CHAPTER 20

SHEET PILE WALLS AND BRACED CUTS

20.1 INTRODUCTION

Sheet pile walls are retaining walls constructed to retain earth, water or any other fill material. These walls are thinner in section as compared to masonry walls described in Chapter 19. Sheet pile walls are generally used for the following:

1. Water front structures, for example, in building wharfs, quays, and piers
2. Building diversion dams, such as cofferdams
3. River bank protection
4. Retaining the sides of cuts made in earth

Sheet piles may be of timber, reinforced concrete or steel. Timber piling is used for short spans and to resist light lateral loads. They are mostly used for temporary structures such as braced sheeting in cuts. If used in permanent structures above the water level, they require preservative treatment and even then, their span of life is relatively short. Timber sheet piles are joined to each other by tongue-and-groove joints as indicated in Fig. 20.1. Timber piles are not suitable for driving in soils consisting of stones as the stones would dislodge the joints.

Figure 20.1 Timber pile wall section
Reinforced concrete sheet piles are precast concrete members, usually with a tongue-and-groove joint. Typical section of piles are shown in Fig. 20.2. These piles are relatively heavy and bulky. They displace large volumes of solid during driving. This large volume displacement of soil tends to increase the driving resistance. The design of piles has to take into account the large driving stresses and suitable reinforcement has to be provided for this purpose.

The most common types of piles used are steel sheet piles. Steel piles possess several advantages over the other types. Some of the important advantages are:

1. They are resistant to high driving stresses as developed in hard or rocky material
2. They are lighter in section
3. They may be used several times
4. They can be used either below or above water and possess longer life
5. Suitable joints which do not deform during driving can be provided to have a continuous wall
6. The pile length can be increased either by welding or bolting

Steel sheet piles are available in the market in several shapes. Some of the typical pile sections are shown in Fig. 20.3. The archweb and Z-piles are used to resist large bending moments, as in anchored or cantilever walls. Where the bending moments are less, shallow-arch piles with corresponding smaller section moduli can be used. Straight-web sheet piles are used where the web will be subjected to tension, as in cellular cofferdams. The ball-and-socket type of joints, Fig. 20.3 (d), offer less driving resistance than the thumb-and-finger joints, Fig. 20.3 (c).

20.2 SHEET PILE STRUCTURES

Steel sheet piles may conveniently be used in several civil engineering works. They may be used as:

1. Cantilever sheet piles
2. Anchored bulkheads
3. Braced sheeting in cuts
4. Single cell cofferdams
5. Cellular cofferdams, circular type
6. Cellular cofferdams (diaphragm)

Anchored bulkheads Fig. 20.4 (b) serve the same purpose as retaining walls. However, in contrast to retaining walls whose weight always represent an appreciable fraction of the weight of the sliding wedge, bulkheads consist of a single row of relatively light sheet piles of which the lower ends are driven into the earth and the upper ends are anchored by tie or anchor rods. The anchor rods are held in place by anchors which are buried in the backfill at a considerable distance from the bulkhead.

Anchored bulkheads are widely used for dock and harbor structures. This construction provides a vertical wall so that ships may tie up alongside, or to serve as a pier structure, which may jet out into the water. In these cases sheeting may be required to laterally support a fill on which railway lines, roads or warehouses may be constructed so that ship cargoes may be transferred to other areas. The use of an anchor rod tends to reduce the lateral deflection, the bending moment, and the depth of the penetration of the pile.

Cantilever sheet piles depend for their stability on an adequate embedment into the soil below the dredge line. Since the piles are fixed only at the bottom and are free at the top, they are called cantilever sheet piles. These piles are economical only for moderate wall heights, since the required section modulus increases rapidly with an increase in wall height, as the bending moment increases with the cube of the cantilevered height of the wall. The lateral deflection of this type of wall, because of the cantilever action, will be relatively large. Erosion and scour in front of the wall, i.e., lowering the dredge line, should be controlled since stability of the wall depends primarily on the developed passive pressure in front of the wall.

20.3 FREE CANTILEVER SHEET PILE WALLS

When the height of earth to be retained by sheet piling is small, the piling acts as a cantilever. The forces acting on sheet pile walls include:

1. The active earth pressure on the back of the wall which tries to push the wall away from the backfill
2. The passive pressure in front of the wall below the dredge line. The passive pressure resists the movements of the wall.

The active and passive pressure distributions on the wall are assumed hydrostatic. In the design of the wall, although the Coulomb approach considering wall friction tends to be more realistic, the Rankine approach (with the angle of wall friction $\delta = 0$) is normally used.

The pressure due to water may be neglected if the water levels on both sides of the wall are the same. If the difference in level is considerable, the effect of the difference on the pressure will have to be considered. Effective unit weights of soil should be considered in computing the active and passive pressures.

![Diagram of sheet pile structures](image)

**Figure 20.4 Use of sheet piles**

(a) Cantilever sheet piles  
(b) Anchored bulk head  
(c) Braced sheeting in cuts  
(d) Single cell cofferdam  
(e) Cellular cofferdam  
(f) Cellular cofferdam, diaphragm type  
(g) Double sheet pile walls
General Principle of Design of Free Cantilever Sheet Piling

The action of the earth pressure against cantilever sheet piling can be best illustrated by a simple case shown in Fig. 20.5 (a). In this case, the sheet piling is assumed to be perfectly rigid. When a horizontal force $P$ is applied at the top of the piling, the upper portion of the piling tilts in the direction of $P$ and the lower portion moves in the opposite direction as shown by a dashed line in the figure. Thus the piling rotates about a stationary point $O'$. The portion above $O'$ is subjected to a passive earth pressure from the soil on the left side of the piles and an active pressure on the right side of the piling, whereas the lower portion $O'g$ is subjected to a passive earth pressure on the right side and an active pressure on the left side of the piling. At point $O'$ the piling does not move and therefore is subjected to equal and opposite earth pressures (at-rest pressure from both sides) with a net pressure equal to zero. The net earth pressure (the difference between the passive and the active) is represented by $abO'c$ in Fig. 20.5 (b). For the purpose of design, the curve $bO'c$ is replaced by a straight line $dc$. Point $d$ is located at such a location on the line $af$ that the sheet piling is in static equilibrium under the action of force $P$ and the earth pressures represented by the areas $ade$ and $ecg$. The position of point $d$ can be determined by a trial and error method.

This discussion leads to the conclusion that cantilever sheet piling derives its stability from passive earth pressure on both sides of the piling. However, the distribution of earth pressure is different between sheet piling in granular soils and sheet piling in cohesive soils. The pressure distribution is likely to change with time for sheet pilings in clay.

20.4 DEPTH OF EMBEDMENT OF CANTILEVER WALLS IN SANDY SOILS

Case 1: With Water Table at Great Depth

The active pressure acting on the back of the wall tries to move the wall away from the backfill. If the depth of embedment is adequate the wall rotates about a point $O'$ situated above the bottom of the wall as shown in Fig. 20.6 (a). The types of pressure that act on the wall when rotation is likely to take place about $O'$ are:

1. Active earth pressure at the back of wall from the surface of the backfill down to the point of rotation, $O'$. The pressure is designated as $P_{at}$. 

![Figure 20.5 Example illustrating earth pressure on cantilever sheet piling](image-url)
2. Passive earth pressure in front of the wall from the point of rotation $O'$ to the dredge line. This pressure is designated as $P_{p1}$.

3. Active earth pressure in front of the wall from the point of rotation to the bottom of the wall. This pressure is designated as $P_{a2}$.

4. Passive earth pressure at the back of wall from the point of rotation $O'$ to the bottom of the wall. This pressure is designated as $P_{p2}$.

The pressures acting on the wall are shown in Fig. 20.6 (a).

If the passive and active pressures are algebraically combined, the resultant pressure distribution below the dredge line will be as given in Fig. 20.6 (b). The various notations used are:

- $D$ = minimum depth of embedment with a factor of safety equal to 1
- $K_A$ = Rankine active earth pressure coefficient
- $K_p$ = Rankine passive earth pressure coefficient
- $K = K_p - K_A$
- $p_a$ = effective active earth pressure acting against the sheet pile at the dredge line
  
  \[ p_a = \gamma HK_A \]
- $p_p$ = effective passive earth pressure at the base of the pile wall and acting towards the backfill
  
  \[ p_p = \gamma D_o K \]

**Figure 20.6** Pressure distribution on a cantilever wall.
\( \tilde{p}'_p \) = effective passive earth pressure at the base of the sheet pile wall acting against the backfill side of the wall = \( p'_p + \gamma KD_0 \)

\( \tilde{p}''_p \) = effective passive earth pressure at level of \( O = \gamma y_0 K + \gamma HK_p \)

\( \gamma \) = effective unit weight of the soil assumed the same below and above dredge line

\( y_0 \) = depth of point \( O \) below dredge line where the active and passive pressures are equal

\( \bar{y} \) = height of point of application of the total active pressure \( P_a \) above point \( O \)

\( h \) = height of point \( G \) above the base of the wall

\( D_0 \) = height of point \( O \) above the base of the wall

**Expression for \( y_0 \)**

At point \( O \), the passive pressure acting towards the right should equal the active pressure acting towards the left, that is

\[ \gamma y_0 K_p = \gamma (H + y_0) K_A \]

Therefore, \( \gamma y_0 (K_p - K_A) = \gamma HK_A \)

\[ y_0 = \frac{\gamma HK_A}{\gamma (K_p - K_A)} = \frac{p_a}{\gamma K} \tag{20.1} \]

**Expression for \( h \)**

For static equilibrium, the sum of all the forces in the horizontal direction must equal zero. That is

\[ P_a - \frac{1}{2} \tilde{p}_p (D - y_0) + \frac{1}{2} (\tilde{p}_p + \tilde{p}'_p) h = 0 \]

Solving for \( h \),

\[ h = \frac{\tilde{p}_p (D - y_0) - 2P_a}{\tilde{p}_p + \tilde{p}'_p} \tag{20.2} \]

Taking moments of all the forces about the bottom of the pile, and equating to zero,

\[ P_a (D_0 + \bar{y}) - \frac{1}{2} \tilde{p}_p x D_0 \times \frac{D_0}{3} + \frac{1}{2} (\tilde{p}_p + \tilde{p}'_p) x h \times \frac{h}{3} = 0 \]

or

\[ 6P_a (D_0 + \bar{y}) - \tilde{p}_p D_0^2 + (\tilde{p}_p + \tilde{p}'_p) h^2 = 0 \tag{20.3} \]

Therefore,

\( \tilde{p}_p = \gamma K D_0 \)

\( \tilde{p}'_p = \tilde{p}''_p + \gamma KD_0 \)

Substituting in Eq. (20.3) for \( \tilde{p}_p, \tilde{p}'_p \) and \( h \) and simplifying,
\[ D_0^4 + C_1 D_0^3 + C_2 D_0^2 + C_3 D_0 + C_4 = 0 \]  \hspace{1cm} (20.4)

where,
\[ C_1 = \frac{\bar{p}_p''}{\gamma K} \]
\[ C_2 = -\frac{8P_a}{\gamma K} \]
\[ C_3 = -\frac{6P_a}{(\gamma K)^2} \left(2\bar{y}K + \bar{p}_p''\right) \]
\[ C_4 = -\frac{6P_a\bar{y}\bar{p}_p'' + 4P_a^2}{(\gamma K)^2} \]

The solution of Eq. (20.4) gives the depth \( D_0 \). The method of trial and error is generally adopted to solve this equation. The minimum depth of embedment \( D \) with a factor of safety equal to 1 is therefore
\[ D = D_0 + y_0 \]  \hspace{1cm} (20.5)

A minimum factor of safety of 1.5 to 2 may be obtained by increasing the minimum depth \( D \) by 20 to 40 percent.

**Maximum Bending Moment**

The maximum moment on section \( AB \) in Fig. 20.6(b) occurs at the point of zero shear. This point occurs below point \( O \) in the figure. Let this point be represented by point \( C \) at a depth \( y_0 \) below point \( O \). The net pressure (passive) of the triangle \( OCC' \) must balance the net active pressure \( P_a \) acting above the dredge line. The equation for \( P_a \) is
\[ P_a = \frac{1}{2} \bar{y}_0^2 (K_p - K_A) = \frac{1}{2} \bar{y}_0^2 \gamma K \]
\[ \text{or} \quad \bar{y}_0 = \sqrt{\frac{2P_a}{\gamma K}} \]  \hspace{1cm} (20.6)

where \( \gamma \) = effective unit weight of the soil. If the water table lies above point \( O \), \( \gamma \) will be equal to \( \gamma_b \), the submerged unit weight of the soil.

Once the point of zero shear is known, the magnitude of the maximum bending moment may be obtained as
\[ M_{\text{max}} = P_a (\bar{y} + \bar{y}_0) - \frac{1}{3} \frac{1}{2} \bar{y}_0^2 \gamma K \quad \bar{y}_0 = P_a (\bar{y} + \bar{y}_0) - \frac{1}{6} \bar{y}_0^3 \gamma K \]  \hspace{1cm} (20.7)

The section modulus \( Z_s \) of the sheet pile may be obtained from the equation
\[ Z_s = \frac{M_{\text{max}}}{f_b} \]  \hspace{1cm} (20.8)

where, \( f_b \) = allowable flexural stress of the sheet pile.
Simplified Method

The solution of the fourth degree equation is quite laborious and the problem can be simplified by assuming the passive pressure $p'_p$ (Fig. 20.6) as a concentrated force $R$ acting at the foot of the pile. The simplified arrangement is shown in Fig. 20.7.

For equilibrium, the moments of the active pressure on the right and passive resistance on the left about the point of reaction $R$ must balance.

$$\frac{1}{3} P_p D - \frac{P_a}{3} (H + D) = 0$$

Now, $P_p = \frac{1}{2} K_p y D^2$ and $P_a = \frac{1}{2} K_A y (H + D)^2$

Therefore, $K_p D^3 - K_A (H + D)^3 = 0$

or $KD^3 - 3HD(H + D)K_A = 0$. (20.9)

The solution of Eq. (20.9) gives a value for $D$ which is at least a guide to the required depth. The depth calculated should be increased by at least 20 percent to provide a factor of safety and to allow extra length to develop the passive pressure $R$. An approximate depth of embedment may be obtained from Table 20.1.

