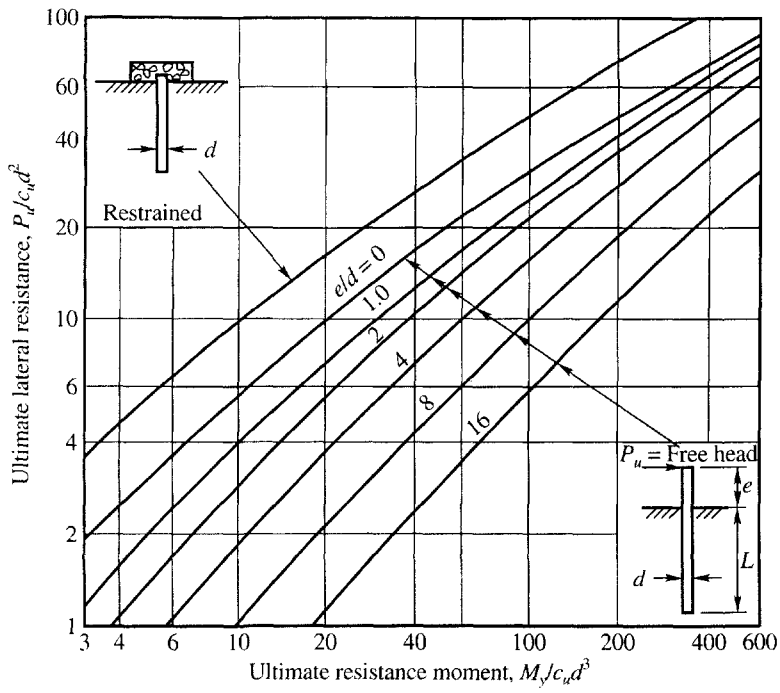

Figure 16.8 Ultimate lateral resistance of a long pile in cohesive soil related to embedded length (after Broms (1964a))

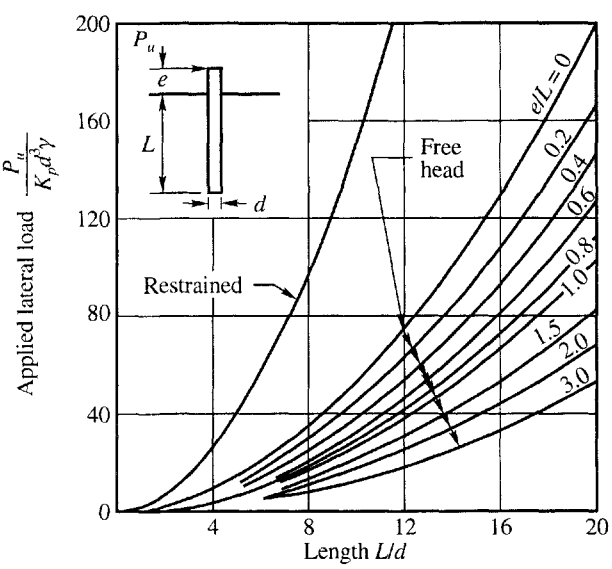

Figure 16.9 Ultimate lateral resistance of a short pile in cohesionless soil related to embedded length (after Broms (1964b))

involves the yield moment M_y for the pile section. The equations suggested by Broms for computing M_y are as follows:

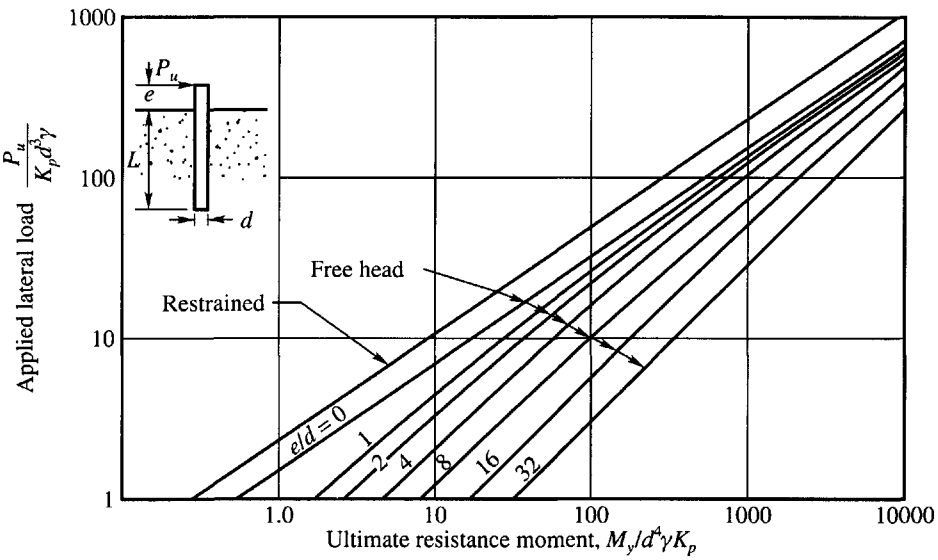

Figure 16.10 Ultimate lateral resistance of a long pile in cohesionless soil related to embedded length (after Broms (1964b))

For a cylindrical steel pipe section

$$M_y = 1.3 f_y Z \tag{16.19a}$$

For an H-section

$$M_y = 1.1 f_y Z_{max} \tag{16.19b}$$

where f_y = yield strength of the pile material
Z = section modulus of the pile section

The ultimate strength of a reinforced concrete pile section can be calculated in a similar manner.

Ultimate Lateral Resistance of Piles in Cohesionless Soils

The ultimate lateral resistance of a short piles embedded in cohesionless soil can be estimated making use of Fig. 16.9 and that of long piles from Fig. 16.10. In Fig. 16.9 the dimensionless quantity $P_u/\gamma d^3 K_P$ is plotted against the L/d ratio for short piles and in Fig. 16.10 $P_u/\gamma d^3 K_P$ is plotted against $M_y/\gamma d^4 K_P$. In both cases the terms used are

γ = effective unit weight of soil
K_P = Rankine's passive earth pressure coefficient = $\tan^2(45°+\phi/2)$

Example 16.4

A steel pipe pile of 61 cm outside diameter with 2.5 cm wall thickness is driven into saturated cohesive soil to a depth of 20 m. The undrained cohesive strength of the soil is 85 kPa. Calculate the ultimate lateral resistance of the pile by Broms' method with the load applied at ground level.

Solution

The pile is considered as a long pile. Use Fig. 16.8 to obtain the ultimate lateral resistance P_u of the pile.

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The non-dimensional yield moment $= \dfrac{M_y}{c_u d^3}$

where M_y = yield resistance of the pile section
= $1.3 f_y Z$
f_y = yield strength of the pile material
= 2800 kg/cm^2 (assumed)

Z = section modulus = $\dfrac{\pi}{64R}[d_0^4 - d_i^4]$
I = moment of inertia,
d_o = outside diameter = 61 cm,
d_i = inside diameter = 56 cm,
R = outside radius = 30.5 cm

$$Z = \frac{3.14}{64 \times 30.5}[61^4 - 56^4] = 6{,}452.6 \text{ cm}^3$$

$M_y = 1.3 \times 2{,}800 \times 6{,}452.6 = 23.487 \times 10^6$ kg-cm.

$$\frac{M_y}{c_u d_0^3} = \frac{23.487 \times 10^6}{0.85 \times 61^3} = 122$$

From Fig. 16.8 for $e/d = 0$, $\dfrac{M_y}{c_u d^3} = 122$, $\dfrac{P_u}{c_u d_0^2} \approx 35$

$P_u = 35\, c_u d_0^2 = 35 \times 85 \times 0.61^2 = 1{,}107$ kN

Example 16.5

If the pile given in Ex. 16.4 is restrained against rotation, calculate the ultimate lateral resistance P_u.

Solution

Per Ex. 16.4 $\dfrac{M_y}{c_u d_0^3} = 122$

From Fig. 16.8, for $\dfrac{M_y}{c_u d_0^3} = 122$, for restrained pile $\dfrac{P_u}{c_u d_0^2} \approx 50$

Therefore $P_u = \dfrac{50}{35} \times 1{,}107 = 1{,}581$ kN

Example 16.6

A steel pipe pile of outside diameter 61 cm and inside diameter 56 cm is driven into a medium dense sand under submerged conditions. The sand has a relative density of 60% and an angle of internal friction of 38°. Compute the ultimate lateral resistance of the pile by Brom`s method. Assume that the yield resistance of the pile section is the same as that given in Ex 16.4. The submerged unit weight of the soil γ_b =8.75 kN/m^3.

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Solution

From Fig. 16.10

$$\text{Non-dimensional yield moment} = \frac{M_y}{\gamma d^4 K_P}$$

where, $K_p = \tan^2(45 + \phi/2) = \tan^2 64 = 4.20$,
$M_y = 23.487 \times 10^6$ kg-cm,
$\gamma = 8.75$ kN/m$^3 \approx 8.75 \times 10^{-4}$ kg/cm^3,
$d = 61$ cm.

Substituting,

$$\frac{M_y}{\gamma d^4 K_P} = \frac{23.487 \times 10^6 \times 10^4}{8.75 \times 61^4 \times 4.2} = 462$$

From Fig. 16.10, for $\frac{M_y}{\gamma d^4 K_P} = 462$, for $e/d = 0$ we have $\frac{P_u}{\gamma d^3 K_P} \approx 80$

Therefore $P_u = 80\ \gamma d^3 K_p = 80 \times 8.75 \times 0.61^3 \times 4.2 = 667$ kN

Example 16.7

If the pile in Ex. 16.6 is restrained, what is the ultimate lateral resistance of the pile?

Solution

From Fig. 16.10, for $\frac{M_y}{\gamma d^4 K_P} = 462$, the value $\frac{P_u}{\gamma d^3 K_P} = 135$

$P_u = 135\ \gamma d^3\ K_p = 135 \times 8.75 \times 0.61^3 \times 4.2 = 1{,}126$ kN.

Example 16.8

Compute the deflection at ground level by Broms' method for the pile given in Ex. 16.1.

Solution

From Eq. (16.18)

$$\eta = \left(\frac{n_h}{EI}\right)^{1/5} = \left(\frac{6}{4.35 \times 10^2}\right)^{1/5} = 0.424$$

$\eta L = 0.424 \times 20 = 8.5$.