Case 2: With Water Table Within the Backfill

Figure 20.8 gives the pressure distribution against the wall with a water table at a depth $h_1$ below the ground level. All the notations given in Fig. 20.8 are the same as those given in Fig. 20.6. In this case the soil above the water table has an effective unit weight $\gamma$ and a saturated unit weight $\gamma_{sat}$ below the water table. The submerged unit weight is
Table 20.1 Approximate penetration (D) of sheet piling

<table>
<thead>
<tr>
<th>Relative density</th>
<th>Depth, D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very loose</td>
<td>2.0 H</td>
</tr>
<tr>
<td>Loose</td>
<td>1.5 H</td>
</tr>
<tr>
<td>Firm</td>
<td>1.0 H</td>
</tr>
<tr>
<td>Dense</td>
<td>0.75 H</td>
</tr>
</tbody>
</table>

Source: Teng, 1969.

\[ \gamma_b = (\gamma_{sat} - \gamma_w) \]

The active pressure at the water table is

\[ p_1 = \gamma h_1 K \]

and \( p_a \) at the dredge line is

\[ p_a = \gamma h_1 K + \gamma_b h_2 K = (\gamma h_1 + \gamma_b h_2) K \]

The other expressions are

\[ p_p^\prime = \gamma_b D_o K \]
\[ p_p^\prime = p_p^\prime + \gamma_b D_o K \]
\[ p_p^\prime = \gamma_b y_o K + (\gamma h_1 + \gamma_b h_2) K \]
\[ y_o = \frac{p_a}{\gamma_b K} \]

\[ P_a = \frac{1}{2} p_p (D - y_o) - \frac{1}{2} ( p_p + p_p^\prime) h \]
\[ h = \frac{p_p (D - y_o) - 2P_a}{p_p + p_p^\prime} \]

The fourth degree equation in terms of \( D_o \) is

\[ D_o^4 + C_1 D_o^3 + C_2 D_o^2 + C_3 D_o + C_4 = 0 \quad (20.10) \]

where,

\[ C_1 = \frac{p_p^\prime}{\gamma_b K} \]
\[ C_2 = -\frac{8P_a}{\gamma_b K} \]
\[ C_3 = -\frac{6P_a}{(\gamma_b K)^2} \left( 2\gamma y_b K + p_p^\prime \right) \]
\[ C_4 = -\frac{6P_a \gamma_b p_p^\prime + 4P_a^2}{(\gamma_b K)^2} \]
The depth of embedment can be determined as in the previous case and also the maximum bending moment can be calculated. The depth $D$ computed should be increased by 20 to 40 percent.

**Case 3: When the Cantilever is Free Standing with No Backfill (Fig. 20.9)**

The cantilever is subjected to a line load of $P$ per unit length of wall. The expressions can be developed on the same lines explained earlier for cantilever walls with backfill. The various expressions are

$$h = \frac{\gamma KD^2 - 2P}{2\gamma DK}$$

where $K = (K_p - K_s)$

The fourth degree equation in $D$ is

$$D^4 + C_1 D^2 + C_2 D + C_3 = 0 \quad (20.11)$$

where

$$C_1 = -\frac{8P}{\gamma K}$$

$$C_2 = -\frac{12PH}{\gamma K}$$
Figure 20.9  Free standing cantilever with no backfill

\[ C_3 = -\frac{4P^2}{(\gamma K)^2} \]

Equation (20.11) gives the theoretical depth \( D \) which should be increased by 20 to 40 percent. Point \( C \) in Fig. 20.9 is the point of zero shear. Therefore,

\[ M_{\text{max}} = P(H + \bar{y}_o) - \frac{\bar{y}_o^3 K}{6} \]  

(20.12)

where \( \gamma \) = effective unit weight of the soil

Example 20.1

Determine the depth of embedment for the sheet-piling shown in Fig. Ex. 20.1a by rigorous analysis. Determine also the minimum section modulus. Assume an allowable flexural stress \( f_b^* = 175 \text{ MN/m}^2 \). The soil has an effective unit weight of 17 kN/m\(^3\) and angle of internal friction of 30°.

Solution

For \( \phi = 30° \), \( K_p = \tan^2 (45° - \phi/2) = \tan^2 30 = \frac{1}{3} \)

\[ K_p = \frac{1}{K_A} = \frac{1}{3}, \quad K = K_p - K_A = 3 - \frac{1}{3} = 2.67. \]

The pressure distribution along the sheet pile is assumed as shown in Fig. Ex. 20.1(b)

\[ p_a = \gamma HK_A = 17 \times 6 \times \frac{1}{3} = 34 \text{ kN/m}^2 \]
From Eq (20.1)
\[ y_0 = \frac{\bar{p}_a}{\gamma(K_p - K_A)} = \frac{34}{17 \times 2.67} = 0.75 \text{ m}. \]

\[ P_a = \frac{1}{2} \bar{p}_a H + \frac{1}{2} \bar{p}_a y_0 = \frac{1}{2} \times 34 \times 6 + \frac{1}{2} \times 34 \times 0.75 \]

\[ = 102 + 12.75 = 114.75 \text{ kN/meter length of wall or say } 115 \text{ kN/m}. \]

\[ \bar{p}_p = \gamma D(K_p - K_A) - \bar{p}_a = 17 \times D \times 2.67 - 34 = 45.4D - 34 \]

\[ \bar{p}_p' = \gamma HK_p + \gamma(D(K_p - K_A)) = 17 \times 6 \times 3 + 17 \times D \times 2.67 = 306 + 45.4D \]

\[ \bar{p}_p'' = \gamma HK_p + \gamma y_0 (K_p - K_A) = 17 \times 6 \times 3 + 17 \times 0.75 \times 2.67 = 340 \text{ kN/m}^2 \]

To find \( y \)

\[ P_a \bar{y} = \frac{1}{2} \bar{p}_a H \left( \frac{H}{3} + y_0 \right) + \frac{1}{2} \bar{p}_a y_0 \left( \frac{2}{3} y_0 \right) \]

\[ = \frac{1}{2} \times 34 \times 6 \times (2 + 0.75) + \frac{1}{2} \times 34 \times 0.75 \times \frac{2}{3} \times 0.75 = 286.9 \]

Therefore, \( \bar{y} = \frac{286.9}{P_a} = \frac{286.9}{115} = 2.50 \text{ m}. \)

Now \( D_0 \) can be found from Eq. (20.4), namely

\[ D_0^4 + C_1 D_0^3 + C_2 D_0^2 + C_3 D_0 + C_4 = 0 \]

\[ C_1 = \frac{\bar{p}_p''}{\gamma K} = \frac{340}{17 \times 2.67} = 7.49, \quad C_2 = \frac{-8 \bar{p}_a}{\gamma K} = \frac{-8 \times 115}{17 \times 2.67} = -20.3 \]

\[ C_3 = -\frac{6 \bar{p}_a}{(\gamma K)^2} (2\bar{y}K + \bar{p}_p'') = -\frac{6 \times 115}{(17 \times 2.67)^2} (2 \times 2.50 \times 17 \times 2.67 + 340) = -189.9 \]

\[ C_4 = -\frac{6 P_a \bar{y} \bar{p}_p'' + 4 \bar{p}_a^2}{(\gamma K)^2} = -\frac{6 \times 115 \times 2.50 \times 340 + 4 \times (115)^2}{(17 \times 2.67)^2} = -310.4 \]

Substituting for \( C_1, C_2, C_3 \) and \( C_4 \), and simplifying we have

\[ D_0^4 + 7.49 D_0^3 - 20.3 D_0^2 - 189.9 D_0 - 310.4 = 0 \]

This equation when solved by the method of trial and error gives

\[ D_0 = 5.3 \text{ m} \]

**Depth of Embedment**

\[ D = D_0 + y_0 = 5.3 + 0.75 = 6.05 \text{ m} \]

Increasing \( D \) by 40%, we have

\[ D \text{ (design)} = 1.4 \times 6.05 = 8.47 \text{ m or say 8.5 m}. \]
(c) Section modulus

From Eq. (20.6) (The point of zero shear)

\[
\bar{y}_o = \frac{2P_a}{\gamma K} = \frac{2 \times 115}{17 \times 2.67} = 2.25 \text{ m}
\]

\[
M_{\text{max}} = P_a \left( \bar{y} + \bar{y}_o \right) - \frac{1}{6} \bar{y}_o^3 \gamma K
\]

\[
= 115(2.50 + 2.25) - \frac{1}{6}(2.25)^3 \times 17 \times 2.67
\]

\[
= 546.3 - 86.2 = 460 \text{ kN-m/m}
\]

From Eq. (20.8)

Section modulus

\[
Z_i = \frac{M_{\text{max}}}{f_b} = \frac{460}{175 \times 10^3} = 26.25 \times 10^{-2} \text{ m}^3/\text{m of wall}
\]

**Example 20.2**

Fig. Ex. 20.2 shows a free standing cantilever sheet pile with no backfill driven into homogeneous sand. The following data are available:

\[
H = 20 \text{ ft}, \quad P = 3000 \text{ lb/ft of wall}, \quad \gamma = 115 \text{ lb/ft}^3, \quad \phi = 36^\circ.
\]

Determine: (a) the depth of penetration, D, and (b) the maximum bending moment \(M_{\text{max}}\).
Solution

\[ K_p = \tan^2 45^\circ + \frac{\phi}{2} = \tan^2 45^\circ + \frac{36^\circ}{2} = 3.85 \]

\[ K_A = \frac{1}{K_p} = \frac{1}{3.85} = 0.26 \]

\[ K = K_p - K_A = 3.85 - 0.26 = 3.59 \]

The equation for \( D \) is (Eq 20.11)

\[ D^4 + C_1 D^2 + C_2 D + C_3 = 0 \]

where

\[ C_1 = -\frac{8P}{\gamma K} = -\frac{8 \times 3000}{115 \times 3.59} = -58.133 \]

\[ C_2 = -\frac{12PH}{\gamma K} = -\frac{12 \times 3000 \times 20}{115 \times 3.59} = -1744 \]

\[ C_3 = -\frac{4P^2}{(\gamma K)^2} = -\frac{4 \times 3000^2}{(115 \times 3.59)^2} = -211.2 \]

Substituting and simplifying, we have

\[ D^4 - 58.133 D^2 - 1744 D - 211.2 = 0 \]

From the above equation \( D = 13.5 \) ft.

From Eq. (20.6)

\[ \bar{y}_0 = \frac{2P}{\gamma K} = \frac{2 \times 3000}{115 \times 3.59} = 3.81 \text{ ft} \]
From Eq. (20.12)

\[
M_{\text{max}} = P_a (H + \gamma_z) - \frac{\gamma \gamma_z^3 K}{6}
\]

\[
= 3000(20 + 3.81) - \frac{115 \times (3.81)^3 \times 3.59}{6}
\]

\[
= 71,430 - 3,806 = 67,624 \text{ lb-ft/ft of wall}
\]

\[
M_{\text{max}} = 67,624 \text{ lb-ft/ft of wall}
\]

20.5 DEPTH OF EMBEDMENT OF CANTILEVER WALLS IN COHESIVE SOILS

Case 1: When the Backfill is Cohesive Soil

The pressure distribution on a sheet pile wall is shown in Fig. 20.10.

The active pressure \( p_a \) at any depth \( z \) may be expressed as

\[
p_a = \sigma_v K_A - 2c \sqrt{K_A}
\]

where

\[
\sigma_v = \text{vertical pressure, } \gamma z
\]

\[
z = \text{depth from the surface of the backfill.}
\]

The passive pressure \( p_p \) at any depth \( y \) below the dredge line may be expressed as

\[
p_p = \sigma_y K_p + 2c \sqrt{K_p}
\]

The active pressure distribution on the wall from the backfill surface to the dredge line is shown in Fig. 20.10. The soil is supposed to be in tension up to a depth of \( z_0 \) and the pressure on the wall is zero in this zone. The net pressure distribution on the wall is shown by the shaded triangle. At the dredge line (at point A)

(a) The active pressure \( \bar{p}_a \) acting towards the left is

\[
\bar{p}_a = \gamma H K_A - 2c \sqrt{K_A}
\]

when \( \phi = 0 \)

\[
\bar{p}_a = \gamma H - 2c = \gamma H - q_u
\]

(20.13a)

where \( q_u = \text{unconfined compressive strength of the clay soil} = 2c. \)

(b) The passive pressure acting towards the right at the dredge line is

\[
\bar{p}_p = 2c \quad \text{since } \phi = 0
\]

or \( \bar{p}_p = q_u \)

The resultant of the passive and active pressures at the dredge line is

\[
\bar{p}_p = \bar{p}_a = q_u - (\gamma H - q_u) = 2q_u - \gamma H - \bar{p}
\]

(20.13b)

The resultant of the passive and active pressures at any depth \( y \) below the dredge line is
passive pressure, \( p_p = \gamma y + q_u \)

active pressure, \( p_a = \gamma (H + y) - q_u \)

The resultant pressure is

\[ p_p - p_a = \bar{p} = (\gamma y + q_u) - (\gamma (H + y) - q_u) = 2q_u - \gamma H \] (20.14)

Equations (20.13b) and (20.14) indicate that the resultant pressure remains constant at \((2q_u - \gamma H)\) at all depths.

If passive pressure is developed on the backfill side at the bottom of the pile (point B), then

\[ p_p = \gamma (H + D) + q_u \] acting towards the left
\[ p_a = \gamma D - q_u \] acting towards the right

The resultant is

\[ p_p - p_a = \gamma (H + D) + q_u - \gamma D + q_u = \gamma H + 2q_u = \bar{p}' \] (20.15)

For static equilibrium, the sum of all the horizontal forces must be equal zero, that is,

\[ P_a - (2q_u - \gamma H)D + \frac{1}{2} (2q_u + 2q_u)h = 0 \]

Simplifying,

\[ P_a + 2q_u h - 2q_u D + \gamma HD = 0 \], therefore,

\[ h = \frac{D(2q_u - \gamma H) - P_a}{2q_u} \] (20.16)

Also, for equilibrium, the sum of the moments at any point should be zero. Taking moments about the base,

\[ P_a (\gamma + D) + \frac{h^2}{6} (2q_u) - \frac{(2q_u - \gamma H)D^2}{2} = 0 \] (20.17)

Substituting for \( h \) in (Eq. 20.17) and simplifying,

\[ C_1 D^2 + C_2 D + C_3 = 0 \] (20.18)

where

\[ C_1 = (2q_u - \gamma H) \]
\[ C_2 = -2P_a \]
\[ C_3 = -\frac{P_a (6q_u \gamma + P_a)}{(q_u + \gamma H)} \]

The depth computed from Eq. (20.18) should be increased by 20 to 40 percent so that a factor of safety of 1.5 to 2.0 may be obtained. Alternatively the unconfined compressive strength \( q_u \) may be divided by a factor of safety.
Figure 20.10  Depth of embedment of a cantilever wall in cohesive soil

Limiting Height of Wall

Equation (20.14) indicates that when \((2q_u - \gamma H) = 0\) the resultant pressure is zero. The wall will not be stable. In order that the wall may be stable, the condition that must be satisfied is

\[
\frac{2q_u}{F} \geq \gamma H
\]

where \(F\) = factor of safety.

Maximum Bending Moment

As per Fig. 20.10, the maximum bending moment may occur within the depth \((D-h)\) below the dredge line. Let this depth be \(\bar{y}_o\) below the dredge line for zero shear. We may write,

\[P_o - \bar{p} \bar{y}_o = 0\]

or

\[\bar{y}_o = \frac{P_o}{\bar{p}}\]  \hspace{1cm} (20.20a)

The expression for maximum bending moment is,

\[M_{\text{max}} = P_o (\bar{y}_o + \bar{y}) - \frac{\bar{p} \bar{y}_o^2}{2}\]  \hspace{1cm} (20.20b)

where \(\bar{p} = 2q_u - \gamma H\)

The section modulus of the sheet pile may now be calculated as before.
Case 2: When the Backfill is Sand with Water Table at Great Depth

Figure 20.11 gives a case where the backfill is sand with no water table within. The following relationships may be written as:

\[
\begin{align*}
\bar{p}_u &= \gamma H K_A \\
p &= 2q_u - \gamma H = 4c - \gamma H \\
p' &= 2q_u + \gamma H = 4c + \gamma H \\
P_a &= \frac{1}{2} \gamma H^2 K_A \\
h &= \frac{D(2q_u - \gamma H) - \frac{1}{2} \gamma H^2 K_A}{2q_u}
\end{align*}
\]  

(20.21)

A second degree equation in \( D \) can be developed as before

\[
C_1 D^2 + C_2 D + C_3 = 0
\]

(20.22)

where

\[
\begin{align*}
C_1 &= (2q_u - \gamma H) \\
C_2 &= -2P_a \\
C_3 &= -\frac{P_a(P_a + 6q_u \bar{\gamma})}{\gamma H + q_u}
\end{align*}
\]

where \( \frac{\gamma}{3} = \frac{H}{3}, \quad q_u = 2c \)

An expression for computing maximum bending moment may be written as

\[
M_{\text{max}} = P_u (\bar{\gamma} + \bar{\gamma}_o) - \frac{(2q_u - \gamma H)\gamma c^3}{2}
\]

(20.23)

Figure 20.11 Sheet pile wall embedded in clay with sand backfill.
Case 3: Cantilever Wall with Sand Backfill and Water Table Above Dredge Line [Fig. 20.12]

The various expressions for this case may be developed as in the earlier cases. The various relationships may be written as

\[
\begin{align*}
\bar{p}_1 &= \gamma h_1 K_A \\
\bar{p}_a &= \gamma h_1 K_A + \gamma_b h_2 K_A = (\gamma h_1 + \gamma_b h_2) K_A \\
\bar{p} &= 2q_a - \gamma H \\
\bar{p}' &= 2q_a + (\gamma h_1 + \gamma_b h_2) \\
\bar{P}_a &= \frac{1}{2} \left( \bar{p}_1 H + \bar{p}_a h_2 \right) \\
h &= \frac{[2q_a - (\gamma h_1 + \gamma_b h_2)]D - P_a}{2q_a}
\end{align*}
\]

(20.25)

(20.26)

The expression for the second degree equation in \( D \) is

\[C_1 D^2 + C_2 D + C_3 = 0\]

(20.27)

where

\[C_1 = [2q_a - (\gamma h_1 + \gamma_b h_2)]\]

\[C_2 = -2 \bar{P}_a\]

Figure 20.12  Cantilever wall with sand backfill and water table
Eq (20.27) may be solved for $D$. The depth computed should be increased by 20 to 40% to obtain a factor of safety of 1.5 to 2.0.

**Case 4: Free-Standing Cantilever Sheet Pile Wall Penetrating Clay**

Figure 20.13 shows a freestanding cantilever wall penetrating clay. An expression for $D$ can be developed as before. The various relationships are given below.

$\bar{p} = 2 q_u = \bar{p}'$

The expression for $D$ is

$$C_1 D^2 + C_2 D + C_3 = 0$$

where

- $C_1 = 2 q_u$
- $C_2 = -2P$
- $C_3 = -\left(\frac{P + 6 q_u H}{q_u}\right)$

The expression for $h$ is

$$h = \frac{2q_u D - P}{2q_u}$$

The maximum moment may be calculated per unit length of wall by using the expression

$$M_{\text{max}} = P(H + \bar{y}_o) - \frac{2q_u \bar{y}_o^2}{2}$$

---

**Figure 20.13** Free standing cantilever wall penetrating clay
where $\bar{y}_o = \frac{P}{2q_u} = \text{depth to the point of zero shear.}$ \hfill (20.31)

**Example 20.3**

Solve Example 20.1, if the soil is clay having an unconfined compressive strength of 70 kN/m$^2$ and a unit weight of 17 kN/m$^3$. Determine the maximum bending moment.

**Solution**

The pressure distribution is assumed as shown in Fig. Ex. 20.3.

For $\phi_u = 0$, $\bar{p}_a = \gamma H - q_u = 17 \times 6 - 70 = 32 \text{ kN/m}^2$

\[
\begin{align*}
\text{Figure Ex. 20.3} \\
z_0 &= \frac{q_u}{\gamma} = \frac{70}{17} = 4.12 \text{ m} \\
P_a &= \frac{1}{2} \bar{p}_a (H - z_0) = \frac{1}{2} \times 32 \times (6.0 - 4.12) = 30 \text{ kN/m of wall} \\
\bar{p} &= 2q_u - \gamma H = 2 \times 70 - 17 \times 6 = 38 \text{ kN/m}^2 \\
\bar{p}' &= 2q_u + \gamma H = 2 \times 70 + 17 \times 6 = 242 \text{ kN/m}^2
\end{align*}
\]
\[ y = \frac{1}{3}(H - z_0) = \frac{1}{3}(6 - 4.12) = 0.63 \text{ m} \]

For the determination of \( h \), equate the summation of all horizontal forces to zero, thus

\[ P_a - \bar{p} \times D + \frac{1}{2}(\bar{p} + \bar{p}' )h = 0 \]

or

\[ 30 - 38 \times D + \frac{1}{2}(38 + 242)h = 0 \]

Therefore

\[ h = \frac{38D - 3}{14} \]

For the determination of \( D \), taking moments of all the forces about the base of the wall, we have

\[ P_a \times (D + y) - \bar{p} \times D^2 + (\bar{p} + \bar{p}') \times \frac{h}{2} \times \frac{h}{3} = 0 \]

or

\[ 30(D + 0.63) - 38 \times \frac{D^2}{2} + (38 + 242) \times \frac{h^2}{6} = 0 \]

Substituting for \( h \) we have,

\[ 3D + 1.89 - 1.9D^2 + 4.7 \times \frac{38D - 3}{14} = 0 \]

Simplifying, we have

\[ D^2 - 1.57D + 1.35 = 0 \]

Solving \( D = 2.2 \text{ m} \); Increasing \( D \) by 40\%, we have \( D = 1.4(2.2) = 3.1 \text{ m} \).

Maximum bending moment

From Eq. (20.20)

\[ M_{\text{max}} = P_a (\bar{y}_0 + \bar{y}) - \frac{\bar{p} \bar{y}_0^2}{2} \]

\[ \bar{y}_0 = \frac{P_a}{\bar{p}} = \frac{30}{38} = 0.79 \text{ m} \]

\[ \bar{y} = 0.63 \text{ m} \]

\[ M_{\text{max}} = 30(0.79 + 0.63) - \frac{38 \times (0.79)^2}{2} = 42.6 - 11.9 = 30.7 \text{ kN-m/m of wall} \]

---

**Example 20.4**

Solve Example 20.1 if the soil below the dredge line is clay having a cohesion of 35 kN/m² and the backfill is sand having an angle of internal friction of 30°. The unit weight of both the soils may be assumed as 17 kN/m³. Determine the maximum bending moment.
Solution
Refer to Fig. Ex. 20.4

\[
\bar{p}_a = \gamma H K_a = 17 \times 6 \times \frac{1}{3} = 34 \text{ kN} / \text{m}^3
\]

\[P_a = \frac{1}{2} \times 34 \times 6 = 102 \text{ kN/m of wall}\]

\[
\bar{p} = 2 q_a - \gamma H = 2 \times 2 \times 35 - 17 \times 6 = 38 \text{ kN} / \text{m}^2
\]

From Eq. (20.22)

\[C_1 D^2 + C_2 D + C_3 = 0\]

where

\[C_1 = (2 q_a - \gamma H) = (2 \times 2 \times 35 - 17 \times 6) = 38 \text{ kN/m}^2 = \bar{p}\]

\[C_2 = -2 P_a = -2 \times 102 = -204 \text{ kN}\]

\[C_3 = \frac{P_a (P_a + 6 q_a \bar{y})}{\gamma H + q_a}
= \frac{102(102 + 6 \times 70 \times 2)}{17 \times 6 + 70} = -558.63\]

Substituting and simplifying

\[38 D^2 - 204 D - 558.63 = 0\]

or

\[D^2 - 5.37 D - 14.7 = 0\]

Solving the equations, we have \(D = 7.37 \text{ m}\)

Increasing \(D\) by 40%, we have

\(D\) (design) = 1.4 \(7.37\) = 10.3 m

---

![Diagram](image)
Maximum Bending Moment

From Eq. (20.23)

\[ M_{\text{max}} = P_a (\bar{y} + \bar{y}_0) - \frac{p \bar{y}_0^2}{2} \]

\[ \bar{y} = \frac{H}{3} = \frac{6}{3} = 2 \text{ m}, \quad p = 38 \text{ kN/m}^2 \]

\[ \bar{y}_0 = \frac{P_a}{\bar{p}} = \frac{\gamma H^2 K_A}{2 \bar{p}} = \frac{17 \times 6^2 \times 0.33}{2 \times 38} = 2.66 \text{ m} \]

\[ M_{\text{max}} = 102(2 + 2.66) - \frac{38 \times (2.66)^2}{2} = 475.32 - 134.44 = 340.9 \text{ kN-m/m of wall.} \]

\[ M_{\text{max}} = 340.9 \text{ kN-m/m of wall} \]

**Example 20.5**

Refer to Fig. Ex. 20.5. Solve the problem in Ex. 20.4 if the water table is above the dredge line.

Given: \( h_1 = 2.5 \text{ m}, \gamma_{\text{sat}} = 17 \text{ kN/m}^3 \)

Assume the soil above the water table remains saturated. All the other data given in Ex. 20.4 remain the same.

---

**Figure Ex. 20.5**
Solution

\[ h_1 = 2.5 \text{ m}, \quad h_2 = 6 - 2.5 = 3.5 \text{ m}, \quad \gamma_b = 17 - 9.81 = 7.19 \text{ kN/m}^3 \]

\[ p_1 = \gamma h_1 k_A = 17 \times 2.5 \times 1/3 = 14.17 \text{ kN/m}^2 \]

\[ p_a = p_1 + \gamma_b h_2 k_A = 14.17 + 7.19 \times 3.5 \times 1/3 = 22.56 \text{ kN/m}^2 \]

\[ P_a = \frac{1}{2} p_1 h_1 + p_1 h_2 + \frac{1}{2} (p_a - p_1) h_2 \]

\[ = \frac{1}{2} \times 14.17 \times 2.5 + 14.17 \times 3.5 + \frac{1}{2} (22.56 - 14.17) \times 3.5 \]

\[ = 17.71 + 49.6 + 14.7 = 82 \text{ kN/m} \]

Determination of \( \bar{y} \) (Refer to Fig. 20.12)

Taking moments of all the forces above dredge line about \( C \) we have

\[ 82 \bar{y} = 17.71 \left( \frac{3.5 + \frac{2.5}{3}}{2} \right) + 49.6 \times \frac{3.5}{2} + 14.7 \times \frac{3.5}{3} \]

\[ = 76.74 + 86.80 + 17.15 = 180.69 \]

\[ \bar{y} = \frac{180.69}{82} = 2.20 \text{ m} \]

From Eq. (20.27), the equation for \( D \) is

\[ C_1 D^2 + C_2 D + C_3 = 0 \]

where

\[ C_1 = \left[ 2 q_u - (\gamma h_1 + \gamma_b h_2) \right] = \left[ 140 - (17 \times 2.5 + 7.19 \times 3.5) \right] = 72.3 \]

\[ C_2 = -2 P_a = -2 \times 30 = -164 \]

\[ C_3 = - \frac{(P_a + 6 q_u \bar{y}) P_a}{q_u + (\gamma h_1 + \gamma_b h_2)} = - \frac{(82 + 6 \times 70 \times 2.2) \times 82}{70 + 17 \times 2.5 + 7.19 \times 3.5} = -599 \]

Substituting we have,

\[ 72.3 D^2 - 164 D - 599 = 0 \]

or

\[ D^2 - 2.27 D - 8.285 = 0 \]

solving we have \( D = 4.23 \text{ m} \)

Increasing \( D \) by 40%; the design value is

\( D \) (design) = 1.4(4.23) = 5.92 m

Example 20.6

Fig. Example 12.6 gives a freestanding sheet pile penetrating clay. Determine the depth of penetration. Given: \( H = 5 \text{ m}, \ P = 40 \text{ kN/m}, \) and \( q_u = 30 \text{ kN/m}^2 \).