From Fig. 16.6, for $\eta L = 8.5$, $e/L = 0$, we have

$$\frac{y_g (EI)^{3/5} (n_h)^{2/5}}{P_t L} = 0.2$$

$$y_g = \frac{0.2 P_t L}{(EI)^{3/5} (n_h)^{2/5}} = \frac{0.2 \times 0.268 \times 20}{(4.35 \times 10^2)^{3/5} (6)^{2/5}} = 0.014 \text{ m} = 1.4 \text{ cm}$$

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Example 16.9

If the pile given in Ex. 16.1 is only 4 m long, compute the ultimate lateral resistance of the pile by Broms' method.

Solution

From Eq. (16.18)

$$\eta = \left(\frac{n_h}{EI}\right)^{1/5} = \left(\frac{6}{4.35\times10^2}\right)^{1/5} = 0.424$$

$\eta L = 0.424 \times 4 = 1.696.$

The pile behaves as an infinitely stiff member since $\eta L < 2.0$, $L/d = 4/0.61 = 6.6$.

From Fig. 16.9, for $L / d = 6.6$, $e / L = 0$, , we have

$P_u / \gamma d^3 K_p = 25.$

$\phi = 33°$, $\gamma = 8.75$ kN/m^3, $d = 61$ cm, $K_p = \tan^2(45° + \phi/2) = 3.4$.

Now $P_u = 25\ \gamma d^3 K_p = 25 \times 8.75 \times (0.61)^3 \times 3.4 = 169$ kN

If the sand is medium dense, as given in Ex. 16.6, then $K_p = 4.20$, and the ultimate lateral resistance P_u is

$$P_u = \frac{4.2}{3.4} \times 169 = 209 \text{ kN}$$

As per Ex. 16.6, P_u for a long pile = 667 kN, which indicates that the ultimate lateral resistance increases with the length of the pile and remains constant for a long pile.

16.7 A DIRECT METHOD FOR SOLVING THE NON-LINEAR BEHAVIOR OF LATERALLY LOADED FLEXIBLE PILE PROBLEMS

Key to the Solution

The key to the solution of a laterally loaded vertical pile problem is the development of an equation for n_h. The present state of the art does not indicate any definite relationship between n_h, the properties of the soil, the pile material, and the lateral loads. However it has been recognized that n_h depends on the relative density of soil for piles in sand and undrained shear strength c for piles in clay. It is well known that the value of n_h decreases with an increase in the deflection of the pile. It was Palmer et al (1948) who first showed that a change of width d of a pile will have an effect on deflection, moment and soil reaction even while EI is kept constant for all the widths. The selection of an initial value for n_h for a particular problem is still difficult and many times quite arbitrary. The available recommendations in this regard (Terzaghi 1955, and Reese 1975) are widely varying.

The author has been working on this problem since a long time (Murthy, 1965). An explicit relationship between n_h and the other variable soil and pile properties has been developed on the principles of dimensional analysis (Murthy and Subba Rao, 1995).

Development of Expressions for n_h

The term n_h may be expressed as a function of the following parameters for piles in sand and clay.

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(a) Piles in sand

$$n_h = f_s\,(EI, \mathrm{d}, P_e, \gamma, \phi) \tag{16.20}$$

(b) Piles in clay

$$n_h = f_c\,(EI, \mathrm{d}, P_e, \gamma, c) \tag{16.21}$$

The symbols used in the above expressions have been defined earlier.

In Eqs (16.20) and (16.21), an equivalent lateral load P_e at ground level is used in place of P_t acting at a height e above ground level. An expression for P_e may be written from Eq. (16.15) as follows.

$$P_e = P_t(1 + 0.67\frac{e}{T}) \tag{16.22}$$

Now the equation for computing groundline deflection y_g is

$$y_g = \frac{2.43 P_e T^3}{EI} \tag{16.23}$$

Based on dimensional analysis the following non-dimensional groups have been established for piles in sand and clay.

Piles in Sand

$$F_n = \frac{n_h P_e^{1/3}}{C_\phi d \gamma^{4/3}} \text{ and } F_p = \frac{(EI)\gamma^{1/3}}{d P_e^{4/3}} \tag{16.24}$$

where C_ϕ = correction factor for the angle of friction ϕ. The expression for C_ϕ has been found separately based on a critical study of the available data. The expression for C_ϕ is

$$C_\phi = 3 \times 10^{-5}\,(1.316)^{\phi^\circ} \tag{16.25}$$

Fig. 16.11 gives a plot of C_ϕ versus ϕ.

Piles in Clay

The nondimensional groups developed for piles in clay are

$$F_n = \frac{n_h \sqrt{p_e}\,(1 + e/d)^{1.5}}{c^{1.5}}; \qquad F_p = \frac{\sqrt{EI\gamma d}}{P_e} \tag{16.26}$$

In any lateral load test in the field or laboratory, the values of EI, γ, ϕ (for sand) and c (for clay) are known in advance. From the lateral load tests, the ground line deflection curve P_t versus y_g is known, that is, for any applied load P_t, the corresponding measured y_g is known. The values of T, n_h and P_e can be obtained from Eqs (16.14a), (16.15) and (16.22) respectively. C_ϕ is obtained from Eq. (16.25) for piles in sand or from Fig. 16.11. Thus the right hand side of functions F_n and F_p are known at each load level.

A large number of pile test data were analyzed and plots of $\sqrt{F_n}$ versus F_p were made on log-log scale for piles in sand, Fig. (16.12) and F_n versus F_p for piles in clay, Fig. (16.13). The method of least squares was used to determine the linear trend. The equations obtained are as given below.

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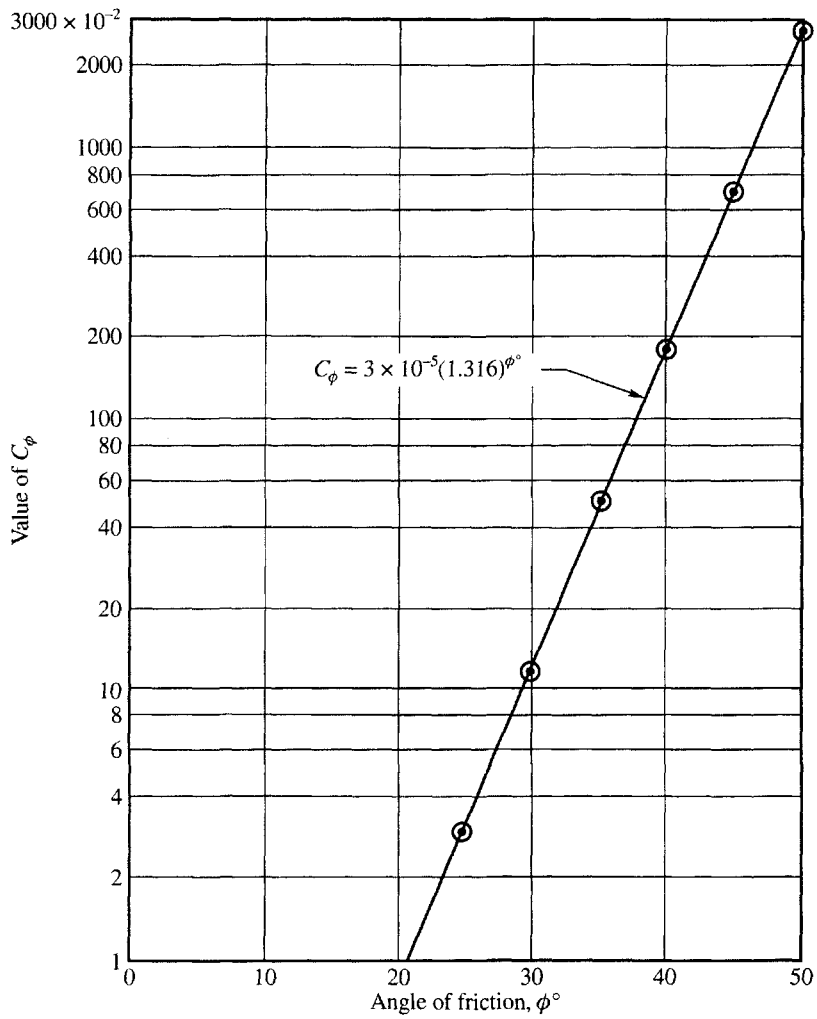

Figure 16.11 C_ϕ versus $\phi°$

Piles in Sand

$$F_n = 150\sqrt{F_p} \tag{16.27}$$

Piles in Clay

$$F_n = 125\ F_p \tag{16.28}$$

By substituting for F_n and F_p, and simplifying, the expressions for n_h for piles in sand and clay are obtained as

for piles in sand, $$n_h = \frac{150 C_\phi \gamma^{1.5}\sqrt{EId}}{P_e} \tag{16.29}$$

for piles in clay, $$n_h = \frac{125 c^{1.5}\sqrt{EI\gamma d}\Big/\left(1 + e/d\right)^{1.5}}{P_e^{1.5}} \tag{16.30}$$

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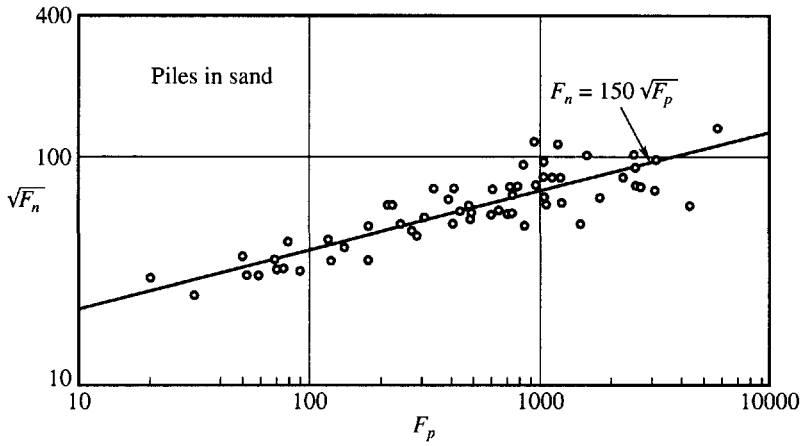

Figure 16.12 Nondimensional plot for piles in sand

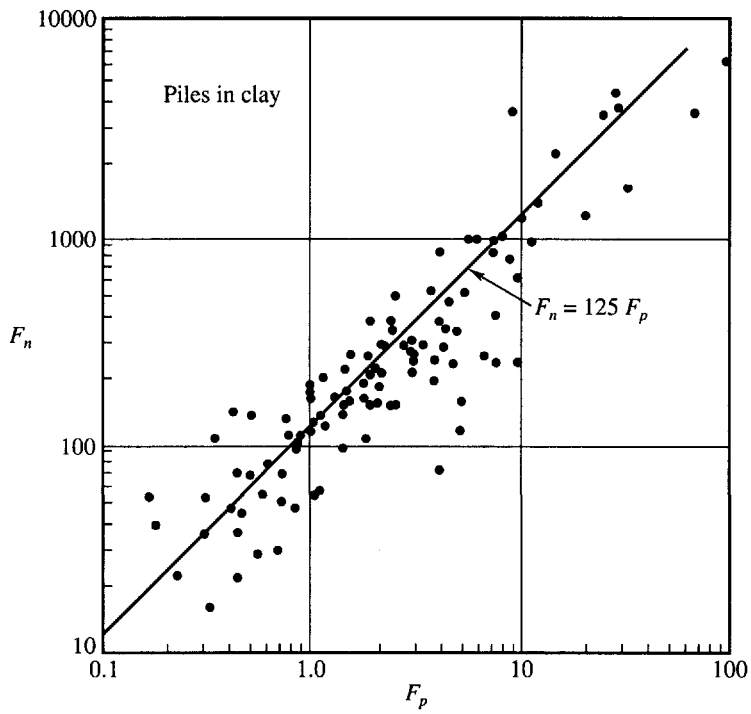

Figure 16.13 Nondimensional plot for piles in clay

It can be seen in the above equations that the numerators in both cases are constants for any given set of pile and soil properties.