Solution

From Eq. (20.13a)

\[ \bar{p} = 2 q_u - \gamma H = 2 q_u = 2 \times 30 = 60 \text{ kN/m}^2 \]
From Eq. (20.28), the expression for $D$ is

$$C_1 D^2 + C_2 D + C_3 = 0$$

where $C_1 = 2 q_u = 60$

$$C_2 = -2 P = -2 \times 40 = -80$$

$$C_3 = \frac{(P + 6 q_u H) P}{q_u} = \frac{(40 + 6 \times 30 \times 5) 40}{30} = -1253$$

Substituting and simplifying

$$60 D^2 - 80 D - 1253 = 0$$

or $D^2 - 1.33 D - 21 = 0$

Solving $D = 5.3$ m. Increasing by 40% we have

$D$ (design) = 1.4(5.3) = 7.42 m
From Eq. (20.29)

\[ h = \frac{2q_u D - P}{2q_u} = \frac{2 \times 30 \times 5.3 - 40}{2 \times 30} = 4.63 \text{ m} \]

### 20.6 ANCHORED BULKHEAD: FREE-EARTH SUPPORT METHOD— DEPTH OF EMBEDMENT OF ANCHORED SHEET PILES IN GRANULAR SOILS

If the sheet piles have been driven to a shallow depth, the deflection of a bulkhead is somewhat similar to that of a vertical elastic beam whose lower end \( B \) is simply supported and the other end is fixed as shown in Fig. 20.14. Bulkheads which satisfy this condition are called bulkheads with free earth support. There are two methods of applying the factor of safety in the design of bulkheads.

1. Compute the minimum depth of embedment and increase the value by 20 to 40 percent to give a factor of safety of 1.5 to 2.
2. The alternative method is to apply the factor of safety to \( K_p \) and determine the depth of embedment.

**Method 1: Minimum Depth of Embedment**

The water table is assumed to be at a depth \( h_1 \) from the surface of the backfill. The anchor rod is fixed at a height \( h_2 \) above the dredge line. The sheet pile is held in position by the anchor rod and the tension in the rod is \( T_a \). The forces that are acting on the sheet pile are

1. Active pressure due to the soil behind the pile,
2. Passive pressure due to the soil in front of the pile, and
3. The tension in the anchor rod.

The problem is to determine the minimum depth of embedment \( D \). The forces that are acting on the pile wall are shown in Fig. 20.15.

The resultant of the passive and active pressures acting below the dredge line is shown in Fig. 20.15. The distance \( y_0 \) to the point of zero pressure is

\[ y_0 = \frac{P_a - \gamma_s K}{\gamma_s} \]

The system is in equilibrium when the sum of the moments of all the forces about any point is zero. For convenience if the moments are taken about the anchor rod,

\[ P_a h_4 = P_a \gamma_a \]

But \( P_a = \frac{1}{2} \gamma_s D_0^2 \)

\[ h_4 = h_3 + y_0 + \frac{2}{3} D_a \]

Therefore,

\[ P_a \gamma_a = \frac{1}{2} \gamma_s D_0^2 \left( h_3 + y_0 + \frac{2}{3} D_0 \right) \]
Figure 20.14 Conditions for free-earth support of an anchored bulkhead

Simplifying the equation,

\[ C_1D_0^3 + C_2D_0^2 + C_3 = 0 \]  \hspace{1cm} (20.32)

Figure 20.15 Depth of embedment of an anchored bulkhead by the free-earth support method (method 1)
where

\[ C_1 = \frac{\gamma_b K}{3} \]

\[ C_2 = \left( \frac{\gamma_b K}{2} \right) (h_2 + y_0) \]

\[ C_3 = -P_a \bar{n} \]

\[ \gamma_b = \text{submerged unit weight of soil} \]

\[ K = K_p - K_A \]

The force in the anchor rod, \( T_a \), is found by summing the horizontal forces as

\[ T_a = P_a - P_p \quad (20.33) \]

The minimum depth of embedment is

\[ D = D_0 + y_0 \quad (20.34) \]

Increase the depth \( D \) by 20 to 40% to give a factor of safety of 1.5 to 2.0.

### Maximum Bending Moment

The maximum theoretical moment in this case may be at a point \( C \) any depth \( h_m \) below ground level which lies between \( h_1 \) and \( H \) where the shear is zero. The depth \( h_m \) may be determined from the equation

\[ \frac{1}{2} \bar{p}_1 h_1 - T_a + \bar{p}_1 (h_m - h_1) + \frac{1}{2} \gamma_b (h_m - h_1)^2 K_A = 0 \quad (20.35) \]

Once \( h_m \) is known the maximum bending moment can easily be calculated.

### Method 2: Depth of Embedment by Applying a Factor of Safety to \( K_p \)

(a) Granular Soil Both in the Backfill and Below the Dredge Line

The forces that are acting on the sheet pile wall are as shown in Fig. 20.16. The maximum passive pressure that can be mobilized is equal to the area of triangle \( ABC \) shown in the figure. The passive pressure that has to be used in the computation is the area of figure \( ABEF \) (shaded). The triangle \( ABC \) is divided by a vertical line \( EF \) such that

\[ \frac{\text{Area} \ ABC}{\text{Factor of safety}} = P'_p \]

The width of figure \( ABEF \) and the point of application of \( P'_p \) can be calculated without any difficulty.

Equilibrium of the system requires that the sum of all the horizontal forces and moments about any point, for instance, about the anchor rod, should be equal to zero.

Hence,

\[ P'_p + T_a - P_a = 0 \quad (20.36) \]

\[ P_a \bar{n} - P'_p h_4 = 0 \quad (20.37) \]

where,

\[ P'_p = \frac{1}{2} \gamma_b K_p D^2 + \frac{1}{F} \]
and \( F_s = \) assumed factor of safety.

The tension in the anchor rod may be found from Eq. (20.36) and from Eq. (20.37) \( D \) can be determined.

(b) Depth of Embedment when the Soil Below Dredge Line is Cohesive and the Backfill Granular

Figure 20.17 shows the pressure distribution.

The surcharge at the dredge line due to the backfill may be written as

\[
q = \gamma h_1 + \gamma_b h_2 = \gamma_e H
\]

(20.38)

where \( h_2 = \) depth of water above the dredge line, \( \gamma_e \) effective equivalent unit weight of the soil, and \( H = h_1 + h_2 \).

The active earth pressure acting towards the left at the dredge line is (when \( \phi = 0 \))

\[
\overline{p}_a = q - q_u
\]

The passive pressure acting towards the right is

\[
\overline{p}_p = q_u
\]

The resultant of the passive and active earth pressures is

\[
\overline{p}_p - \overline{p}_a = 2q_u - q = \overline{p}
\]

(20.39)
Figure 20.17  Depth of embedment when the soil below the dredge line is cohesive

The pressure remains constant with depth. Taking moments of all the forces about the anchor rod,

\[ P_a \tilde{y}_a - D(2q_v - q)(h_3 + D/2) = 0 \]  \hspace{1cm} (20.40)

where \( \tilde{y}_a \) = the distance of the anchor rod from \( P_a \).

Simplifying Eq. (15.40),

\[ D^2 + C_1 D + C_2 = 0 \]  \hspace{1cm} (20.41)

where \( C_1 = 2h_3 \)

\[ C_2 = \frac{2\tilde{y}_a P_a}{2q_v - q} \]

The force in the anchor rod is given by Eq. (20.33).

It can be seen from Eq. (20.39) that the wall will be unstable if

\[ 2q_v - q = 0 \]

or \[ 4c - q = 0 \]

For all practical purposes \( q = \gamma' H = \gamma H \), then Eq. (20.39) may be written as

\[ 4c - \gamma H = 0 \]

or \[ N_s = \frac{c}{\gamma H} = \frac{1}{4} = 0.25 \]  \hspace{1cm} (20.42)
Eq. (20.42) indicates that the wall is unstable if the ratio \( c/\gamma H \) is equal to 0.25. \( N_s \) is termed is Stability Number. The stability is a function of the wall height \( H \), but is relatively independent of the material used in developing \( q \). If the wall adhesion \( c_a \) is taken into account the stability number \( N_s \) becomes

\[
N_s = \frac{c}{\gamma H} \sqrt{1 + \frac{c_a}{c}}
\]

(20.43)

At passive failure \( \sqrt{1 + c_a/c} \) is approximately equal to 1.25.

The stability number for sheet pile walls embedded in cohesive soils may be written as

\[
N_s = \frac{1.25c}{\gamma H}
\]

(20.44)

When the factor of safety \( F_s = 1 \) and \( \frac{c}{\gamma H} = 0.25 \), \( N_s = 0.30 \).

The stability number \( N_s \) required in determining the depth of sheet pile walls is therefore

\[
N_s = 0.30 \times F_s
\]

(20.45)

The maximum bending moment occurs as per Eq. (20.35) at depth \( h_m \) which lies between \( h_i \) and \( H \).

### 20.7 DESIGN CHARTS FOR ANCHORED BULKHEADS IN SAND

Hagerty and Nofal (1992) provided a set of design charts for determining

1. The depth of embedment
2. The tensile force in the anchor rod and
3. The maximum moment in the sheet piling

The charts are applicable to sheet piling in sand and the analysis is based on the free-earth support method. The assumptions made for the preparation of the design charts are:

1. For active earth pressure, Coulomb's theory is valid
2. Logarithmic failure surface below the dredge line for the analysis of passive earth pressure.
3. The angle of friction remains the same above and below the dredge line
4. The angle of wall friction between the pile and the soil is \( \phi/2 \)

The various symbols used in the charts are the same as given in Fig. 20.15 where,

- \( h_a \) = the depth of the anchor rod below the backfill surface
- \( h_i \) = the depth of the water table from the backfill surface
- \( h_2 \) = depth of the water above dredge line
- \( H \) = height of the sheet pile wall above the dredge line
- \( D \) = the minimum depth of embedment required by the free-earth support method
- \( T_a \) = tensile force in the anchor rod per unit length of wall

Hagerty and Nofal developed the curves given in Fig. 20.18 on the assumption that the water table is at the ground level, that is \( h_i = 0 \). Then they applied correction factors for \( h_i > 0 \). These correction factors are given in Fig. 20.19. The equations for determining \( D, T_a \) and \( M_{(max)} \) are
Figure 20.18 Generalized (a) depth of embedment, $G_d$, (b) anchor force $G_f$, and (c) maximum moment $G_m$ (after Hagerty and Nofal, 1992)
Figure 20.19  Correction factors for variation of depth of water $h_t$, (a) depth correction $C_d$, (b) anchor force correction $C_f$ and (c) moment correction $C_m$ (after Hagerty and Nofal, 1992)
\[ D = G_d C_d H \]  
\[ T_d = G_t C_t \gamma_d H^2 \]  
\[ M_{(\text{max})} = G_m C_m \gamma_d H^3 \]

where,

- \( G_d \) = generalized non-dimensional embedment = \( D/H \) for \( h_1 = 0 \)
- \( G_t \) = generalized non-dimensional anchor force = \( T_d / (\gamma_d H^2) \) for \( h_1 = 0 \)
- \( G_m \) = generalized non-dimensional moment = \( M_{(\text{max})} / \gamma_d (H^3) \) for \( h_1 = 0 \)
- \( C_d, C_t, C_m \) = correction factors for \( h_1 > 0 \)
- \( \gamma_a \) = average effective unit weight of soil
- = \( (\gamma_m h_1^2 + \gamma_b h_2^2 + 2\gamma_m h_1 h_2) / H^2 \)
- \( \gamma_m \) = moist or dry unit weight of soil above the water table
- \( \gamma_b \) = submerged unit weight of soil

The theoretical depth \( D \) as calculated by the use of design charts has to be increased by 20 to 40% to give a factor of safety of 1.5 to 2.0 respectively.

### 20.8 MOMENT REDUCTION FOR ANCHORED SHEET PILE WALLS

The design of anchored sheet piling by the free-earth method is based on the assumption that the piling is perfectly rigid and the earth pressure distribution is hydrostatic, obeying classical earth pressure theory. In reality, the sheet piling is rather flexible and the earth pressure differs considerably from the hydrostatic distribution.

As such the bending moments \( M_{(\text{max})} \) calculated by the lateral earth pressure theories are higher than the actual values. Rowe (1952) suggested a procedure to reduce the calculated moments obtained by the free earth support method.

### Anchored Piling in Granular Soils

Rowe (1952) analyzed sheet piling in granular soils and stated that the following significant factors are required to be taken in the design:

1. The relative density of the soil
2. The relative flexibility of the piling which is expressed as

\[ \rho = 109 \times 10^{-6} \frac{H^4}{E I} \]  
\[ \rho = \frac{H^4}{E I} \]

where,
- \( \rho \) = flexibility number
- \( H \) = the total height of the piling in m
- \( E I \) = the modulus of elasticity and the moment of inertia of the piling (MN-m²) per m of wall

Eq. (20.47a) may be expressed in English units as

\[ \rho = \frac{H^4}{E I} \]  
\[ \rho = \frac{H^4}{E I} \]

where, \( H \) is in ft, \( E \) is in lb/in² and \( I \) is in in⁴/ft-of wall
Anchored Piling in Cohesive Soils

For anchored piles in cohesive soils, the most significant factors are (Rowe, 1957)

1. The stability number
2. The relative height of piling $\alpha$

where,

$H = \text{height of piling above the dredge line in meters}$

$\gamma = \text{effective unit weight of the soil above the dredge line = moist unit weight above water level and buoyant unit weight below water level, kN/m}^3$

$c = \text{the cohesion of the soil below the dredge line, kN/m}^2$

$\gamma = \text{adhesion between the soil and the sheet pile wall, kN/m}^2$

$\sqrt{1 + \frac{c}{\gamma}} = 1.25 \text{ for design purposes}$

$\alpha = \text{ratio between } H \text{ and } H$

$M_d = \text{design moment}$

$M_{\text{max}} = \text{maximum theoretical moment}$

Fig. 20.20 gives charts for computing design moments for pile walls in granular and cohesive soils.