The above two equations can be used to predict the non-linear behavior of piles subjected to lateral loads very accurately.

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Example 16.10

Solve the problem in Example 16.1 by the direct method (Murthy and Subba Rao, 1995). The soil is loose sand in a submerged condition.

Given; $EI = 4.35 \times 10^{11}$ kg- cm^2 = 4.35×10^5 kN-m^2

d = 61 cm, L = 20 m, γ_b = 8.75 kN/m^3

ϕ = 33°, P_t = 268 kN (since e = 0)

Required y_g at ground level

Solution

For a pile in sand for the case of $e = 0$, use Eq. (16.29)

$$n_h = \frac{150 C_\phi \gamma^{1.5} \sqrt{EId}}{P_e}$$

For $\phi = 33°$, $C_\phi = 3 \times 10^{-5} (1.316)^{33} = 0.26$ from Eq. (16.25)

$$n_h = \frac{150 \times 0.26 \times (8.75)^{1.5} \sqrt{4.65 \times 10^5 \times 0.61}}{P_e} = \frac{54 \times 10^4}{P_e} = \frac{54 \times 10^4}{268} = 2{,}015 \text{ kN/m}^3$$

$$T = \left(\frac{EI}{n_h}\right)^{1/5} = \left(\frac{43.5 \times 10^4}{2015}\right)^{1/5} = 2.93 \text{ m}$$

Now, using Eq. (16.23)

$$y_g = \frac{2.43 \times 268 \times (2.93)^3}{4.35 \times 10^5} = 0.0377 \text{ m} = 3.77 \text{ cm}$$

It may be noted that the direct method gives a greater ground line deflection (= 3.77 cm) as compared to the 1.96 cm in Ex. 16.1.

Example 16.11

Solve the problem in Example 16.2 by the direct method. In this case P_t is applied at a height 2 m above ground level All the other data remain the same.

Solution

From Example 16.10

$$n_h = \frac{54 \times 10^4}{P_e}$$

For $P_e = P_t = 268$ kN, we have $n_h = 2{,}015$ kN/m^3, and $T = 2.93$ m

From Eq. (16.22)

$$P_e = P_t \left(1 + 0.67 \frac{e}{T}\right) = 268 \left(1 + 0.67 \times \frac{2}{2.93}\right) = 391 \text{ kN}$$

For $P_e = 391$ kN, $n_h = \dfrac{54 \times 10^4}{391} = 1{,}381$ kN/m^3

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Now $T = \left(\dfrac{43.5\times10^4}{1{,}381}\right)^{1/5} = 3.16$ m

As before $P_e = 268\ \left(1+0.67\times\dfrac{2}{3.16}\right) = 382$ kN

For $P_e = 382$ kN, $n_h = 1{,}414$ kN/m^3, $T = 3.14$ m

Convergence will be reached after a few trials. The final values are

$P_e = 387$ kN, $n_h = 1718$ kN/m^3, $T = 3.025$ m

Now from Eq. (16.23)

$$y_g = \frac{2.43P_eT^3}{EI} = \frac{2.43\times382\times(3.14)^3}{4.35\times10^5} = 0.066 \text{ m} = 6.6 \text{ cm}$$

The n_h value from the direct method is 1,414 kN/m^3 whereas from Fig. 16.3 it is 6,000 kN/m^3. The n_h from Fig. 16.3 gives y_g which is 50 percent of the probable value and is on the unsafe side.

Example 16.12

Compute the ultimate lateral resistance for the pile given in Example 16.4 by the direct method. All the other data given in the example remain the same.

Given: EI = 4.35×10^5 kN-m^2, $d = 61$ cm, $L = 20$ m

c_u = 85 kN/m^3, $\gamma_b = 10$ kN/m^3 (assumed for clay)

M_y = 2,349 kN-m; $e = 0$

Required: The ultimate lateral resistance P_u.

Solution

Use Eqs (16.30) and (16.14)

$$n_h = \frac{125c^{1.5}\sqrt{EI\gamma d}}{P_t^{1.5}} \quad \text{for } e = 0$$

$$T = \left(\frac{EI}{n_h}\right)^{0.2} \tag{a}$$

Substituting the known values and simplifying

$$n_h = \frac{1{,}600\times10^5}{P_t^{1.5}} \tag{b}$$

Step 1

Let $P_t = 1{,}000$ kN, $n_h = \dfrac{1{,}600\times10^5}{(1000)^{1.5}} = 5{,}060$ kN/m^3

$$T = \left(\frac{4.35\times10^5}{5060}\right)^{0.2} = 2.437 \text{ m}$$

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For $e = 0$, from Table 16.2 and Eq. (16.11) we may write

$$M_{max} = 0.77[P_t T]$$

where $A_m = 0.77$ (max) correct to two decimal places.

For $P_t = 1000$ kN, and $T = 2.437$ m

$$M_{max} = 0.77 \times 1000 \times 2.437 = 1876 \text{ kN-m} < M_y.$$

Step 2

Let P_t	=	1500 kN
n_h	=	2754 kN/m^3 from Eq. (b)
and T	=	2.75 m from Eq. (a)
Now M_{max}	=	$0.77 \times 1500 \times 2.75 = 3179$ kN-m $> M_y$.
P_u for M_y	=	2349 kN-m can be determined as

$$P_u = 1{,}000 + (1{,}500 - 1{,}000) \times \frac{(2{,}349 - 1{,}876)}{(3{,}179 - 1{,}876)} = 1{,}182 \text{ kN}$$

$P_u = 1{,}100$ kN by Brom's method which agrees with the direct method.

16.8 CASE STUDIES FOR LATERALLY LOADED VERTICAL PILES IN SAND

Case 1: Mustang Island Pile LoadTest (Reese et al., 1974)

Data:

Pile diameter, d	=	24 in, steel pipe (driven pile)
EI	=	4.854×10^{10} lb-in^2
L	=	69 ft
e	=	12 in.
ϕ	=	39°
γ	=	66 lb/ft^3 (= 0.0382 lb/in^3)
M_y	=	7×10^6 in-lbs

The soil was fine silty sand with WT at ground level

Required:

(a) Load-deflection curve (P_t vs. y_g) and n_h vs. y_g curve

(b) Load-max moment curve (P_t Vs M_{max})

(c) Ultimate load P_u

Solutions:

For pile in sand, $$n_h = \frac{150\, C_\phi \gamma^{1.5} \sqrt{EId}}{P_e}$$

For $\phi = 39°$, $$C_\phi = 3 \times 10^{-5} (1.316)^{39°} = 1.34$$

After substitution and simplifying

$$n_h = \frac{1631 \times 10^3}{P_e} \qquad \text{(a)}$$

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From Eqs (16.22) and (16.14a), $$P_e = P_t \; 1 + 0.67\frac{e}{T} \tag{b}$$

$$T = \frac{EI}{n_h}^{\frac{1}{5}} \tag{c}$$

(a) Calculation of Groundline Deflection, y_g

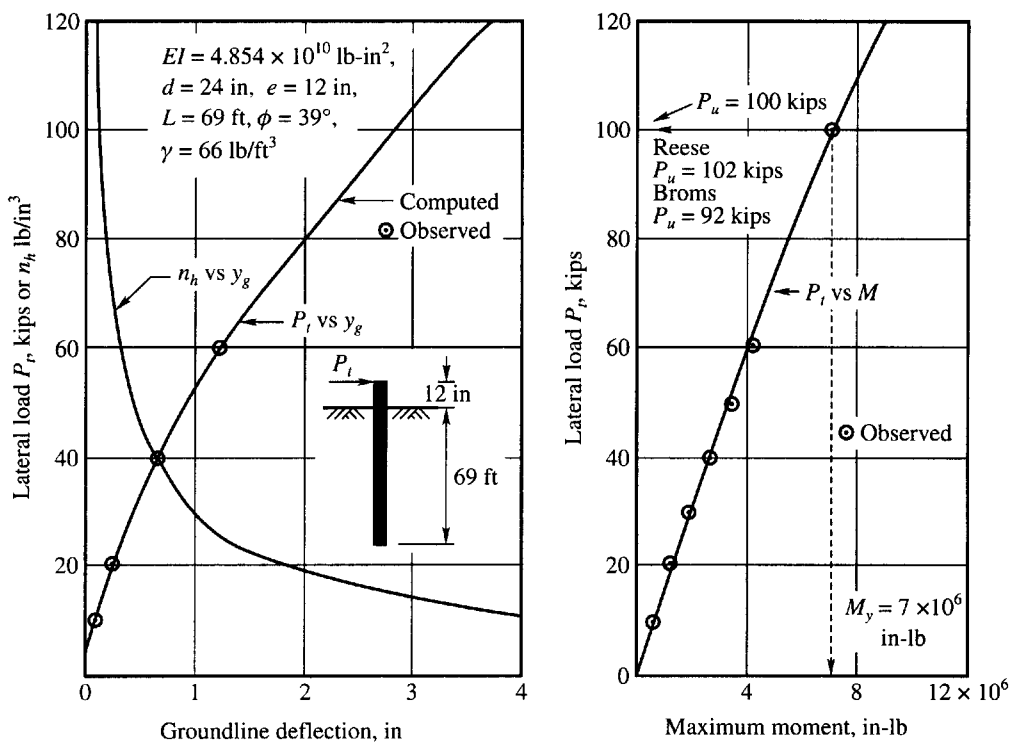
Step 1

Since T is not known to start with, assume $e = 0$, and $P_e = P_t = 10{,}000$ lbs