Example 20.7

Determine the depth of embedment and the force in the tie rod of the anchored bulkhead shown in Fig. Ex. 20.7(a). The backfill above and below the dredge line is sand, having the following properties

$G_s = 2.67, \gamma_{sat} = 18 \text{ kN/m}^3, \gamma_d = 13 \text{ kN/m}^3 \text{ and } \phi = 30^\circ$

Solve the problem by the free-earth support method. Assume the backfill above the water table remains dry.

Solution

Assume the soil above the water table is dry

For $\phi = 30^\circ$, $K_A = \frac{1}{3}, K_P = 3.0$

and $K = K_P - K_A = 3 - \frac{1}{3} = 2.67$

$\gamma_b = \gamma_{sat} - \gamma_w = 18 - 9.81 = 8.19 \text{ kN/m}^3$

where $\gamma_w = 9.81 \text{ kN/m}^3$.

The pressure distribution along the bulkhead is as shown in Fig. Ex. 20.7(b)

$\bar{p}_1 = \gamma_d h_1 K_A = 13 \times 2 \times \frac{1}{3} = 8.67 \text{ kN/m}^2 \text{ at GW level}$

$\bar{p}_a = \bar{p}_1 + \gamma_b h_2 K_A = 8.67 + 8.19 \times 3 \times \frac{1}{3} = 16.86 \text{ kN/m}^2 \text{ at dredge line level}$
Sheet Pile Walls and Braced Cuts

\[
y_0 = -\frac{\bar{p}_a}{\gamma_b \times K} = \frac{16.86}{8.19 \times 2.67} = 0.77 \text{ m}
\]

\[
P_a = \frac{1}{2} \times \bar{p}_1 \times h_1 + \bar{p}_1 \times h_2 + \frac{1}{2} (\bar{p}_a - \bar{p}_1) h_2 + \frac{1}{2} \bar{p}_a y_0
\]

\[
= \frac{1}{2} \times 8.67 \times 2 + 8.67 \times 3 + \frac{1}{2} (16.86 - 8.67)3
\]

\[
+ \frac{1}{2} \times 16.86 \times 0.77 = 53.5 \text{ kN/m of wall}
\]

To find \( \bar{y} \), taking moments of areas about 0, we have

\[
53.5 \times \bar{y} = \frac{1}{2} \times 8.67 \times 2 \times \frac{2}{3} + 3 + 0.77 + 8.67 \times 3(3/2 + 0.77)
\]

\[
+ \frac{1}{2} (16.86 - 8.67) \times 3(3/3 + 0.77) + \frac{1}{2} \times 16.86 \times \frac{2}{3} \times 0.77^2 = 122.6
\]

We have \( \bar{y} = 122.6 \times \frac{53.5}{53.5} = 2.3 \text{ m}, \quad \bar{y}_a = 4 + 0.77 - 2.3 = 2.47 \text{ m} \)

Now \( P_p = \frac{1}{2} \times \gamma_b \times K \times D_0^2 = \frac{1}{2} \times 8.19 \times 2.67 \times D_0^2 = 10.93 D_0^2 \)

and its distance from the anchor rod is

\[
h_4 = h_3 + y_0 + 2/3 D_0 = 4 + 0.77 + 2/3 D_0 = 4.77 + 0.67 D_0
\]

Figure Ex. 20.7
Now, taking the moments of the forces about the tie rod, we have

\[
P_a \times y_a = P_p \times h
\]

\[
53.5 \times 2.47 = 10.93 D_0^2 \times (4.77 + 0.67 D_0)
\]

Simplifying, we have

\[
D_0 \approx 1.5 \text{ m}, \quad D = y_0 + D_0 = 0.77 + 1.5 = 2.27 \text{ m}
\]

\[
D \text{ (design)} = 1.4 \times 2.27 = 3.18 \text{ m}
\]

For finding the tension in the anchor rod, we have

\[
P_a - P_p - T_a = 0
\]

Therefore, \( T_a = P_a - P_p = 53.5 - 10.93(1.5)^2 = 28.9 \text{ kN/m of wall for the calculated depth } D_0. \)

**Example 20.8**

Solve Example 20.7 by applying \( F_v = 2 \) to the passive earth pressure.

**Solution**

Refer to Fig. Ex. 20.8

The following equations may be written

\[
P'_p = \frac{1}{2} y_b K_p D^2 \frac{1}{F_v} = \frac{1}{2} \times 8.19 \times 3 D^2 \times \frac{1}{2} = 6.14 D^2
\]

\[p_p = y_b K_p D = 8.19 \times 3 D \approx 24.6 D
\]

\[
\frac{FG}{BC} = \frac{\alpha p_p}{p_p} = \frac{D - h}{D} \quad \text{or} \quad h = D(1 - \alpha)
\]

Area \( ABEF = \frac{D + h}{2} \alpha p_p = \frac{D + h}{2} \alpha \times 24.6 D
\]

or \( 6.14 D^2 = \alpha (D + h) \times 12.3 D \)

Substituting for \( h = D (1 - \alpha) \) and simplifying we have

\[2 \alpha^2 - 4 \alpha + 1 = 0\]

Solving the equation, we get \( \alpha = 0.3. \)

Now \( h = D - 0.3 \text{ D and } AG = D - 0.7 \text{ D} = 0.3 \text{ D} \)

Taking moments of the area \( ABEF \) about the base of the pile, and assuming \( \alpha p_p = 1 \) in Fig. Ex. 20.8 we have

\[
\frac{1}{2} (1) \times 0.3D + \frac{1}{3} \times 0.3D \times 0.7D + (1) \times 0.7D \times \frac{0.7D}{2}
\]
simplifying we have \( \bar{y}_p = 0.44D \)

\[
\bar{y}_p = 0.44D
\]

Now \( h_4 = h_3 + (D - \bar{y}_p) = 4 + (D - 0.44D) = 4 + 0.56D \)

From the active earth pressure diagram (Fig. Ex. 20.8) we have

\[
\bar{p}_1 = \gamma_d h_1 K_A = 13 \times 2 \times \frac{1}{3} = 8.67 \text{ kN/m}^2
\]

\[
\bar{p}_a = p_1 + \gamma_p (h_2 + D) K_A = 8.67 + \frac{8.19(3+D)}{3} = 16.86 + 2.73D
\]

\[
P_a = \frac{1}{2} p_1 h_1 + \frac{(\bar{p}_1 + \bar{p}_a)}{2} (h_2 + D)
\]

\[
= \frac{1}{2} \times 8.67 \times 2 + \frac{8.67 + 16.86 + 2.73D}{2} (3 + D) = 13.6D^2 + 16.86D + 47
\]

Taking moments of active and passive forces about the tie rod, and simplifying, we have

(a) for moments due to active forces = \( 0.89D^3 + 13.7D^2 + 66.7D + 104 \)

(b) for moments due to passive forces = \( 6.14D^2 (4 + 0.56D) = 24.56D^2 + 3.44D^3 \)
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Since the sum of the moments about the anchor rod should be zero, we have
\[0.89 \, D^3 + 13.7 \, D^2 + 66.7 \, D + 104 = 24.56 \, D^2 + 3.44 \, D^3\]
Simplifying we have
\[D^3 + 4.26 \, D^2 - 26.16 \, D - 40.8 = 0\]
By solving the equation we obtain \(D = 4.22\) m with \(F_s = 2.0\)

Force in the anchor rod
\[T_a = P_a - P_p\]
where \(P_a = 1.36 \, D^2 + 16.86 \, D + 47 = 1.36 \times (4.22)^2 + 16.86 \times 4.22 + 47 = 143\) kN
\(P_p = 6.14 \, D^2 = 6.14 \times (4.22)^2 = 109\) kN
Therefore \(T_a = 143 - 109 = 34\) kN/m length of wall.

**Example 20.9**

Solve Example 20.7, if the backfill is sand with \(\phi = 30^\circ\) and the soil below the dredge line is clay having \(c = 20\) kN/m². For both the soils, assume \(G_v = 2.67\).

**Solution**

The pressure distribution along the bulkhead is as shown in Fig. Ex. 20.9.

\[\bar{p}_1 = 8.67\ \text{kN/m}^2\ \text{as in Ex. 20.7,}\ \bar{p}_a = 16.86\ \text{kN/m}^2\]

\[P_a = \frac{1}{2} \bar{p}_1 h_1 + \bar{p}_1 h_2 + \frac{1}{2} (\bar{p}_a - \bar{p}_1) h_2\]

\[= \frac{1}{2} \times 8.67 \times 2 + 8.67 \times 3 + \frac{1}{2} (16.86 - 8.67) \times 3 = 47.0\ \text{kN/m}\]

\[\bar{p} = 2 \, q_a - q\]

\[c = 0\ \text{Clay}\]

\[\bar{p}_a = 20\ \text{kN/m}^2\]

\[\phi = 0\]

**Figure Ex. 20.9**
To determine \( y_a \), take moments about the tie rod.

\[
P_a \times \bar{y}_a = \frac{1}{2} \times 8.67 \times 2 \times \frac{2}{3} \times 2 - 1 + 8.67 \times 3 \times 2.5 + \frac{1}{2} (16.86 - 8.67) \times 3 \times 3 = 104.8
\]

Therefore \( \bar{y}_a = \frac{104.8}{46.7} = 2.23 \) m

Now, \( q = \gamma_d h_1 + \gamma_b h_2 = 13 \times 3 + 8.19 \times 3 = 50.6 \) kN/m²

\( \bar{p} = 2 \times 2 \times 20 - 50.6 = 29.4 \) kN/m²

Therefore, \( P_p = \bar{p} \times D = 29.4 \) D kN/m

and \( h_4 = \left( 4 + \frac{D}{2} \right) \)

Taking moments of forces about the tie rod, we have

\[
P_a \times \bar{y}_a - P_p \times h_4 = 0
\]

or \( 47 \times 2.23 - 29.4 D \times \frac{4 + \frac{D}{2}}{2} = 0 \)

or \( 1.47D^2 + 11.76D - 10.48 = 0 \) or \( D = 0.8 \) m.

\( D = 0.8 \) m is obtained with a factor of safety equal to one. It should be increased by 20 to 40 percent to increase the factor of safety from 1.5 to 2.0. For a factor of safety of 2, the depth of embedment should be at least 1.12 m. However the suggested depth (design) = 2 m.

Hence \( P_p \) for \( D \) (design) = 2 m is

\( P_p = 29.4 \times 2 = 58.8 \) kN/m length of wall

The tension in the tie rod is

\( T_a = P_a - P_p = 47 - 59 = -12 \) kN/m of wall.

This indicates that the tie rod will not be in tension under a design depth of 2 m. However there is tension for the calculated depth \( D = 0.8 \) m.

---

**Example 20.10**

Determine the depth of embedment for the sheet pile given in Fig. Example 20.7 using the design charts given in Section 20.7.

**Given:** \( H = 5 \) m, \( h_1 = 2 \) m, \( h_2 = 3 \) m, \( h_3 = 1 \) m, \( h_4 = 4 \) m, \( \phi = 30^\circ \)
Solution

\[ \frac{h_a}{H} = \frac{1}{5} = 0.2 \]

From Fig. 20.18

For \( h_a / H = 0.2 \), and \( \phi = 30^\circ \) we have

\[ G_d = 0.26, \quad G_t = 0.084, \quad G_m = 0.024 \]

From Fig. 20.19 for \( \phi = 30^\circ \), \( h_a / H = 0.2 \) and \( h_t / H = 0.4 \) we have

\[ C_d = 1.173, \quad C_t = 1.073, \quad C_m = 1.036 \]

Now from Eq. (20.46 a)

\[ D = G_d C_d h = 0.26 \times 1.173 \times 5 = 1.52 \text{ m} \]

\[ D \text{ (design)} = 1.4 \times 1.52 = 2.13 \text{ m}. \]

\[ T_a = G_t \gamma_a H^2 \]

where

\[ \gamma_a = \frac{(\gamma_m h_t^2 + \gamma_b h_b^2 + 2 \gamma_m h_t h_b)}{H^2} \]

\[ = \frac{13 \times 2^2 + 8.19 \times 3^2 + 2 \times 13 \times 2 \times 3}{5^2} = 11.27 \text{ kN/m}^2 \]

Substituting and simplifying

\[ T_a = 0.084 \times 1.073 \times 11.27 \times 5^2 = 25.4 \text{ kN} \]

\[ M_{max} = G_m C_m \gamma_a H^3 \]

\[ = 0.024 \times 1.036 \times 11.27 \times 5^3 = 35.03 \text{ kN-m/m of wall} \]

The values of \( D \text{ (design)} \) and \( T_a \) from Ex. 20.7 are

\[ D \text{ (design)} = 3.18 \text{ m} \]

\[ T_a = 28.9 \text{ kN} \]

The design chart gives less by 33\% in the value of \( D \text{ (design)} \) and 12\% in the value of \( T_a \).

Example 20.11

Refer to Example 20.7. Determine for the pile (a) the bending moment \( M_{max} \), and (b) the reduced moment by Rowe’s method.

Solution

Refer to Fig. Ex. 20.7. The following data are available

\[ H = 5 \text{ m}, \quad h_1 = 2 \text{ m}, \quad h_2 = 3 \text{ m}, \quad h_3 = 4 \text{ m} \]

\[ \gamma_d = 13 \text{ kN/m}^3, \quad \gamma_b = 8.19 \text{ kN/m}^3 \text{ and } \phi = 30^\circ \]

The maximum bending moment occurs at a depth \( h_m \) from ground level where the shear is zero. The equation which gives the value of \( h_m \) is

\[ \frac{1}{2} \bar{p}_1 h_1 - T_a + \bar{p}_1 (h_m - h_1) + \frac{1}{2} \gamma_b K_A (h_m - h_1)^2 = 0 \]
where \( p_1 = 8.67 \text{kN/m}^2 \)

- \( h_1 = 2 \text{ m}, T_a = 28.9 \text{kN/m} \)

Substituting

\[
\frac{1}{2} \times 8.67 \times 2 - 28.9 + 8.67(h_m - 2) + \frac{1}{2} \times 8.19 \times \frac{1}{3}(h_m - 2)^2 = 0
\]

Simplifying, we have

\[
h_m^2 + 2.35h_m - 23.5 = 0
\]

or \( h_m = 3.81 \text{ m} \)

Taking moments about the point of zero shear,

\[
M_{\text{max}} = -\frac{1}{2}p_1 h_1 h_m - \frac{2}{3} h_1 + T_a (h_m - h_a) - \frac{p_1}{2} (h_m - h_1)^2 - \frac{1}{6} y_b K_h (h_m - h_1)^3
\]

\[
= -\frac{1}{2} \times 8.67 \times 2 \times 3.81 - \frac{2}{3} \times 2 + 28.9(3.81 - 1) - 8.67 \frac{(3.81 - 2)^2}{2} - \frac{1}{6} 8.19 \times \frac{1}{3}(3.81 - 2)^3
\]

\[
= -21.41 + 81.2 - 14.2 - 2.70 = 42.8 \text{kN-m/m}
\]

From Ex. 20.7

\( D \) (design) = 3.18 \text{ m}, \( H = 5 \text{ m} \)

Therefore \( H = 8.18 \text{ m} \)

From Eq. (20.47a) \( \rho = 109 \times 10^{-5} \frac{H^4}{EI} \)

Assume \( E = 20.7 \times 10^4 \text{ MN/m}^2 \)

For a section \( P_z = 27, I = 25.2 \times 10^{-5} \text{ m}^4/\text{m} \)

Substituting \( \rho = \frac{10.9 \times 10^{-5} \times (8.18)^4}{207 \times 10^5 \times 25.2 \times 10^{-5}} = 9.356 \times 10^{-3} \)

\( \log \rho = \log \frac{9.36}{10^3} = -2.0287 \text{ or say } 2.00 \)

Assuming the sand backfill is loose, we have from Fig. 20.20 (a)

\[
\frac{M_d}{M_{\text{max}}} = 0.32 \quad \text{for } \log \rho = -2.00
\]

Therefore \( M_d \) (design) = 0.32 \times 42.8 = 13.7 \text{kN-m/m} \)

### 20.9 ANCHORAGE OF BULKHEADS

Sheet pile walls are many times tied to some kind of anchors through tie rods to give them greater stability as shown in Fig. 20.21. The types of anchorage that are normally used are also shown in the same figure.
Chapter 20

Anchors such as anchor walls and anchor plates which depend for their resistance entirely on passive earth pressure must be given such dimensions that the anchor pull does not exceed a certain fraction of the pull required to produce failure. The ratio between the tension in the anchor $T_a$ and the maximum pull which the anchor can stand is called the factor of safety of the anchor.

The types of anchorages given in Fig. 20.21 are:

1. **Deadmen, anchor plates, anchor beams etc.**: Deadmen are short concrete blocks or continuous concrete beams deriving their resistance from passive earth pressure. This type is suitable when it can be installed below the level of the original ground surface.

2. **Anchor block supported by battered piles**: Fig (20.21b) shows an anchor block supported by two battered piles. The force $T_a$ exerted by the tie rod tends to induce compression in pile $P_1$ and tension in pile $P_2$. This type is employed where firm soil is at great depth.

3. **Sheet piles**: Short sheet piles are driven to form a continuous wall which derives its resistance from passive earth pressure in the same manner as deadmen.

4. **Existing structures**: The rods can be connected to heavy foundations such as buildings, crane foundations etc.

**Figure 20.21** Types of anchorage: (a) deadman; (b) braced piles; (c) sheet piles; (d) large structure (after Teng, 1969)
Location of Anchorage

The minimum distance between the sheet pile wall and the anchor blocks is determined by the failure wedges of the sheet pile (under free-earth support condition) and deadmen. The anchorage does not serve any purpose if it is located within the failure wedge ABC shown in Fig. 20.22a.

If the failure wedges of the sheet pile and the anchor interfere with each other, the location of the anchor as shown in Fig. 20.22b reduces its capacity. Full capacity of the anchorage will be available if it is located in the shaded area shown in Fig. 20.22c. In this case

1. The active sliding wedge of the backfill does not interfere with the passive sliding wedge of the deadman.
2. The deadman is located below the slope line starting from the bottom of the sheet pile and making an angle $\phi$ with the horizontal, $\phi$ being the angle of internal friction of the soil.

Capacity of Deadman (After Teng, 1969)

A series of deadmen (anchor beams, anchor blocks or anchor plates) are normally placed at intervals parallel to the sheet pile walls. These anchor blocks may be constructed near the ground surface or at great depths, and in short lengths or in one continuous beam. The holding capacity of these anchorages is discussed below.

Continuous Anchor Beam Near Ground Surface (Teng, 1969)

If the length of the beam is considerably greater than its depth, it is called a continuous deadman. Fig. 20.23(a) shows a deadman. If the depth to the top of the deadman, $h$, is less than about one-third to one-half of $H$ (where $H$ is depth to the bottom of the deadman), the capacity may be calculated by assuming that the top of the deadman extends to the ground surface. The ultimate capacity of a deadman may be obtained from (per unit length)

For granular soil ($c = 0$)

$$T_u = P_p - P_a = \frac{1}{2} \gamma H^2 K_p - \frac{1}{2} \gamma H^2 K_A$$

or

$$T_u = \frac{1}{2} \gamma H^2 (K_p - K_A) \quad (20.49)$$

For clay soil ($\phi = 0$)

$$T_u = P_p - P_a = q_u H + \frac{1}{2} \gamma H^2 - \frac{1}{2} \gamma H^2 - q_u H + \frac{2c^2}{\gamma} = 2q_u H - \frac{2c^2}{\gamma} \quad (20.50)$$

where $q_u$ = unconfined compressive strength of soil,
$\gamma$ = effective unit weight of soil, and
$K_p, K_A$ = Rankine's active and passive earth pressure coefficients.

It may be noted here that the active earth pressure is assumed to be zero at a depth $= 2c/\gamma$ which is the depth of the tension cracks. It is likely that the magnitude and distribution of earth pressure may change slowly with time. For lack of sufficient data on this, the design of deadmen in cohesive soils should be made with a conservative factor of safety.
Figure 20.22 Location of deadmen: (a) offers no resistance; (b) efficiency greatly impaired; (c) full capacity. (after Teng, 1969)

Short Deadman Near Ground Surface in Granular Soil (Fig. 20.23b)

If the length of a deadman is shorter than $5h$ ($h = \text{height of deadman}$) there will be an end effect with regards to the holding capacity of the anchor. The equation suggested by Teng for computing the ultimate tensile capacity $T_u$ is

$$T_u = L(P_p - P_a) + \frac{1}{3}K_p\gamma(K_p + K_A)H^3\tan\phi$$  (20.51)

where

- $h$ = height of deadman
- $\bar{h}$ = depth to the top of deadman
- $L$ = length of deadman
- $H$ = depth to the bottom of the dead man from the ground surface
- $P_p, P_a$ = total passive and active earth pressures per unit length
Sheet Pile Walls and Braced Cuts

Ground surface

Anchor pull

Deadman

Active wedge

Passive wedge

Figure 20.23 Capacity of deadmen: (a) continuous deadmen near ground surface ($h/H < 1/3 \sim 1/2$); (b) short deadmen near ground surface; (c) deadmen at great depth below ground surface (after Teng, 1969)

Ko = coefficient of earth pressures at-rest, taken equal to 0.4
γ = effective unit weight of soil
Kp, KA = Rankine’s coefficients of passive and active earth pressures
φ = angle of internal friction

Anchor Capacity of Short Deadman in Cohesive Soil Near Ground Surface

In cohesive soils, the second term of Eq. (20.51) should be replaced by the cohesive resistance

$$T_a = L (P_p - P_d) + q_u H^2$$

where $q_u =$ unconfined compressive strength of soil.

Deadman at Great Depth

The ultimate capacity of a deadman at great depth below the ground surface as shown in Fig. (20.23c) is approximately equal to the bearing capacity of a footing whose base is located at a depth ($\tilde{h} + h/2$), corresponding to the mid height of the deadman (Terzaghi, 1943).
Ultimate Lateral Resistance of Vertical Anchor Plates in Sand

The load-displacement behavior of horizontally loaded vertical anchor plates was analyzed by Ghaly (1997). He made use of 128 published field and laboratory test results and presented equations for computing the following:

1. Ultimate horizontal resistance $T_u$ of single anchors
2. Horizontal displacement $u$ at any load level $T$

The equations are

\[
T_u = \frac{C \gamma A H}{\tan \phi} \frac{H^2}{A} \alpha
\]

(20.53)

\[
\frac{T}{T_u} = 2.2 \left[ \frac{u}{H} \right]^{0.3}
\]

(20.54)

where

- $A = \text{area of anchor plate} = h_L$
- $H = \text{depth from the ground surface to the bottom of the plate}$
- $h = \text{height of plate and } L = \text{width}$
- $\gamma = \text{effective unit weight of the sand}$
- $\phi = \text{angle of friction}$

![Figure 20.24 Relationship of pullout-capacity factor versus geometry factor (after Ghaly, 1997)](image-url)
Sheet Pile Walls and Braced Cuts

1.0

Figure 20.25  Relationship of load ratio versus displacement ratio [from data reported by Das and Seeley, 1975)] (after Ghaly, 1997)

\[ C_A = \begin{cases} 
5.5 \text{ for a rectangular plate} \\
5.4 \text{ for a general equation} \\
3.3 \text{ for a square plate.} 
\end{cases} \\
\alpha = \begin{cases} 
0.31 \text{ for a rectangular plate} \\
0.28 \text{ for a general equation} \\
0.39 \text{ for a square plate} 
\end{cases} \]

The equations developed are valid for relative depth ratios \((H/h) \leq 5\). Figs 20.24 and 20.25 give relationships in non-dimensional form for computing \(T_u\) and \(u\) respectively. Non-dimensional plots for computing \(T_u\) for square and rectangular plates are also given in Fig. 20.24.

20.10 BRACED CUTS

General Considerations

Shallow excavations can be made without supporting the surrounding material if there is adequate space to establish slopes at which the material can stand. The steepest slopes that can be used in a given locality are best determined by experience. Many building sites extend to the edges of the property lines. Under these circumstances, the sides of the excavation have to be made vertical and must usually be supported by bracings.

Common methods of bracing the sides when the depth of excavation does not exceed about 3 m are shown in Figs 20.26(a) and (b). The practice is to drive vertical timber planks known as sheeting along the sides of the excavation. The sheeting is held in place by means of horizontal
beams called *wales* that in turn are commonly supported by horizontal *struts* extending from side to side of the excavation. The struts are usually of timber for widths not exceeding about 2 m. For greater widths metal pipes called *trench braces* are commonly used.

When the excavation depth exceeds about 5 to 6 m, the use of vertical timber sheeting will become uneconomical. According to one procedure, *steel sheet piles* are used around the boundary of the excavation. As the soil is removed from the enclosure, wales and struts are inserted. The wales are commonly of steel and the struts may be of steel or wood. The process continues until the

![Diagram](image-url)

*Figure 20.26* Cross sections, through typical bracing in deep excavation. (a) sides retained by steel sheet piles, (b) sides retained by *H* piles and lagging, (c) one of several tieback systems for supporting vertical sides of open cut. Several sets of anchors may be used, at different elevations (Peck, 1969).
excavation is complete. In most types of soil, it may be possible to eliminate sheet piles and to replace them with a series of \( H \) piles spaced 1.5 to 2.5 m apart. The \( H \) piles, known as soldier piles or soldier beams, are driven with their flanges parallel to the sides of the excavation as shown in Fig. 20.26(b). As the soil next to the piles is removed horizontal boards known as lagging are introduced as shown in the figure and are wedged against the soil outside the cut. As the general depth of excavation advances from one level to another, wales and struts are inserted in the same manner as for steel sheeting.

If the width of a deep excavation is too great to permit economical use of struts across the entire excavation, tiebacks are often used as an alternative to cross-bracings as shown in Fig. 20.26(c). Inclined holes are drilled into the soil outside the sheeting or \( H \) piles. Tensile reinforcement is then inserted and concreted into the hole. Each tieback is usually prestressed before the depth of excavation is increased.

**Example 20.12**

Fig. Ex. 20.12 gives an anchor plate fixed vertically in medium dense sand with the bottom of the plate at a depth of 3 ft below the ground surface. The size of the plate is 2 \( \times \) 12 ft. Determine the ultimate lateral resistance of the plate. The soil parameters are \( \gamma = 115 \text{ lb/ft}^3, \phi = 38^\circ \).