Now, from Eq. (a), $$n_h = \frac{1631 \times 10^3}{10 \times 10^3} = 163 \text{ lb/in}^3$$

from Eq. (c), $$T = \frac{4.854 \times 10^{10}}{163}^{\frac{1}{5}} = 49.5 \text{ in}$$

from Eq. (b), $$P_e = 10 \times 10^3 \; 1 + 0.67 \times \frac{12}{49.5} = 11.624 \times 10^3 \text{ lbs}$$



(a) P_t vs y_g and n_h vs y_g (b) P_t vs M_{max}

Figure 16.14 Mustang Island lateral load test

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Step 2

For $P_e = 11.62 \times 10^3$ lb, $n_h = \dfrac{1631 \times 10^3}{11.624 \times 10^3} = 140$ lb / in^3

As in Step 1 $T = 51$ ins, $P_e = 12.32 \times 10^3$ lbs

Step 3

Continue Step 1 and Step 2 until convergence is reached in the values of T and P_e. The final values obtained for $P_t = 10 \times 10^3$ lb are $T = 51.6$ in, and $P_e = 12.32 \times 10^3$ lbs

Step 4

The ground line deflection may be obtained from Eq (16.23).

$$y_g = \frac{2.43 P_e T^3}{EI} = \frac{2.43 \times 12.32 \times 10^3 \times (51.6)^3}{4.854 \times 10^{10}} = 0.0845 \text{ in}$$

This deflection is for $P_t = 10 \times 10^3$ lbs. In the same way the values of y_g can be obtained for different stages of loadings. Fig. 16.14(a) gives a plot P_t vs. y_g. Since n_h is known at each stage of loading, a curve of n_h vs. y_g can be plotted as shown in the same figure.

(b) Maximum Moment

The calculations under (a) above give the values of T for various loads P_t. By making use of Eq. (16.11) and Table 16.2, moment distribution along the pile for various loads P_t can be calculated. From these curves the maximum moments may be obtained and a curve of P_t vs. M_{max} may be plotted as shown in Fig. 16.14b.

(c) Ultimate Load P_u

Figure 16.14(b) is a plot of M_{max} vs. P_t. From this figure, the value of P_u is equal to 100 kips for the ultimate pile moment resistance of 7×10^6 in-lb. The value obtained by Broms' method and by computer (Reese, 1986) are 92 and 102 kips respectively

Comments:

Figure 16.14a gives the computed P_t vs. y curve by the direct approach method (Murthy and Subba Rao 1995) and the observed values. There is an excellent agreement between the two. In the same way the observed and the calculated moments and ultimate loads agree well.

Case 2: Florida Pile Load Test (Davis, 1977)

Data

Pile diameter, d = 56 in steel tube filled with concrete
EI = 132.5×10^{10} lb-in^2
L = 26 ft
e = 51 ft
ϕ = 38°,
γ = 60 lb/ft^3
M_y = 4630 ft-kips.

The soil at the site was medium dense and with water table close to the ground surface.

Required

(a) P_t vs. y_g curve and n_h vs. y_g curve
(b) Ultimate lateral load P_u

Solution

The same procedure as given for the Mustang Island load test has been followed for calculating the P_t vs. y_g and n_h vs. y_g curves. For getting the ultimate load P_u the P_t vs. M_{max} curve

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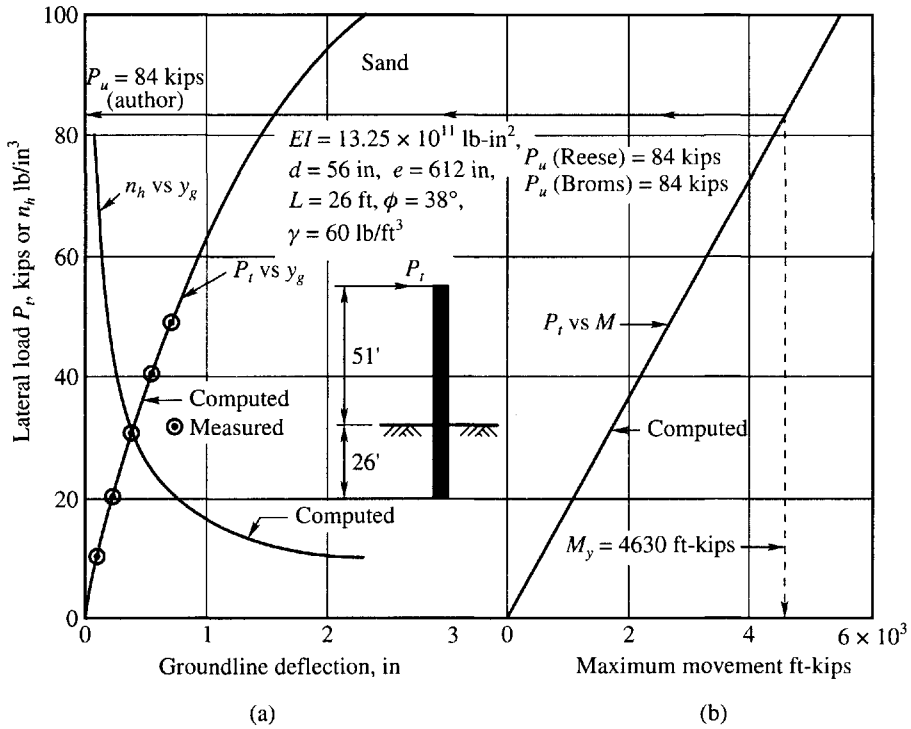

Figure 16.15 Florida pile test (Davis, 1977)

is obtained. The value of P_u obtained is equal to 84 kips which is the same as the ones obtained by Broms (1964) and Reese (1985) methods. There is a very close agreement between the computed and the observed test results as shown in Fig. 16.15.

Case 3: Model Pile Tests in Sand (Murthy, 1965)

Data

Model pile tests were carried out to determine the behavior of vertical piles subjected to lateral loads. Aluminum alloy tubings, 0.75 in diameter and 0.035 in wall thickness, were used for the test. The test piles were instrumented. Dry clean sand was used for the test at a relative density of 67%. The other details are given in Fig. 16.16.

Solution

Fig. 16.16 gives the predicted and observed

(a) load-ground line deflection curve
(b) deflection distribution curves along the pile
(c) moment and soil reaction curves along the pile

There is an excellent agreement between the predicted and the observed values. The direct approach method has been used.

16.9 CASE STUDIES FOR LATERALLY LOADED VERTICAL PILES IN CLAY

Case 1: Pile load test at St. Gabriel (Capazzoli, 1968)

Data

Pile diameter, d = 10 in, steel pipe filled with concrete

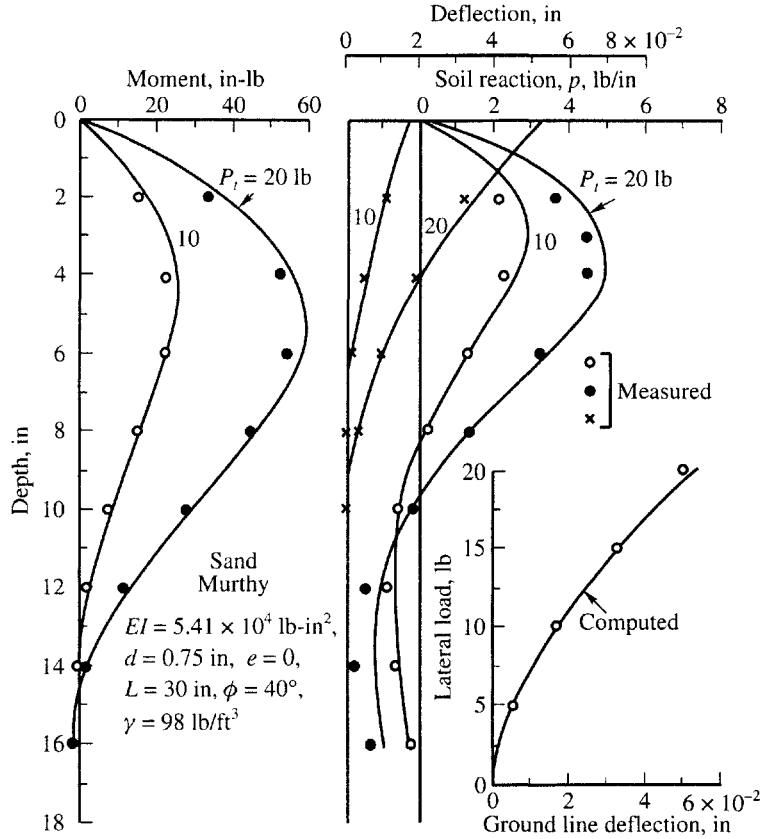

Figure 16.16 Curves of bending moment, deflection and soil reaction for a model pile in sand (Murthy, 1965)

$$\begin{aligned} EI &= 38 \times 10^8 \text{ lb-in}^2 \\ L &= 115 \text{ ft} \\ e &= 12 \text{ in.} \\ c &= 600 \text{ lb/ft}^2 \\ \gamma &= 110 \text{ lb/ft}^3 \\ M_y &= 116 \text{ ft-kips} \end{aligned}$$

Water table was close to the ground surface.

Required

(a) P_t vs. y_g curve

(b) the ultimate lateral load, P_u

Solution

We have,

$$\text{(a)}\ n_h = \frac{125c^{1.5}\sqrt{EI\gamma d}}{(1+e/d)^{1.5}P_e^{1.5}} \qquad \text{(b)}\ P_e = P_t\ 1+0.67\frac{e}{T} \qquad \text{(c)}\ T = \frac{EI}{n_h}^{\frac{1}{5}}$$

After substituting the known values in Eq. (a) and simplifying, we have

$$n_h = \frac{16045 \times 10^3}{P_e^{1.5}}$$

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(a) Calculation of groundline deflection

1. Let $P_e = P_t = 500$ lbs
   From Eqs (a) and (c), $n_h = 45$ lb/in^3, $T = 38.51$ in
   From Eq. (d), $P_e = 6044$ lb.
2. For $P_e = 6044$ lb, $n_h = 34$ lb/in^3 and $T = 41$ in
3. For $T = 41$ in, $P_e = 5980$ lb, and $n_h = 35$ lb/in^3
4. For $n_h = 35$ lb/in^3, $T = 40.5$ in, P_e =5988 lb
5. $$y_g = \frac{2.43 P_e T^3}{EI} = \frac{2.43 \times 5988 \times (40.5)^3}{38 \times 10^8} = 0.25 \text{ in}$$
6. Continue steps 1 through 5 for computing y_g for different loads P_t. Fig. 16.17 gives a plot of P_t vs. y_g which agrees very well with the measured values.