![Image of anchor plate](image)

**Solution**

For all practical purposes if \( L/h \geq 5 \), the plate may be considered as a long beam. In this case \( L/h = 12/2 = 6 > 5 \). If the depth \( h \) to the top of the plate is less than about 1/3 to 1/2 of \( H \) (where \( H \) is the depth to the bottom of the plate), the lateral capacity may be calculated using Eq. (20.49). In this case \( h/H = 1/3 \). As such

\[
T_u = \frac{1}{2} \gamma H^2 (K_p - K_A)
\]

where \( K_p = \tan^2 \left( 45^\circ + \frac{38^\circ}{2} \right) = 4.204 \)

\[
K_A = \frac{1}{4.204} = 0.238
\]

\[
T_u = \frac{1}{2} \times 115 \times 3^2 (4.204 - 0.238) = 2051 \text{ lb/ft}
\]

**Example 20.13**

Solve Example 20.12 if \( \phi = 0 \) and \( c = 300 \text{ lb/ft}^2 \). All the other data remain the same.
Solution
Use Eq. (20.50)

\[ T_u = 4cH - \frac{2c^2}{\gamma} = 4 \times 300 \times 3 - \frac{2 \times 300^2}{115} = 2035 \text{ lb/ft} \]

**Example 20.14**
Solve the problem in Example 20.12 for a plate length of 6 ft. All the other data remain the same.

**Solution**
Use Eq. (20.51) for a shorter length of plate

\[ T_u = L(P_p - P_w) + \frac{1}{3} K_p \gamma \left( \sqrt{K_p} + \sqrt{K_A} \right) H^3 \tan \phi \]

where \((P_p - P_w) = 2051 \text{ lb/ft from Ex. 20.12}\)

\[ K_p = 1 - \sin \phi = 1 - \sin 38^\circ = 0.384 \]

\[ \tan \phi = \tan 38^\circ = 0.78 \]

\[ \sqrt{K_p} = \sqrt{4.204} = 2.05, \quad \sqrt{K_A} = \sqrt{0.238} = 0.488 \]

substituting

\[ T_u = 6 \times 2051 + \frac{1}{3} \times 0.384 \times 115 \times (2.05 + 0.488) \times 3^3 \times 0.78 = 12,306 + 787 = 13,093 \text{ lb} \]

**Example 20.15**
Solve Example 20.13 if the plate length \(L = 6 \text{ ft}\). All the other data remain the same.

**Solution**
Use Eq. (20.52)

\[ T_u = L (P_p - P_w) + q_u H^2 \]

where \((P_p - P_w) = 2035 \text{ lb/ft from Ex. 20.13}\)

\[ q_u = 2 \times 300 = 600 \text{ lb/ft}^2 \]

Substituting

\[ T_u = 6 \times 2035 + 600 \times 3^2 = 12,210 + 5,400 = 17,610 \text{ lb} \]

**Example 20.16**
Solve the problem in Example 20.14 using Eq. (20.53). All the other data remain the same.

**Solution**
Use Eq. (20.53)

\[ T_u = \frac{5.4 \gamma AH}{\tan \phi} \frac{H^2}{A}^{0.28} \]

where \(A = 2 \times 6 = 12 \text{ sq ft}, H = 3 \text{ ft}, C_A = 5.4\) and \(\alpha = 0.28\) for a general equation
tan \( \phi = \tan 38^\circ = 0.78 \)

Substituting

\[
T_u = \frac{5.4 \times 115 \times 12 \times 3 \times 0.28}{0.78^2} = 26,443 \text{ lb/ft}
\]

### 20.11 LATERAL EARTH PRESSURE DISTRIBUTION ON BRACED-CUTS

Since most open cuts are excavated in stages within the boundaries of sheet pile walls or walls consisting of soldier piles and lagging, and since struts are inserted progressively as the excavation precedes, the walls are likely to deform as shown in Fig. 20.27. Little inward movement can occur at the top of the cut after the first strut is inserted. The pattern of deformation differs so greatly from that required for Rankine's state that the distribution of earth pressure associated with retaining walls is not a satisfactory basis for design (Peck et al., 1974). The pressures against the upper portion of the walls are substantially greater than those indicated by the equation.

\[
p_a = \frac{1 - \sin \phi}{1 + \sin \phi} p_v
\]

for Rankine's condition

where

\( p_v = \) vertical pressure,

\( \phi = \) friction angle

#### Apparent Pressure Diagrams

Peck (1969) presented pressure distribution diagrams on braced cuts. These diagrams are based on a wealth of information collected by actual measurements in the field. Peck called these pressure diagrams *apparent pressure envelopes* which represent fictitious pressure distributions for estimating strut loads in a system of loading. Figure 20.28 gives the apparent pressure distribution diagrams as proposed by Peck.

#### Deep Cuts in Sand

The apparent pressure diagram for sand given in Fig. 20.28 was developed by Peck (1969) after a great deal of study of actual pressure measurements on braced cuts used for subways.
The pressure diagram given in Fig. 20.28(b) is applicable to both loose and dense sands. The struts are to be designed based on this apparent pressure distribution. The most probable value of any individual strut load is about 25 percent lower than the maximum (Peck, 1969). It may be noted here that this apparent pressure distribution diagram is based on the assumption that the water table is below the bottom of the cut.

The pressure \( p_a \) is uniform with respect to depth. The expression for \( p_a \) is

\[
p_a = 0.65 \gamma H K_A
\]

where,
\[
K_A = \tan^2 \left( 45^\circ - \phi/2 \right)
\]
\[
\gamma = \text{unit weight of sand}
\]

**Cuts in Saturated Clay**

Peck (1969) developed two apparent pressure diagrams, one for soft to medium clay and the other for stiff fissured clay. He classified these clays on the basis of non-dimensional factors (stability number \( N_s \)) as follows.

**Stiff Fissured clay**

\[
N_s = \frac{\gamma H}{c} \leq 4 \quad (20.57a)
\]

**Soft to Medium clay**

\[
N_s = \frac{\gamma H}{c} > 4 \quad (20.57b)
\]

where \( \gamma \) = unit weight of clay, \( c \) = undrained cohesion (\( \phi = 0 \))

**Figure 20.28** Apparent pressure diagram for calculating loads in struts of braced cuts: (a) sketch of wall of cut, (b) diagram for cuts in dry or moist sand, (c) diagram for clays if \( \gamma H/c \) is less than 4 (d) diagram for clays if \( \gamma H/c \) is greater than 4 where \( c \) is the average undrained shearing strength of the soil (Peck, 1969)
The pressure diagrams for these two types of clays are given in Fig. 20.28(c) and (d) respectively. The apparent pressure diagram for soft to medium clay (Fig. 20.28(d)) has been found to be conservative for estimating loads for design supports. Fig. 20.28(c) shows the apparent pressure diagram for stiff-fissured clays. Most stiff clays are weak and contain fissures. Lower pressures should be used only when the results of observations on similar cuts in the vicinity so indicate. Otherwise a lower limit for $p_a = 0.3 \gamma H$ should be taken. Fig. 20.29 gives a comparison of measured and computed pressures distribution for cuts in London, Oslo and Houston clays.

**Cuts in Stratified Soils**

It is very rare to find uniform deposits of sand or clay to a great depth. Many times layers of sand and clays overlying one another the other are found in nature. Even the simplest of these conditions does not lend itself to vigorous calculations of lateral earth pressures by any of the methods.
available. Based on field experience, empirical or semi-empirical procedures for estimating apparent pressure diagrams may be justified. Peck (1969) proposed the following unit pressure for excavations in layered soils (sand and clay) with sand overlying as shown in Fig. 20.30.

When layers of sand and soft clay are encountered, the pressure distribution shown in Fig. 20.28(d) may be used if the unconfined compressive strength  \( q_u \) is substituted by the average \( \bar{q}_u \) and the unit weight of soil \( \gamma \) by the average value \( \bar{\gamma} \) (Peck, 1969). The expressions for \( \bar{q}_u \) and \( \bar{\gamma} \) are

\[
\bar{q}_u = \frac{1}{H} \left[ \gamma_1 K_s h_1^2 \tan \phi + h_2 n q_u \right]
\]

\[
\bar{\gamma} = \frac{1}{H} \left[ \gamma_1 h_1 + \gamma_2 h_2 \right]
\]

where

- \( H \) = total depth of excavation
- \( \gamma_1, \gamma_2 \) = unit weights of sand and clay respectively
- \( h_1, h_2 \) = thickness of sand and clay layers respectively
- \( K_s \) = hydrostatic pressure ratio for the sand layer, may be taken as equal to 1.0 for design purposes
- \( \phi \) = angle of friction of sand
- \( n \) = coefficient of progressive failure varies from 0.5 to 1.0 which depends upon the creep characteristics of clay. For Chicago clay \( n \) varies from 0.75 to 1.0.
- \( q_u \) = unconfined compression strength of clay

### 20.12 STABILITY OF BRACED CUTS IN SATURATED CLAY

A braced-cut may fail as a unit due to unbalanced external forces or heaving of the bottom of the excavation. If the external forces acting on opposite sides of the braced cut are unequal, the stability of the entire system has to be analyzed. If soil on one side of a braced cut is removed due to some unnatural forces the stability of the system will be impaired. However, we are concerned here about the stability of the bottom of the cut. Two cases may arise. They are

1. Heaving in clay soil
2. Heaving in cohesionless soil

#### Heaving in Clay Soil

The danger of heaving is greater if the bottom of the cut is soft clay. Even in a soft clay bottom, two types of failure are possible. They are

**Case 1:** When the clay below the cut is homogeneous at least up to a depth equal 0.7 \( B \) where \( B \) is the width of the cut.

**Case 2:** When a hard stratum is met within a depth equal to 0.7 \( B \).

In the first case a full plastic failure zone will be formed and in the second case this is restricted as shown in Fig. 20.31. A factor of safety of 1.5 is recommended for determining the resistance here. Sheet piling is to be driven deeper to increase the factor of safety. The stability analysis of the bottom of the cut as developed by Terzaghi (1943) is as follows.
Case 1: Formation of Full Plastic Failure Zone Below the Bottom of Cut.

Figure 20.31(a) is a vertical section through a long cut of width $B$ and depth $H$ in saturated cohesive soil ($\phi = 0$). The soil below the bottom of the cut is uniform up to a considerable depth for the formation of a full plastic failure zone. The undrained cohesive strength of soil is $c$. The weight of the blocks of clay on either side of the cut tends to displace the underlying clay toward the excavation. If the underlying clay experiences a bearing capacity failure, the bottom of the excavation heaves and the earth pressure against the bracing increases considerably.

The anchorage load block of soil $a b c d$ in Fig. 20.31(a) of width $\overline{B}$ (assumed) at the level of the bottom of the cut per unit length may be expressed as

\[
Q = \gamma H \overline{B} - cH = \overline{B} H \left( \gamma - \frac{c}{\overline{B}} \right) \tag{20.60}
\]

The vertical pressure $q$ per unit length of a horizontal, $ba$, is

\[
q = \frac{Q}{\overline{B}} = H \left( \gamma - \frac{c}{\overline{B}} \right) \tag{20.61}
\]

Figure 20.31  Stability of braced cut: (a) heave of bottom of timbered cut in soft clay if no hard stratum interferes with flow of clay, (b) as before, if clay rests at shallow depth below bottom of cut on hard stratum (after Terzaghi, 1943)
The bearing capacity $q_u$ per unit area at level $ab$ is

$$q_u = N_c c = 5.7c$$

(20.62)

where $N_c = 5.7$

The factor of safety against heaving is

$$F_s = \frac{q_u}{q} = \frac{5.7c}{H \gamma - \frac{c}{B}}$$

(20.63)

Because of the geometrical condition, it has been found that the width $\bar{B}$ cannot exceed $0.7B$. Substituting this value for $\bar{B}$,

$$F_s = \frac{5.7c}{H \left( \gamma - \frac{c}{0.7B} \right)}$$

(20.64)

This indicates that the width of the failure slip is equal to $\bar{B}\sqrt{2} = 0.7B$.

**Case 2: When the Formation of Full Plastic Zone is Restricted by the Presence of a Hard Layer**

If a hard layer is located at a depth $D$ below the bottom of the cut (which is less than $0.7B$), the failure of the bottom occurs as shown in Fig. 20.31(b). The width of the strip which can sink is also equal to $D$.

Replacing $0.7B$ by $D$ in Eq. (20.64), the factor of safety is represented by

$$F_s = \frac{5.7c_u}{H \gamma - \frac{c}{D}}$$

(20.65)

For a cut in soft clay with a constant value of $c_u$ below the bottom of the cut, $D$ in Eq. (20.65) becomes large, and $F_s$ approaches the value

$$F_s = \frac{5.7c_u}{\gamma H} = \frac{5.7}{N_s}$$

(20.66)

where $N_s = \frac{\gamma H}{c_u}$

(20.67)

is termed the *stability number*. The stability number is a useful indicator of potential soil movements. The soil movement is smaller for smaller values of $N_s$.

The analysis discussed so far is for long cuts. For short cuts, square, circular or rectangular, the factor of safety against heave can be found in the same way as for footings.

**20.13 BJERRUM AND EIDE (1956) METHOD OF ANALYSIS**

The method of analysis discussed earlier gives reliable results provided the width of the braced cut is larger than the depth of the excavation and that the braced cut is very long. In the cases where the braced cuts are rectangular, square or circular in plan or the depth of excavation exceeds the width of the cut, the following analysis should be used.
In this analysis the braced cut is visualized as a deep footing whose depth and horizontal dimensions are identical to those at the bottom of the braced cut. This deep footing would fail in an identical manner to the bottom braced cut failed by heave. The theory of Skempton for computing $N_c$ (bearing capacity factor) for different shapes of footing is made use of. Figure 20.32 gives values of $N_c$ as a function of $H/B$ for long, circular or square footings. For rectangular footings, the value of $N_c$ may be computed by the expression

$$N_c^{(\text{rect})} = (0.84 + 0.16 \frac{B}{L}) N_c^{(\text{sq})}$$  \hspace{1cm} (20.68)$$

where

- $L = \text{length of excavation}$
- $B = \text{width of excavation}$

The factor of safety for bottom heave may be expressed as

$$F_s = \frac{c N_c}{gH + q} \geq 1.5$$

where

- $\gamma = \text{effective unit weight of the soil above the bottom of the excavation}$
- $q = \text{uniform surcharge load (Fig. 20.32)}$

**Example 20.17**

A long trench is excavated in medium dense sand for the foundation of a multistorey building. The sides of the trench are supported with sheet pile walls fixed in place by struts and wales as shown in Fig. Ex. 20.17. The soil properties are:

- $\gamma = 18.5 \ \text{kN/m}^3$, $c = 0$ and $\phi = 38^\circ$

Determine: (a) The pressure distribution on the walls with respect to depth.
(b) Strut loads. The struts are placed horizontally at distances $L = 4 \ \text{m center to center}$.
(c) The maximum bending moment for determining the pile wall section.
(d) The maximum bending moments for determining the section of the wales.