(b) Ultimate load P_u

A curve of M_{max} vs. P_t is given in Fig. 16.17 following the procedure given for the Mustang Island Test. From this curve $P_u = 23$ k for $M_y = 116$ ft kips. This agrees well with the values obtained by the methods of Reese (1985) and Broms (1964a).

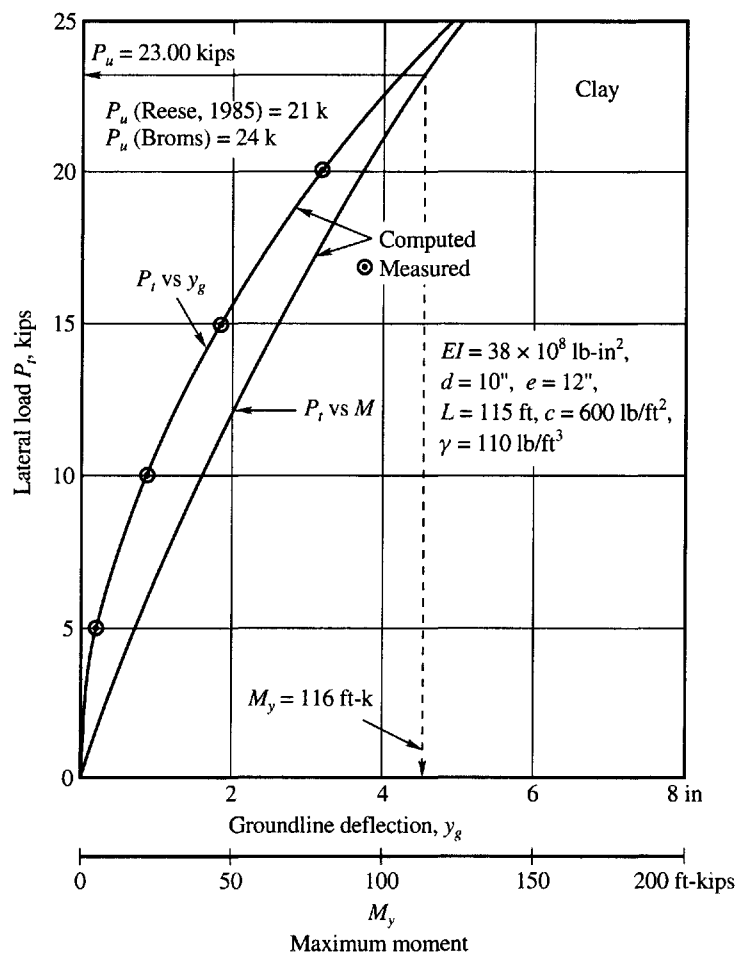

Figure 16.17 St. Gabriel pile load test in clay

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Case 2: Pile Load Test at Ontario (Ismael and Klym, 1977)

Data

Pile diameter, d = 60 in, concrete pile (Test pile 38)

EI = 93×10^{10} lb-in^2

L = 38 ft

e = 12 in.

c = 2000 lb/ft^2

γ = 60 lb/ft^3

The soil at the site was heavily overconsolidated

Required:

(a) P_t vs. y_g curve

(b) n_h vs. y_g curve

Solution

By substituting the known quantities in Eq. (16.30) and simplifying,

$$n_h = \frac{68495 \times 10^5}{P_e^{1.5}}, \quad T = \left(\frac{EI}{n_h}\right)^{\frac{1}{5}}, \text{ and } P_e = P_t \left(1 + 0.67\frac{e}{T}\right)$$

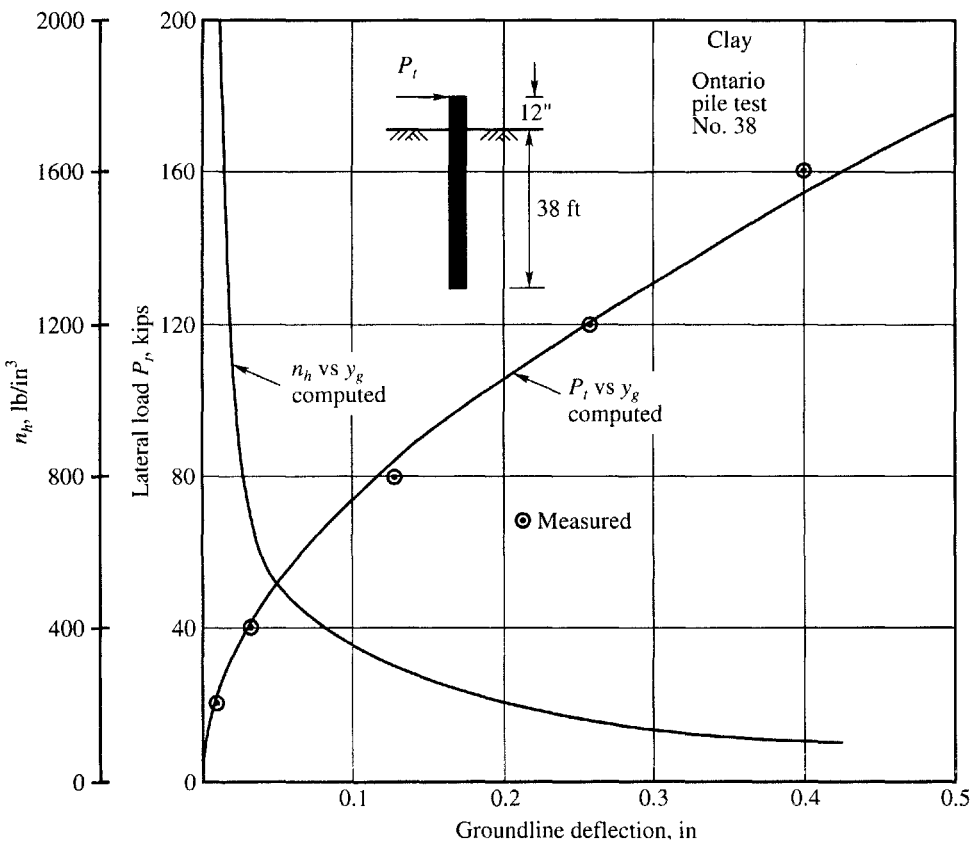

Figure 16.18 Ontario pile load test (38)

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Follow the same procedure as given for Case 1 to obtain values of y_g for the various loads P_t. The load deflection curve can be obtained from the calculated values as shown in Fig. 16.18. The measured values are also plotted. It is clear from the curve that there is a very close agreement between the two. The figure also gives the relationship between n_h and y_g.

Case 3: Restrained Pile at the Head for Offshore Structure (Matlock and Reese, 1961)

Data

The data for the problem are taken from Matlock and Reese (1961). The pile is restrained at the head by the structure on the top of the pile. The pile considered is below the sea bed. The undrained shear strength c and submerged unit weights are obtained by working back from the known values of n_h and T. The other details are

$$\begin{aligned} \text{Pile diameter, } d &= 33 \text{ in, pipe pile} \\ EI &= 42.35 \times 10^{10} \text{ lb-in}^2 \\ c &= 500 \text{ lb/ft}^2 \\ \gamma &= 40 \text{ lb/ft}^3 \\ P_t &= 150{,}000 \text{ lbs} \end{aligned}$$

$$\text{(a) } \frac{M_t}{P_t T} = \frac{-T}{12.25 + 1.078T}, \quad \text{(b) } T = \frac{EI}{n_h}^{\frac{1}{5}}, \quad \text{(c) } P_e = P_t \; 1 - 0.67\frac{e}{T}$$

Required

(a) deflection at the pile head

(b) moment distribution diagram

Solution

Substituting the known values in Eq (16.30) and simplifying,

$$n_h = \frac{458 \times 10^6}{P_e^{1.5}(1 + e/d)^{1.5}} \tag{d}$$

Calculations

1. Assume $e = 0$, $P_e = P_t = 150{,}000$ lb

From Eqs (d) and (b) $n_h = 7.9 \text{ lb/in}^2$, $T = 140$ in

From Eq. (a) $$\frac{M_t}{P_t T} = \frac{-140}{12.25 + 1.078 \times 140} = -0.858$$

or $$M_t = -0.858 \; P_t T = P_t e$$

Therefore $$e = 0.858 \times 140 = 120 \text{ in}$$

2. $$P_e = P_t \; 1 - 0.67\frac{e}{T} = 1.5 \times 10^5 \; 1 - 0.67 \times \frac{120}{140} = 63{,}857 \text{ lb}$$

$$1 + \frac{e}{d}^{1.5} = 1 + \frac{120}{33}^{1.5} = 10$$

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Now from Eq. (d), $n_h = 2.84$ lb/in^3, from Eq. (b) $T = 171.64$ in
After substitution in Eq. (a)

$$\frac{M_t}{P_t T} = -0.875 \text{, and } e = 0.875 \times 171.64 = 150.2 \text{ in}$$

$$P_e = 1 - 0.67 \times \frac{150.2}{171.64} \times 1.5 \times 10^5 = 62{,}205 \text{ lbs}$$

3. Continuing this process for a few more steps there will be convergence of values of n_h, T and P_e. The final values obtained are

$$n_h = 2.1 \text{ lb/in}^3, \; T = 182.4 \text{ in, and } P_e = 62{,}246 \text{ lb}$$

$$M_t = -P_t e = -150{,}000 \times 150.2 = -22.53 \times 10^6 \text{ lb-in}^2$$

$$y_g = \frac{2.43\, P_e T^3}{EI} = \frac{2.43 \times 62{,}246 \times (182.4)^3}{42.35 \times 10^{10}} = 2.17 \text{ in}$$

Moment distribution along the pile may now be calculated by making use of Eq. (16.11) and Table 16.2. Please note that M_t has a negative sign. The moment distribution curve is given in Fig. 16.19. There is a very close agreement between the computed values by direct method and the Reese and Matlock method. The deflection and the negative bending moment as obtained by Reese and Matlock are

$y_m = 2.307$ in and $M_t = -24.75 \times 10^6$ lb-in^2

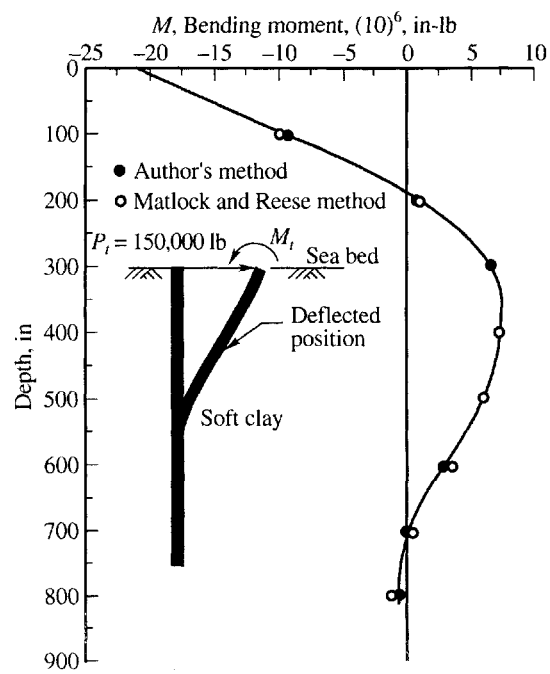

Figure 16.19 Bending moment distribution for an offshore pile supported structure (Matlock and Reese, 1961)

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16.10 BEHAVIOR OF LATERALLY LOADED BATTER PILES IN SAND

General Considerations

The earlier sections dealt with the behavior of long vertical piles. The author has so far not come across any rational approach for predicting the behavior of batter piles subjected to lateral loads. He has been working on this problem for a long time (Murthy, 1965). Based on the work done by the author and others, a method for predicting the behavior of long batter piles subjected to lateral load has now been developed.

Model Tests on Piles in Sand (Murthy, 1965)

A series of seven instrumented model piles were tested in sand with batters varying from 0 to ±45°. Aluminum alloy tubings of 0.75 in outside diameter and 30 in long were used for the tests. Electrical resistance gauges were used to measure the flexural strains at intervals along the piles at different load levels. The maximum load applied was 20 lbs. The pile had a flexural rigidity $EI = 5.14 \times 10^4$ lb-in^2. The tests were conducted in dry sand, having a unit weight of 98 lb/ft^3 and angle of friction ϕ equal to 40°. Two series of tests were conducted-one series with loads horizontal and the other with loads normal to the axis of the pile. The batters used were 0°, ± 15°, ±30° and ±45°. Pile movements at ground level were measured with sensitive dial gauges. Flexural strains were converted to moments. Successive integration gave slopes and deflections and successive differentiations gave shears and soil reactions respectively. A very high degree of accuracy was maintained throughout the tests. Based on the test results a relationship was established between the $n^b{}_h$ values of batter piles and $n^o{}_h$

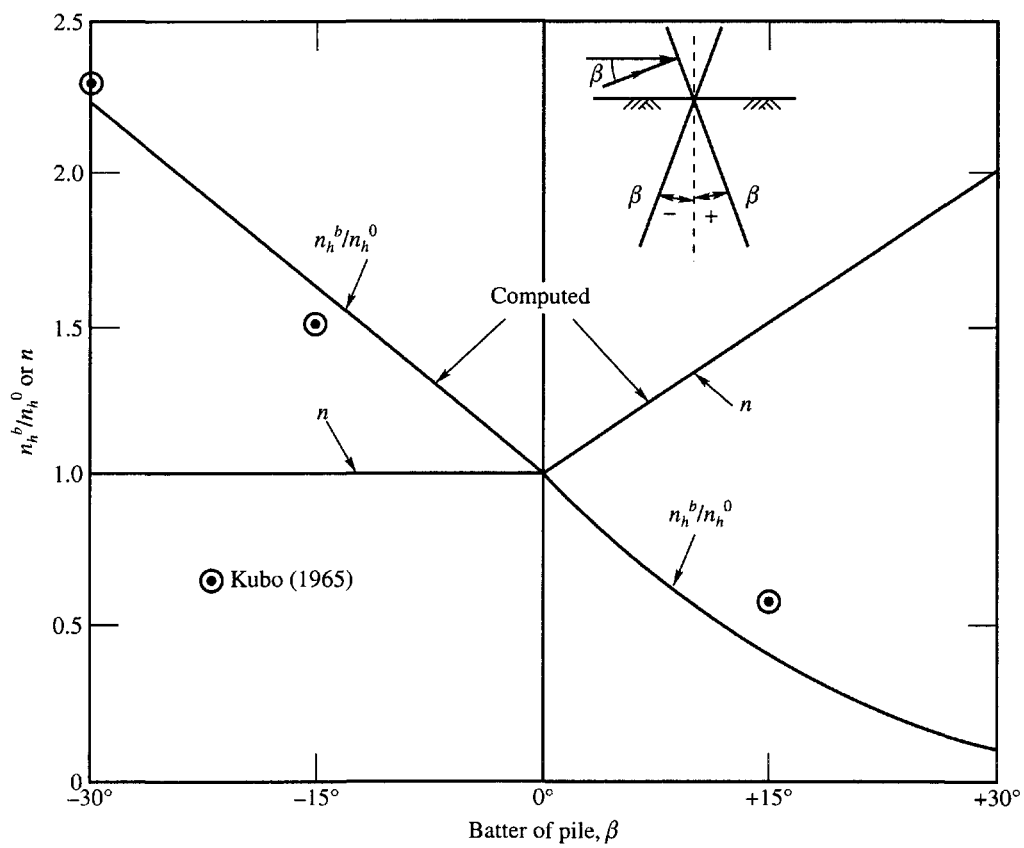

Figure 16.20 Effect of batter on n_h^b/n_h^o and n (after Murthy, 1965)

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values of vertical piles. Fig. 16.20 gives this relationship between n^b_h/n^o_h and the angle of batter β. It is clear from this figure that the ratio increases from a minimum of 0.1 for a positive 30° batter pile to a maximum of 2.2 for a negative 30° batter pile. The values obtained by Kubo (1965) are also shown in this figure. There is close agreement between the two.

The other important factor in the prediction is the value of n in Eq. (16.8a). The values obtained from the experimental test results are also given in Fig. 16.20. The values of n are equal to unity for vertical and negative batter piles and increase linearly for positive batter piles up to a maximum of 2.0 at + 30° batter.

In the case of batter piles the loads and deflections are considered normal to the pile axis for the purpose of analysis. The corresponding loads and deflections in the horizontal direction may be written as

$$P_t(\text{Hor}) = \frac{P_t(Nor)}{\cos\beta} \tag{16.31}$$

$$y_g(\text{Hor}) = \frac{y_g(\text{Nor})}{\cos\beta} \tag{16.32}$$

where P_t and y_g are normal to the pile axis; P_t(Hor) and y_g(Hor) are the corresponding horizontal components.

Application of the Use of n^b_h/n^o_h and n

It is possible now to predict the non-linear behavior of laterally loaded batter piles in the same way as for vertical piles by making use of the ratio n^b_h/n^o_h and the value of n. The validity of this method is explained by considering a few case studies.

Case Studies

Case 1: Model Pile Test (Murthy, 1965).

Piles of +15° and +30° batters have been used here to predict the P_t vs. y_g and P_t vs. $M_{\max}$ relationships. The properties of the pile and soil are given below.

$EI = 5.14 \times 10^4$ lb in^2, $d = 0.75$ in, $L = 30$ in; $e = 0$

$\gamma = 98$ lb/ft^3 and $\phi = 40°$

For $\phi = 40°$, $C_\phi = 1.767$ [$= 3 \times 10^{-5}\,(1.316)^\phi$]

From Eq. (16.29), $n^o_h = \dfrac{150 C_\phi \gamma^{1.5}\sqrt{EId}}{P_t}$

After substituting the known values and simplifying we have

$$n^o_h = \frac{700}{P_t}$$

Solution: +15° batter pile

From Fig. 16.20 $n^b_h/n^o_h = 0.4$, $n = 1.5$

From Eq. (16.14b), $T_b = \left[\dfrac{EI}{n^b_h}\right]^{\frac{1}{1.5+4}} = 5.33\left[\dfrac{5.14}{n^b_h}\right]^{0.1818}$

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Calculations of Deflection y_g

For $P_t = 5$ lbs, $n^o{}_h = 141$ lbs/in^3, $n^b{}_h = 141 \times 0.4 = 56$ lb/in^3 and $T_b = 3.5$ in

$$y_g = \frac{2.43 P_t^3 T_b^3}{5.14 \times 10^4} = 0.97 \times 10^{-2} \text{in}$$

Similarly, y_g can be calculated for $P_t = 10$, 15 and 20 lbs.

The results are plotted in Fig. 16.21 along with the measured values of y_g. There is a close agreement between the two.

Calculation of Maximum Moment, M_{max}

For $P_t = 5$ lb, $T_b = 3.5$ in, The equation for M is [Eq. (16.11)]

$$M = A_m P_t T_b = 0.77 P_t T_b$$

where $A_m = 0.77$ (max) from Table 16.2

By substituting and calculating, we have

$$M_{(max)} = 13.5 \text{ in-lb}$$

Similarly $M_{(max)}$ can be calculated for other loads. The results are plotted in Fig. 16.21 along with the measured values of $M_{(max)}$. There is very close agreement between the two.

+30° Batter Pile

From Fig. 16.20, $n^b{}_h/n^o{}_h = 0.1$, and $n = 2$; $T_b = \left(\frac{EI}{n_h^b}\right)^{\frac{1}{2+4}} = 4.64 \left(\frac{5.14}{n_h^b}\right)^{0.1667}$, $n_h^o = \frac{700}{P_t}$

For $P_t = 5$ lbs, $n^o{}_h = 141$ lbs/in^3, $n^b{}_h = 0.1 \times 141 = 14.1$ lb/in^3, $T_b = 3.93$ in.

For $P_t = 5$ lbs, $T_b = 3.93$ in, we have, $y_g = 1.43 \times 10^{-2}$ in

As before, $M_{(max)} = 0.77 \times 5 \times 3.93 = 15$ in-lb.