**Solution**

(a) For a braced cut in sand use the apparent pressure envelope given in Fig. 20.28 b. The equation for $p_a$ is

$$p_a = 0.65 \ \gamma H K_A = 0.65 \times 18.5 \times 8 \tan^2 (45 - 38/2) = 23 \ \text{kN/m}^2$$
Fig. Ex. 20.17b shows the pressure envelope.

(b) Strut loads
The reactions at the ends of struts A, B and C are represented by $R_A$, $R_B$ and $R_C$ respectively

For reaction $R_A$, take moments about $B$

\[
R_A \times 3 = 4 \times 23 \times \frac{4}{2} \quad \text{or} \quad R_A = \frac{184}{3} = 61.33 \text{ kN}
\]

\[
R_{B_1} = 23 \times 4 - 61.33 = 30.67 \text{ kN}
\]

Due to the symmetry of the load distribution,

\[
R_{B_1} = R_{B_2} = 30.67 \text{ kN}, \quad \text{and} \quad R_A = R_C = 61.33 \text{ kN}.
\]

Now the strut loads are (for $L = 4$ m)

Strut $A$, $P_A = 61.33 \times 4 = 245$ kN

Strut $B$, $P_B = (R_{B_1} + R_{B_2}) \times 4 = 61.34 \times 4 \approx 245$ kN

Strut $C$, $P_C = 245$ kN
(c) Moment of the pile wall section

To determine moments at different points it is necessary to draw a diagram showing the shear force distribution.

Consider sections DB and BE of the wall in Fig. Ex. 20.17(b). The distribution of the shear forces are shown in Fig. 20.17(c) along with the points of zero shear.

The moments at different points may be determined as follows:

\[ M_A = \frac{1}{2} \times 1 \times 23 = 11.5 \text{ kN-m} \]

\[ M_C = \frac{1}{2} \times 1 \times 23 = 11.5 \text{ kN-m} \]

\[ M_m = \frac{1}{2} \times 1.33 \times 30.67 = 20.4 \text{ kN-m} \]

\[ M_n = \frac{1}{2} \times 1.33 \times 30.67 = 20.4 \text{ kN-m} \]

The maximum moment \( M_{\text{max}} = 20.4 \text{ kN-m} \). A suitable section of sheet pile can be determined as per standard practice.

(d) Maximum moment for wales

The bending moment equation for wales is

\[ M_{\text{max}} = \frac{R L^2}{8} \]

where \( R = \) maximum strut load = 245 kN

\( L = \) spacing of struts = 4 m

\[ M_{\text{max}} = \frac{245 \times 4^2}{8} = 490 \text{ kN-m} \]

A suitable section for the wales can be determined as per standard practice.

---

**Example 20.18**

Fig. Ex. 20.18a gives the section of a long braced cut. The sides are supported by steel sheet pile walls with struts and wales. The soil excavated at the site is stiff clay with the following properties:

\[ c = 800 \text{ lb/ft}^2, \ \phi = 0, \ \gamma = 115 \text{ lb/ft}^3 \]

Determine:

(a) The earth pressure distribution envelope.

(b) Strut loads.

(c) The maximum moment of the sheet pile section.

The struts are placed 12 ft apart center to center horizontally.

**Solution**

(a) The stability number \( N_s \) from Eq. (20.57a) is

\[ N_s = \frac{\gamma H}{c} = \frac{115 \times 25}{800} = 3.6 < 4 \]
The soil is stiff fissured clay. As such the pressure envelope shown in Fig. 20.28(c) is applicable. Assume \( p_a = 0.3 \gamma H \)

\[
p_a = 0.3 \times 115 \times 25 = 863 \text{ lb/ft}^2
\]

The pressure envelope is drawn as shown in Fig. Ex. 20.18(b).

(b) Strut loads

Taking moments about the strut head \( B_1 \) (\( B \))

\[
R_A \times 7.5 = \frac{1}{2} \times 863 \times 6.25 \left( \frac{6.25}{3} + 6.25 \right) + 863 \times \left( \frac{6.25}{2} \right)^2
\]

\[
= 22.47 \times 10^3 + 16.85 \times 10^3 = 39.32 \times 10^3
\]

\[
R_A = 5243 \text{ lb/ft}
\]

\[
R_{B_1} = \frac{1}{2} \times 863 \times 6.25 + 863 \times 6.25 - 5243 = 2848 \text{ lb/ft}
\]

Due to symmetry

\[
R_A = R_C = 5243 \text{ lb/ft}
\]

\[
R_{B_2} = R_{B_1} = 2848 \text{ lb/ft}
\]
Strut loads are:

\[ P_A = 5243 \times 12 = 62,916 \text{ lb} = 62.92 \text{ kips} \]
\[ P_B = 2 \times 2848 \times 12 = 68,352 \text{ lb} = 68.35 \text{ kips} \]
\[ P_C = 62.92 \text{ kips} \]

(c) Moments

The shear force diagram is shown in Fig. 20.18c for sections \(DB_1\) and \(B_2E\)

\[ \text{Moment at } A = \frac{1}{2} \times 5 \times 690 \times \frac{5}{3} = 2,875 \text{ lb-ft/ft of wall} \]

\[ \text{Moment at } m = 2848 \times 3.3 - 863 \times 3.3 \times \frac{33}{2} = 4699 \text{ lb-ft/ft} \]

Because of symmetrical loading

\[ \text{Moment at } A = \text{Moment at } C = 2875 \text{ lb-ft/ft of wall} \]
\[ \text{Moment at } m = \text{Moment at } n = 4699 \text{ lb-ft/ft of wall} \]

Hence, the maximum moment = 4699 lb-ft/ft of wall.

The section modulus and the required sheet pile section can be determined in the usual way.

### 20.14 PIPING FAILURES IN SAND CUTS

Sheet piling is used for cuts in sand and the excavation must be dewatered by pumping from the bottom of the excavation. Sufficient penetration below the bottom of the cut must be provided to reduce the amount of seepage and to avoid the danger of piping.

Piping is a phenomenon of water rushing up through pipe-shaped channels due to large upward seepage pressure. When piping takes place, the weight of the soil is counteracted by the upward hydraulic pressure and as such there is no contact pressure between the grains at the bottom of the excavation. Therefore, it offers no lateral support to the sheet piling and as a result the sheet piling may collapse. Further, the soil will become very loose and may not have any bearing power. It is therefore, essential to avoid piping. For further discussions on piping, see Chapter 4 on Soil Permeability and Seepage. Piping can be reduced by increasing the depth of penetration of sheet piles below the bottom of the cut.

### 20.15 PROBLEMS

20.1 Figure Prob. 20.1 shows a cantilever sheet pile wall penetrating medium dense sand with the following properties of the soil:
\[ \gamma = 115 \text{ lb/ft}^3, \phi = 38^\circ, \]
All the other data are given in the figure.
Determine: (a) the depth of embedment for design, and (b) the maximum theoretical moment of the sheet pile.

20.2 Figure Prob. 20.2 shows a sheet pile penetrating medium dense sand with the following data:
\[ h_1 = 6 \text{ ft}, h_2 = 18 \text{ ft}, \gamma_{\text{sat}} = 120 \text{ lb/ft}^3, \phi = 38^\circ, \]
Determine: (a) the depth of embedment for design, and (b) the maximum theoretical moment of the sheet pile. The sand above the water table is saturated.
20.3 Figure Prob. 20.3 shows a sheet pile penetrating loose to medium dense sand with the following data:

\[ h_1 = 2 \text{ m}, \ h_2 = 4 \text{ m}, \ \gamma_{\text{moist}} = 17 \text{ kN/m}^3, \ \gamma_{\text{sat}} = 19.5 \text{ kN/m}^3, \ \phi = 34^\circ, \ c = 0 \]

Determine: (a) the depth of embedment, and (b) the maximum bending moment of the sheet pile.

20.4 Solve Problem 20.3 for the water table at great depth. Assume \( \gamma = 17 \text{ kN/m}^3 \). All the other data remain the same.

20.5 Figure Problem 20.5 shows freestanding cantilever wall with no backfill. The sheet pile penetrates medium dense sand with the following data:

\[ H = 5 \text{ m}, \ P = 20 \text{ kN/m}, \ \gamma = 17.5 \text{ kN/m}^3, \ \phi = 36^\circ \]

Determine: (a) the depth of embedment, and (b) the maximum moment

20.6 Solve Prob 20.5 with the following data:

\[ H = 20 \text{ ft}, \ P = 3000 \text{ lb/ft}, \ \gamma = 115 \text{ lb/ft}^3, \ \phi = 36^\circ \]

20.7 Figure Prob. 20.7 shows a sheet pile wall penetrating clay soil and the backfill is also clay. The following data are given.

\[ H = 5 \text{ m}, \ c = 30 \text{ kN/m}^2, \ \gamma = 17.5 \text{ kN/m}^3 \]

Determine: (a) the depth of penetration, and (b) the maximum bending moment.

20.8 Figure Prob. 20.8 has sand backfill and clay below the dredge line. The properties of the Backfill are:

\( \phi = 32^\circ, \ \gamma = 17.5 \text{ kN/m}^3 \)

Determine the depth of penetration of pile.

20.9 Solve Prob. 20.8 with the water table above dredge line: Given \( h_1 = 3 \text{ m}, \ h_2 = 3 \text{ m}, \ \gamma_{\text{sat}} = 18 \text{ kN/m}^3 \), where \( h_1 \) = depth of water table below backfill surface and \( h_2 = (H - h_1) \).

The soil above the water table is also saturated. All the other data remain the same.

20.10 Figure Prob. 20.10 shows a free-standing sheet pile penetrating clay. The following data are available:

\[ H = 5 \text{ m}, \ P = 50 \text{ kN/m}, \ c = 35 \text{ kN/m}^2, \ \gamma = 17.5 \text{ kN/m}^3 \]

Determine: (a) the depth of embedment, and (b) the maximum bending moment.
20.11 Solve Prob. 20.10 with the following data:

\[ H = 15 \text{ ft}, \quad P = 3000 \text{ lb/ft}, \quad c = 300 \text{ lb/ft}^2, \quad \gamma = 100 \text{ lb/ft}^3. \]

20.12 Figure Prob. 20.12 shows an anchored sheet pile wall for which the following data are given

\[ H = 8 \text{ m}, \quad h_a = 1.5 \text{ m}, \quad h_1 = 3 \text{ m}, \quad \gamma_{sat} = 19.5 \text{ kN/m}^3, \quad \phi = 38^\circ. \]

Determine: the force in the tie rod.

Solve the problem by the free-earth support method.
20.13 Solve the Prob. 20.12 with the following data: $H = 24$ ft, $h_1 = 9$ ft, $h_2 = 15$ ft, and $h_a = 6$ ft. Soil properties: $\phi = 32^{\circ}$, $\gamma_{sat} = 120$ lb/ft$^3$. The soil above the water table is also saturated.

20.14 Figure Prob. 20.14 gives an anchored sheet pile wall penetrating clay. The backfill is sand. The following data are given:

$H = 24$ ft, $h_a = 6$ ft, $h_1 = 9$ ft,

For sand: $\phi = 36^{\circ}$, $\gamma_{sat} = 120$ lb/ft$^3$,

The soil above the water table is also saturated.

For clay: $\phi = 0$, $c = 600$ lb/ft$^2$

Determine: (a) the depth of embedment, (b) the force in the tie rod, and (c) the maximum moment.

20.15 Solve Prob. 20.14 with the following data:

$H = 8$ m, $h_a = 1.5$ m, $h_1 = 3$ m

For sand: $\gamma_{sat} = 19.5$ kN/m$^3$, $\phi = 36^{\circ}$

The sand above the WT is also saturated.

For clay: $c = 30$ kN/m$^2$

20.16 Solve Prob. 20.12 by the use of design charts given in Section 20.7.

20.17 Refer to Prob. 20.12. Determine for the pile (a) the bending moment $M_{max}$, and (b) the reduced moment by Rowe’s method.

20.18 Figure Prob. 20.18 gives a rigid anchor plate fixed vertically in medium dense sand with the bottom of the plate at a depth of 6 ft below the ground surface. The height ($h$) and length ($L$) of the plate are 3 ft and 18 ft respectively. The soil properties are: $\gamma = 120$ lb/ft$^3$, and $\phi = 38^{\circ}$. Determine the ultimate lateral resistance per unit length of the plate.

20.19 Solve Prob. 20.18 for a plate of length = 9 ft. All the other data remain the same.

20.20 Solve Prob. 20.18 for the plate in clay ($\phi = 0$) having $c = 400$ lb/ft$^2$. All the other data remain the same.
20.21 Solve Prob. 20.20 for a plate of length 6 ft. All the other data remain the same.
20.22 Solve the prob. 20.19 by Eq (20.53). All the other determine the same.
20.23 Figure Prob. 20.23 shows a braced cut in medium dense sand. Given $\gamma = 18.5 \text{ kN/m}^3$, $c = 0$ and $\phi = 38^\circ$.
    (a) Draw the pressure envelope, (b) determine the strut loads, and (c) determine the maximum moment of the sheet pile section.
    The struts are placed laterally at 4 m center to center.
20.24 Figure Prob. 20.24 shows the section of a braced cut in clay. Given: $c = 650 \text{ lb/ft}^2$, $\gamma = 115 \text{ lb/ft}^3$.
    (a) Draw the earth pressure envelope, (b) determine the strut loads, and (c) determine the maximum moment of the sheet pile section.
    Assume that the struts are placed laterally at 12 ft center to center.