The values of y_g and $M_{(max)}$ for other loads can be calculated in the same way. Fig. 16.21 gives P_t vs. y_g and P_t vs. $M_{(max)}$ along with measured values. There is close agreement up to about

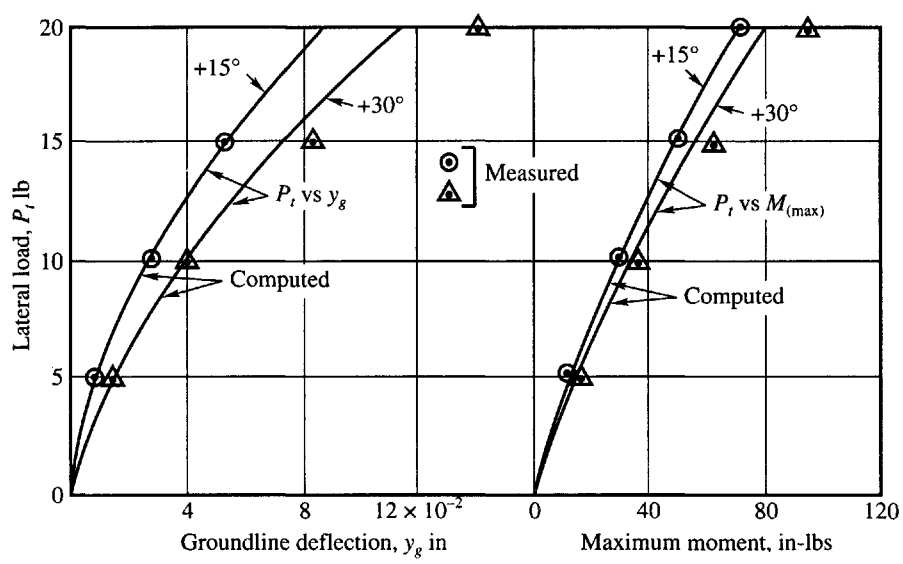

Figure 16.21 Model piles of batter +15° and +30° (Murthy, 1965)

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P_t = 10 lb, and beyond this load, the measured values are greater than the predicted by about 25 percent which is expected since the soil yields at a load higher than 10 lb at this batter and there is a plastic flow beyond this load.

Case 2: Arkansas River Project (Pile 12) (Alizadeh and Davisson, 1970).

Given:

$EI = 278.5 \times 10^8$ lb–in^2, $d = 14$ in, $e = 0$.

$\phi = 41°$, $\gamma = 63$ lb/ft^3, $\beta = 18.4°$ (–ve)

From Fig. 16.11, $C_\phi = 2.33$, from Fig. 16.20 $n^b_h/n^o_h = 1.7$, $n = 1.0$

From Eq. (16.29), after substituting the known values and simplifying, we have,

$$\text{(a) } n^o_h = \frac{1528 \times 10^3}{P_t}, \text{ and (b) } T_b = 39.8\left[\frac{278.5}{n^b_h}\right]^{0.2}$$

Calculation for P_t = 12.6^k

From Eq. (a), $n^o_h = 121$ lb/in^3; now $n^b_h = 1.7 \times 121 = 206$ lb/in^3

From Eq. (b), $T_b = 42.27$ in

$$y_g = \frac{2.43 \times 12{,}600(42.27)^3}{278.5 \times 10^8} = 0.083 \text{ in}$$

$M_{(\max)} = 0.77\, P_t T = 0.77 \times 12.6 \times 3.52 = 34$ ft-kips.

The values of y_g and $M_{(\max)}$ for P_t = 24.1^k, 35.5^k, 42.0^k, 53.5^k, 60^k can be calculated in the same way the results are plotted the Fig. 16.22 along with the measured values. There is a very close agreement between the computed and measured values of y but the computed values of $M_{\max}$

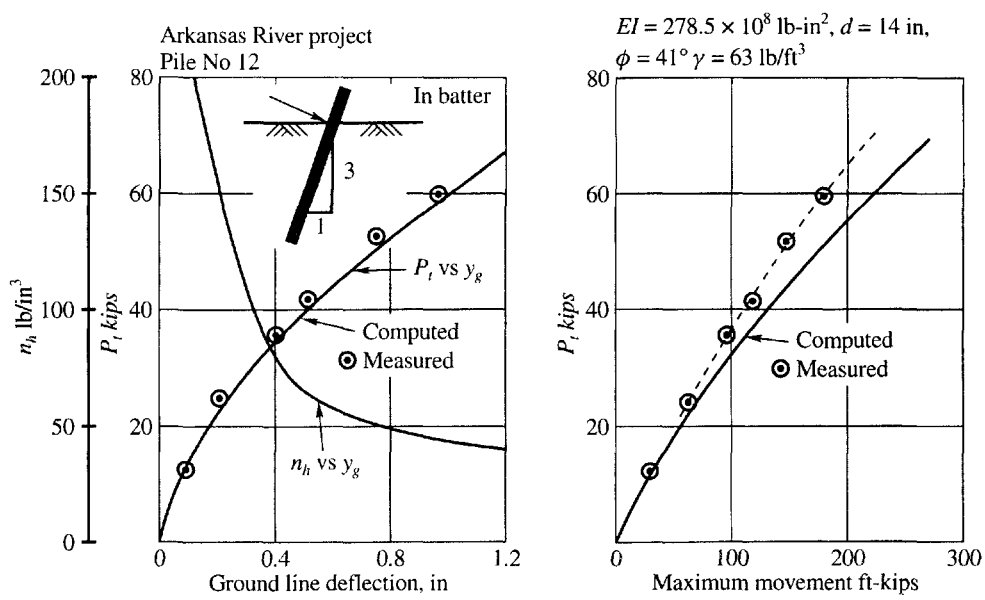

Figure 16.22 Lateral load test-batter pile 12-Arkansas River Project (Alizadeh and Davisson, 1970)

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are higher than the measured values at higher loads. At a load of 60 kips, $M_{(max)}$ is higher than the measured by about 23% which is quite reasonable.

Case 3: Arkansas River Project (Pile 13) (Alizadeh and Davisson, 1970).

Given:

$EI = 288 \times 10^8$ lb-ins, $d = 14''$, $e = 6$ in.

$\gamma = 63$ lbs/ft^3, $\phi = 41°(C_\phi = 2.33)$

$\beta = 18.4°$ (+ve), $n = 1.6$, $n^b{}_h/n^o{}_h = 0.3$

$$T_b = \frac{EI}{n_h^b}^{\frac{1}{1.6+4}} = 27\,\frac{288}{n_h^b}^{0.1786} \tag{a}$$

After substituting the known values in the equation for $n^o{}_h$ [Eq. (16.29)] and simplifying, we have

$$n_h^0 = \frac{1597 \times 10^3}{P_e} \tag{b}$$

Calculations for y_g for P_t = 141.4k

1. From Eq (b), $n^o{}_h = 39$ lb/in^3, hence $n^b{}_h = 0.3 \times 39 = 11.7$ lb/in^3
   From Eq. (a), $T_b \approx 48$ in.

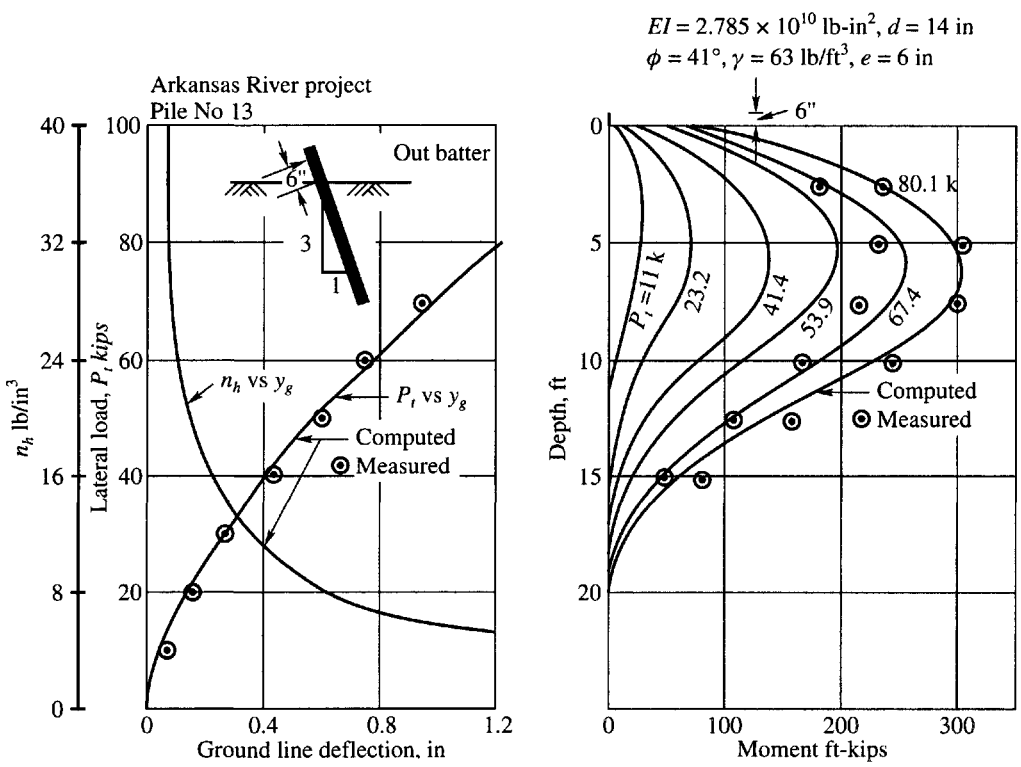

Figure 16.23 Lateral load test-batter pile 13-Arkansas River Project (Alizadeh and Davisson, 1970)

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2. $P_e = P_t \ 1 + 0.67\frac{e}{T} \ = 41.4 \ 1 + 0.67 \times \frac{6}{48} \ = 44.86$ kips

3. For $P_e = 44.86$ kips, $n_h^o = 36$ lb/in^3, and $n_h^b = 11$ lb/in^3, $T_b \approx 48$ in

4. Final values: $P_e = 44.86$ kips, $n_h^b = 11$ lb/in^3, and $T_b = 48$ in

5. $y_g = \frac{2.43 P_e T_b^3}{EI} = \frac{2.43 \times 44{,}860 \times (48)^3}{288 \times 10^8} = 0.42$ in.

6. Follow Steps 1 through 5 for other loads. Computed and measured values of y are plotted in Fig. 16.23 and there is a very close agreement between the two. The n_h values against y_g are also plotted in the same figure.

Calculation of Moment Distribution

The moment at any distance x along the pile may be calculated by the equation

$$M = [P_t T] A_m + [M_t] B_m$$

As per the calculations shown above, the value of T will be known for any lateral load level P_t. This means $[P_t T]$ will be known. The values of A_m and B_m are functions of the depth coefficient Z which can be taken from Table 16.2 for the distance $x(Z = x/T)$. The moment at distance x will be known from the above equation. In the same way moments may be calculated for other distances. The same procedure is followed for other load levels. Fig. 16.23 gives the computed moment distribution along the pile axis. The measured values of M are shown for two load levels $P_t = 67.4$ and 80.1 kips. The agreement between the measured and the computed values is very good.

Example 16.13

A steel pipe pile of 61 cm diameter is driven vertically into a medium dense sand with the water table close to the ground surface. The following data are available:

Pile: $EI = 43.5 \times 10^4$ kN-m^2, $L = 20$ m, the yield moment M_y of the pile material = 2349 kN-m.

Soil: Submerged unit weight $\gamma_b = 8.75$ kN/m^3, $\phi = 38°$.

Lateral load is applied at ground level ($e = 0$)

Determine:

(a) The ultimate lateral resistance P_u of the pile

(b) The groundline deflection y_g at the ultimate lateral load level.

Solution

From Eq. (16.29) the expression for n_h is

$$n_h = \frac{150 C_\phi \gamma^{1.5} \sqrt{EId}}{P_t} \quad \text{since } P_e = P_t \text{ for } e = 0 \tag{a}$$

From Eq. (16.25) $C_\phi = 3 \times 10^{-5} (1.326)^{38°} = 1.02$

Substituting the known values for n_h we have

$$n_h = \frac{150 \times 1.02 \times (8.75)^{1.5} \sqrt{43.5 \times 10^4 \times 0.61}}{P_t} = \frac{204 \times 10^4}{P_t} \text{ kN/m}^3$$

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(a) Ultimate lateral load P_u

Step 1:

Assume $P_u = P_t = 1000$ kN (a)

Now from Eq. (a) $n_h = \dfrac{204 \times 10^4}{1000} = 2040 \text{ kN/m}^3$

From Eq. (16.14a) $T = \left(\dfrac{EI}{n_h}\right)^{\frac{1}{n+4}} = \left(\dfrac{EI}{n_h}\right)^{\frac{1}{1+4}} = \left(\dfrac{EI}{n_h}\right)^{\frac{1}{5}}$

Substituting and simplyfiing $T = \left(\dfrac{43.5 \times 10^4}{2040}\right)^{\frac{1}{5}} = 2.92$ m

The moment equation for $e = 0$ may be written as (Eq. 16.11)

$M = A_m[P_t T]$

Substituting and simplifying we have (where $A_m(\max) = 0.77$)

$M_{\max} = 0.77(1000 \times 2.92) = 2248$ kN - m

which is less than $M_y = 2349$ kN-m.

Step 2:

Try $P_t = 1050$ kN.

Following the procedure given in Step 1

$T = 2.95$ m for $P_t = 1050$ kN

Now $M_{\max} = 0.77(1050 \times 2.95) = 2385$ kN - m

which is greater than $M_y = 2349$ kN-m.

The actual value P_u lies between 1000 and 1050 kN which can be obtained by proportion as

$$P_u = 1000 + (1050 - 1000) \times \frac{(2349 - 2248)}{(2385 - 2248)} = 1037 \text{ kN}$$

(b) Groundline deflection for P_u = 1037 kN

For this the value T is required at $P_u = 1037$ kN. Following the same procedure as in Step 1, we get $T = 2.29$ m.

Now from Eq. (16.15a) for $e = 0$

$$y_g = 2.43\frac{P_t T^3}{EI} = \frac{2.43 \times 1037 \times (2.944)^3}{43.5 \times 10^4} = 0.1478 \text{ m} = 14.78 \text{ cm}$$

Example 16.14

Refer to Ex. 16.13. If the pipe pile is driven at an angle of 30° to the vertical, determine the ultimate lateral resistance and the corresponding groundline deflection for the load applied (a) against batter, and (b) in the direction of batter.

In both the cases the load is applied normal to the pile axis.

All the other data given in Ex. 16.13 remain the same.

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Solution

From Ex. 16.13, the expression for n_h for vertical pile is

$$n_h = n_h^o = \frac{204 \times 10^4}{P_t} \text{ kN/m}^3 \tag{a}$$

+ 30° Batter pile

$$\text{From Fig. 16.20} \quad \frac{n_h^{+b}}{n_h^o} = 0.1 \text{ and } n = 2 \tag{b}$$

$$\text{From Eq. (16.14b)} \quad T = T_b = \left(\frac{EI}{n_h^b}\right)^{\frac{1}{n+4}} = \left(\frac{EI}{n_h^b}\right)^{\frac{1}{2+4}} = \left(\frac{EI}{n_h^b}\right)^{\frac{1}{6}} \tag{c}$$

Determination of P_u

Step 1

Assume $P_e = P_t = 500$ kN.

Following the Step 1 in Ex. 16.13, and using Eq. (a) above

$n_h^o = 4{,}083$ kN/m^3, hence $n_h^{+b} = 4083 \times 0.1 \approx 408$ kN/m^3

$$\text{Form Eq (c)}, \quad T_b = \left(\frac{43.5 \times 10^4}{408}\right)^{\frac{1}{6}} = 3.2 \text{ m}$$

As before, $M_{\max} = 0.77 P_t T_b = 0.77 \times 500 \times 3.2 = 1232$ kN-m $< M_y$

Step 2

Try $P_t = 1{,}000$ kN

Proceeding in the same way as given in Step 1 we have $T_b = 3.59$ m, $M_{\max} = 2764$ kN-m which is more than M_y. The actual P_u is

$$P_u = 500 + (1000 - 500) \times \frac{(2349 - 1232)}{(2764 - 1232)} = 865 \text{ kN}$$

Step 3

As before the corresponding T_b for $P_u = 865$ kN is 3.5 m.

Step 4

The groundline deflection is

$$y_g^{+b} = \frac{2.43 \times 865 \times (3.5)^3}{43.5 \times 10^4} = 0.2072 \text{ m} = 20.72 \text{ cm}$$

– 30° Batter pile

$$\text{From Fig. 16.20,} \quad \frac{n_h^{-b}}{n_h^o} = 2.2 \text{ and } n = 1.0 \tag{d}$$

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$$\text{and } T_b = \left(\frac{EI}{n_h^{-b}}\right)^{\frac{1}{n+4}} = \left(\frac{EI}{n_h^{-b}}\right)^{\frac{1}{1+4}} = \left(\frac{EI}{n_h^{-b}}\right)^{\frac{1}{5}} \tag{e}$$

Determination of P_u

Step 1

Try $P_t = 1000$

From Eq. (a) $n_h^o = 2040$ kN/m^3 and from Eq. (d) $n_h^{-b} = 2.2 \times 2040 = 4488$ kN/m^3

Now from Eq. (e), $T_b = 2.5$ m

As before $M_{max} = 0.77 \times 1000 \times 2.5 = 1925$ kN-m

which is less than $M_y = 2349$ kN-m

Step 2

Try $P_t = 1{,}500$ kN

Proceeding as in Step 1, $T_b = 2.71$ m, and $M_{max} = 0.77 \times 1500 \times 2.71 = 3130$ kN-m which is greater than M_y.

Step 3

The actual value of P_u is therefore

$$P_u = 1000 + (1500 - 1000) \times \frac{(2349 - 1925)}{(2764 - 1925)} = 1253 \text{ kN}$$

Step 4

Groundline deflection

$T_b = 2.58$ m for $P_u = 1253$ kN

$$\text{Now } y_g^{-b} = \frac{2.43 \times 1176 \times (2.58)^3}{43.5 \times 10^4} = 0.1202 \text{ m} = 12.0 \text{ cm}$$

The above calculations indicate that the negative batter piles are more resistant to lateral loads than vertical or positive batter piles. Besides, the groundline deflections of the negative batter piles are less than the vertical and corresponding positive batter piles.

16.11 PROBLEMS

16.1 A reinforced concrete pile 50 cm square in section is driven into a medium dense sand to a depth of 20 m . The sand is in a submerged state. A lateral load of 50 kN is applied on the pile at a height of 5 m above the ground level. Compute the lateral deflection of the pile at ground level. Given: $n_h = 15$ MN/m^3, $EI = 115 \times 10^9$ kg-cm^2. The submerged unit weight of the soil is 8.75 kN/m^3.

16.2 If the pile given in Prob. 16.1 is fully restrained at the top, what is the deflection at ground level?

16.3 If the pile given in Prob. 16.1 is 3 m long, what will be the deflection at ground level (a) when the top of the pile is free, and (b) when the top of the pile is restrained? Use Broms' method.

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16.4 Refer to Prob. 16.1. Determine the ultimate lateral resistance of the pile by Broms' method. Use $\phi = 38°$. Assume $M_y = 250$ kN-m.

16.5 If the pile given in Prob. 16.1 is driven into saturated normally consolidated clay having an unconfined compressive strength of 70 kPa, what would be the ultimate lateral resistance of the soil under (a) a free-head condition, and (b) a fixed-condition? Make necessary assumptions for the yield strength of the material.

16.6 Refer to Prob. 16.1. Determine the lateral deflection of the pile at ground level by the direct method. Assume $m_y = 250$ kN-m and $e = 0$

16.7 Refer to Prob. 16.1. Determine the ultimate lateral resistance of the pile by the direct method.

16.8 A precast reinforced concrete pile of 30 cm diameter is driven to a depth of 10 m in a vertical direction into a medium dense sand which is in a semi-dry state. The value of the coefficient of soil modulus variation (n_h) may be assumed as equal to 0.8 kg/cm^3. A lateral load of 40 kN is applied at a height of 3 m above ground level. Compute (a) the deflection at ground level, and (b) the maximum bending moment on the pile (assume $E = 2.1 \times 10^5$ kg/cm^2).

16.9 Refer to Prob. 16.8. Solve the problem by the direct method. All the other data remain the same. Assume $\phi = 38°$ and $\gamma = 16.5$ kN/m^3.

16.10 If the pile in Prob. 16.9 is driven at a batter of 22.5° to the vertical, and lateral load is applied at ground level, compute the normal deflection at ground level, for the cases of the load acting in the direction of batter and against the batter.

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